TABLES OF INTEGRAL TRANSFORMS

Volume II

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СПРАВОЧНАЯ МАТЕМАТИЧЕСКАЯ БИБЛИОТЕКА

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при участии

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ТАБЛИЦЫ ИНТЕГРАЛЬНЫХ ПРЕОБРАЗОВАНИЙ

ТОМ II

ПРЕОБРАЗОВАНИЯ БЕССЕЛЯ. ИНТЕГРАЛЫ ОТ СПЕЦИАЛЬНЫХ ФУНКЦИЙ

Перевод с английского
Н. Я. ВИЛЕНКИНА

ИЗДАТЕЛЬСТВО «НАУКА»
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АННОТАЦИЯ

«Таблицы интегральных преобразований» состоят из двух томов. Они вышли в США в 1954 г. и являются естественным дополнением и завершением трехтомного издания «Высшие трансцендентные функции» тех же авторов, перевод которого на русский язык вышел в этой же серии в 1965—67 гг. Перевод первого тома «Таблиц интегральных преобразований» вышел в свет в 1969 г.

Настоящая книга представляет собой перевод второго тома «Таблиц интегральных преобразований». Этот том содержит таблицы преобразований Бесселя, Римана—Лиувилля, Вейля, Стилтьеса, Гильберта, а также таблицы интегралов от специальных функций.

По полноте охвата материала это издание уникально.

«Таблицы» явятся настольной книгой для физиков теоретиков и экспериментаторов, инженеров-исследователей, математиков-прикладников и др.

ШТАБ ПО ОСУЩЕСТВЛЕНИЮ ПРОЕКТА БЕЙТМЕНА

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ОГЛАВЛЕНИЕ

ПРЕОБРАЗОВАНИЯ БЕССЕЛЯ

Глава VIII

ПРЕОБРАЗОВАНИЯ ГАНКЕЛЯ

Преобразования Ганкеля порядка ν

Глава IX

У-ПРЕОБРАЗОВАНИЯ

Глава X

K-ПРЕОБРАЗОВАНИЯ

Глава XI

H-ПРЕОБРАЗОВАНИЯ

Глава XII

ПРЕОБРАЗОВАНИЯ КОНТОРОВИЧА — ЛЕБЕДЕВА

РАЗЛИЧНЫЕ ПРЕОБРАЗОВАНИЯ

Глава XIII

ИНТЕГРАЛЫ ДРОБНОГО ПОРЯДКА

Глава XIV

ПРЕОБРАЗОВАНИЯ СТИЛТЬЕСА

Глава XV

ПРЕОБРАЗОВАНИЯ ГИЛЬБЕРТА

ИНТЕГРАЛЫ ОТ ВЫСШИХ ТРАНСЦЕНДЕНТНЫХ ФУНКЦИЙ

Глава XVI

ОРТОГОНАЛЬНЫЕ МНОГОЧЛЕНЫ

Глава XVII

ГАММА-ФУНКЦИЯ, НЕПОЛНЫЕ ГАММА-ФУНКЦИИ И РОДСТВЕННЫЕ ИМ ФУНКЦИИ

Глава XVIII

ФУНКЦИИ ЛЕЖАНДРА

Глава XIX

ФУНКЦИИ БЕССЕЛЯ

Глава XX

ГИПЕРГЕОМЕТРИЧЕСКИЕ ФУНКЦИИ

ПРИЛОЖЕНИЕ

ПРЕДИСЛОВИЕ

Цели, история и организация «Таблиц интегральных преобразований» были описаны во введении к первому тому. Несколько больше половины данного второго (и последнего) тома составляют таблицы интегральных преобразований, не вошедших в первый том, а остальная часть тома содержит интегралы от высших трансцендентных функций.

Под общим названием «Преобразования Бесселя» мы даем не только хорошо известные преобразования Ганкеля, но и другие преобразования, ядрами которых служат цилиндрические и родственные им функции. Кроме того, даны таблицы интегралов дробного порядка, а также преобразований Гильберта и Стилтьеса. Насколько нам известно, не существует достаточно обширных таблиц для всех преобразований, включенных в этот том. Для некоторых из них вообще известно очень мало пар двойственных функций. Перечень преобразований, включенных в оба тома книги, приведен на стр. 9 и 10.

Вторая часть тома содержит различные интегралы от высших трансцендентных функций. Некоторые из этих интегралов не могут быть представлены в виде интегральных преобразований каких-то функций, другие не были включены в таблицы интегральных преобразований и даны здесь. Вообще говоря, интеграл, который может быть записан в виде интегрального преобразования некоторой функции, более вероятно найти в таблицах таких преобразований, чем среди интегралов от высших трансцендентных функций. Эти интегралы расположены в соответствии с видом подынтегральной функции. Мы пользуемся здесь данной на стр. 12 первого тома иерархией функций, причем, как и в первом томе, сложные функции классифицируются в соответствии с «наивысшей» входящей в них функцией. Список определений высших трансцендентных функций дан в Приложении.

Мы выражаем признательность и благодарность тем же людям и организациям, что и в связи с первым томом. Мы благодарим также Джона Б. Джонстона, который читал гранки и оказывал другую ценную техническую помощь.

Исправления ошибок, добавления и предложения об улучшении будут с благодарностью приняты авторами.

А. Эрдейи

СТАНДАРТНЫЕ ФОРМЫ ИНТЕГРАЛЬНЫХ ПРЕОБРАЗОВАНИЙ

Косинус-преобразование Фурье ($\mathfrak{F}_c$, глава I)

$$\int_0^\infty f(x)\cos(xy)\,dx.$$

Синус-преобразование Фурье ($\mathfrak{F}_s$, глава II)

$$\int_0^\infty f(x)\sin(xy)\,dx.$$

Экспоненциальное преобразование Фурье ($\mathfrak{F}_e$, глава III)

$$\int_{-\infty}^{\infty} f(x)\,e^{-ixy}\,dx.$$

Преобразование Лапласа ($\mathfrak{L}$, глава IV)

$$\int_0^\infty f(t)\,e^{-pt}\,dt.$$

Обратное преобразование Лапласа (глава V)

$$\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} g(p)\,e^{pt}\,dp.$$

Преобразование Меллина ($\mathfrak{M}$, глава VI)

$$\int_0^\infty f(x)\,x^{s-1}\,dx.$$

Обратное преобразование Меллина (глава VII)

$$\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} g(s)\,x^{-s}\,ds.$$

Преобразование Ганкеля ($\mathfrak{H}_\nu$, глава VIII)

$$\int_0^\infty f(x)\, J_\nu(xy)\,(xy)^{1/2}\,dx.$$

Y-преобразование ($\mathfrak{Y}_\nu$, глава IX)

$$\int_0^\infty f(x)\, Y_\nu(xy)\,(xy)^{1/2}\,dx.$$

K-преобразование ($\mathfrak{K}_\nu$, глава X)

$$\int_0^\infty f(x)\, K_\nu(xy)\,(xy)^{1/2}\,dx.$$

$\mathbf{H}$-преобразование (глава XI)

$$\int_0^\infty f(x)\, \mathbf{H}_\nu(xy)\,(xy)^{1/2}\,dx.$$

Преобразование Конторовича — Лебедева (глава XII)

$$\int_0^\infty f(x)\, K_{ix}(y)\,dx.$$

Интегралы дробного порядка в смысле Римана — Лиувилля ($\mathfrak{R}_\mu$, глава XIII)

$$\frac{1}{\Gamma(\mu)}\int_0^y f(x)\,(y-x)^{\mu-1}\,dx.$$

Интегралы дробного порядка в смысле Вейля ($\mathfrak{W}_\mu$, глава XIII)

$$\frac{1}{\Gamma(\mu)}\int_y^\infty f(x)\,(x-y)^{\mu-1}\,dx.$$

Преобразование Стилтьеса ($\mathfrak{S}$, глава XIV)

$$\int_0^\infty \frac{f(x)}{x+y}\,dx.$$

Обобщенное преобразование Стилтьеса ($\mathfrak{S}_\rho$, глава XIV)

$$\int_0^\infty \frac{f(x)}{(x+y)^\rho}\,dx.$$

Преобразование Гильберта (глава XV)

$$\frac{1}{\pi}\,\text{P}\!\!\int_{-\infty}^\infty \frac{f(x)}{x-y}\,dx.$$

ПРЕОБРАЗОВАНИЯ БЕССЕЛЯ,

ИЛИ

ИНТЕГРАЛЬНЫЕ ПРЕОБРАЗОВАНИЯ, ЯДРАМИ КОТОРЫХ ЯВЛЯЮТСЯ ФУНКЦИИ БЕССЕЛЯ ИЛИ РОДСТВЕННЫЕ ИМ ФУНКЦИИ

ГЛАВА VIII

ПРЕОБРАЗОВАНИЯ ГАНКЕЛЯ

Мы называем функцию

$$g(y;\nu)=\mathfrak{H}_\nu\{f(x);y\}=\int_0^\infty f(x)\,J_\nu(xy)\,(xy)^{1/2}\,dx$$

преобразованием Ганкеля порядка ν функции $f(x)$. Здесь y пробегает положительные значения. Для краткости мы будем часто писать $g(y)$ вместо $g(y;\nu)$. Эта форма преобразования Ганкеля имеет то преимущество, что при $\nu=\pm{}^1/_2$ она приводится соответственно к синус- и косинус-преобразованиям Фурье. Многие авторы определяют преобразование Ганкеля порядка ν функции $f(x)$ формулами

$$\int_0^\infty f(x)\,J_\nu(xy)\,x\,dx$$

или

$$\int_0^\infty f(x)\,J_\nu\left[2(xy)^{1/2}\right]dx.$$

Преобразование Ганкеля обратно самому себе (см. 8.1 (1)), а потому для него не требуется таблиц обратных преобразований.

В книге Г. Н. Ватсона (1949) дано детальное доказательство теоремы обращения для преобразования Ганкеля и вычислены преобразования Ганкеля для многих функций. Теория и приложения преобразований Ганкеля описаны во многих книгах по интегралам Фурье, среди которых отметим книги Снеддона (1955) и Титчмарша (1948).

Из приведенных в этой главе пар преобразований можно вывести новые пары преобразований с помощью методов, указанных во введении к тому I этой книги, а также с помощью общих формул, указанных в п. 8.1. Трикоми (1935) открыл соотношение

$$\mathfrak{L}\left\{t^{\nu/2-1/4}g\left[(2t)^{1/2};\nu\right];\ s\right\}=s^{-\nu-1}\,\mathfrak{L}\left\{t^{\nu/2-1/4}f\left[(2t)^{1/2}\right];\ s^{-1}\right\}$$

между преобразованиями Ганкеля и Лапласа. Это соотношение можно использовать, чтобы вычислять преобразования Ганкеля с помощью таблиц преобразований Лапласа и обратных преобразований Лапласа, содержащихся в главах IV и V тома I.

8.1. Общие формулы

	$f(x)$	$\int\limits_0^\infty f(x)\,J_\nu(xy)\,(xy)^{1/2}\,dx = g(y;\nu),\quad y>0$
(1)	$\int\limits_0^\infty g(y)\,J_\nu(xy)\,(xy)^{1/2}\,dy,\quad \operatorname{Re}\nu > -{}^1/_2$	$g(y)$
(2)	$f(ax),\quad a>0$	$a^{-1}\,g(a^{-1}y;\,\nu)$
(3)	$x^m f(x),\quad m=0,\,1,\,2,\,\ldots$	$y^{1/2-\nu}\times \times\left(\frac{d}{y\,dy}\right)^m\left[y^{\nu-1/2+m}\,g(y;\,\nu+m)\right]$
(4)	$x^m f(x),\quad m=0,\,1,\,2,\,\ldots$	$(-1)^m\,y^{1/2+\nu}\left(\frac{d}{y\,dy}\right)^m z,$ где $z=y^{-1/2+m-\nu}\,g(y;\,\nu-m)$
(5)	$2\nu x^{-1} f(x)$	$y\,g(y;\,\nu-1)+y\,g(y;\,\nu+1)$
(6)	$x^{-1} f(x)$	$y^{1/2-\nu}\int\limits_0^y \eta^{\nu-1/2}\,g(\eta;\,\nu-1)\,d\eta$
(7)	$x^{-1} f(x)$	$y^{1/2+\nu}\int\limits_y^\infty \eta^{-\nu-1/2}\,g(\eta,\,\nu+1)\,d\eta$
(8)	$x^{-\mu} f(x),\quad \operatorname{Re}\nu+1>\operatorname{Re}\mu>0$	$2^{1-\mu}\,[\Gamma(\mu)]^{-1}\,y^{1/2-\nu}\times \times\int\limits_0^y \eta^{\nu-\mu+1/2}\,(y^2-\eta^2)^{\mu-1}\times \times\, g(\eta;\,\nu-\mu)\,d\eta$
(9)	$x^{-\mu} f(x),\quad \operatorname{Re}\nu-{}^3/_2>\operatorname{Re}\mu>0$	$2^{1-\mu}\,[\Gamma(\mu)]^{-1}\,y^{1/2+\nu}\times \times\int\limits_y^\infty \eta^{1/2-\mu-\nu}\,(\eta^2-y^2)^{\mu-1}\times \times\, g(\eta;\,\nu+\mu)\,d\eta$
(10)	$2\nu f'(x)$	$(\nu-{}^1/_2)\,y\,g(y;\,\nu+1)-{}-(\nu+{}^1/_2)\,y\,g(y;\,\nu-1)$

	$f(x)$	$\int\limits_0^\infty f(x)\, J_\nu(xy)\,(xy)^{1/2}\,dx = g(y;\nu),\quad y>0$
(11)	$x^{1/2-\nu}\left(\frac{d}{x\,dx}\right)^m\left[x^{\nu+m-1/2} f(x)\right],$ $m=0,1,2,\ldots$	$y^m g(y;\nu+m)$
(12)	$x^{1/2+\nu}\left(\frac{d}{x\,dx}\right)^m\left[x^{m-\nu-1/2} f(x)\right],$ $m=0,1,2,\ldots$	$(-y)^m g(y;\nu-m)$
(13)	$x^{1/2-\nu}\int\limits_0^x \xi^{\nu-\mu+1/2}(x^2-\xi^2)^{\mu-1}\times$ $\times f(\xi)\,d\xi,$ $\operatorname{Re}\nu+1/2>\operatorname{Re}\mu>0$	$2^{\mu-1}\Gamma(\mu)\,y^{-\mu} g(y;\nu-\mu)$
(14)	$x^{1/2+\nu}\int\limits_x^\infty \xi^{1/2-\mu-\nu}(\xi^2-x^2)^{\mu-1}\times$ $\times f(\xi)\,d\xi,$ $\operatorname{Re}\nu+1>\operatorname{Re}\mu>0$	$2^{\mu-1}\Gamma(\mu)\,y^{-\mu} g(y;\nu+\mu)$
(15)	$2^\lambda\Gamma(\lambda)\,x^{1/2-\nu}\times$ $\times\int\limits_0^x \xi^{1/2-\lambda-\mu+\nu}(x^2-\xi^2)^{\mu-1}\times$ $\times f(\xi)\,d\xi,$ $\operatorname{Re}\lambda>0,\quad \operatorname{Re}\mu>0$ $\operatorname{Re}\nu>\operatorname{Re}(\lambda+\mu)-1/2$	$2^\mu\Gamma(\mu)\,y^{1/2-\nu}\times$ $\times\int\limits_0^y \eta^{1/2-\lambda-\mu+\nu}(y^2-\eta^2)^{\lambda-1}\times$ $\times g(\eta;\nu-\lambda-\mu)\,d\eta$
(16)	$2^\lambda\Gamma(\lambda)\,x^{1/2+\nu}\times$ $\times\int\limits_x^\infty \xi^{1/2-\lambda-\mu-\nu}(\xi^2-x^2)^{\mu-1}\times$ $\times f(\xi)\,d\xi,\quad \operatorname{Re}\lambda>0$ $\operatorname{Re}\mu>0,\ \operatorname{Re}\nu>\lvert\operatorname{Re}(\lambda-\mu)\rvert-1$	$2^\mu\Gamma(\mu)\,y^{1/2+\nu}\times$ $\times\int\limits_y^\infty \eta^{1/2-\lambda-\mu-\nu}(\eta^2-y^2)^{\lambda-1}\times$ $\times g(\eta;\nu+\lambda+\mu)\,d\eta$

8.2. Преобразования Ганкеля нулевого порядка; элементарные функции

	$f(x)$	$\int_0^\infty f(x)\, J_0(xy)\,(xy)^{1/2}\,dx, \quad y>0$
(1)	$x^{-1/2}$	$y^{-1/2}$
(2)	$x^{-1/2}, \quad 0<x<1$ $0, \quad 1<x<\infty$	$y^{1/2} J_0(y) + 2^{-1}\pi y^{1/2}[J_1(y)\,\mathbf{H}_0(y) - J_0(y)\,\mathbf{H}_1(y)]$
(3)	$0, \quad 0<x<1$ $x^{-1/2}, \quad 1<x<\infty$	$y^{-1/2} - y^{1/2} J_0(y) + 2^{-1}\pi y^{1/2}[J_0(y)\,\mathbf{H}_1(y) - J_1(y)\,\mathbf{H}_0(y)]$
(4)	$x^{1/2}(\alpha^2+x^2)^{-1/2}, \quad \operatorname{Re}\alpha>0$	$y^{-1/2} e^{-\alpha y}$
(5)	$x^{1/2}(a^2-x^2)^{-1/2}, \quad 0<x<a$ $0, \quad a<x<\infty$	$y^{-1/2}\sin(ay)$
(6)	$0, \quad 0<x<a$ $x^{1/2}(x^2-a^2)^{-1/2}, \quad a<x<\infty$	$y^{-1/2}\cos(ay)$
(7)	$x^{1/2}(x^2+\alpha^2)^{-3/2}, \quad \operatorname{Re}\alpha>0$	$\alpha^{-1} y^{1/2} e^{-\alpha y}$
(8)	$x^{1/2}(x^4+\alpha^4)^{-1}, \quad \lvert\arg\alpha\rvert<\pi/4$	$-\alpha^{-2} y^{1/2}\operatorname{kei}_0(\alpha y)$
(9)	$x^{5/2}(x^4+\alpha^4)^{-1}, \quad \lvert\arg\alpha\rvert<\pi/4$	$y^{1/2}\operatorname{ker}_0(\alpha y)$
(10)	$x^{5/2}(x^4-a^4)^{-1}, \quad a>0$	$2^{-1} y^{1/2}[K_0(ay) - 2^{-1}\pi\, Y_0(ay)]$ Интеграл понимается в смысле главного значения по Коши.
(11)	$x^{-1/2}\dfrac{(x^2+\alpha^2)^{1/2}-x}{(x^2+\alpha^2)^{1/2}+x}, \quad \operatorname{Re}\alpha>0$	$y^{-1/2} + 2\alpha^{-2} y^{-5/2} \times (\alpha y\, e^{-\alpha y} + e^{-\alpha y} - 1)$
(12)	$x^{1/2}(x^4+2a^2x^2+b^4)^{-1/2}, \quad a>b>0$	$y^{1/2} J_0[2^{-1/2}(a^2-b^2)^{1/2} y] \times K_0[2^{-1/2}(a^2+b^2)^{1/2} y]$
(13)	$x^{1/2}(x^4+2a^2x^2+b^4)^{-1/2}, \quad b>a>0$	$y^{1/2} J_0[2^{-1/2}(b^2-a^2)^{1/2} y] \times K_0[2^{-1/2}(a^2+b^2)^{1/2} y]$

	$f(x)$	$\int\limits_0^\infty f(x)\, J_0(xy)\,(xy)^{1/2}\, dx,\ y>0$
(14)	$x^{1/2}(b^2-x^2)(x^4 \pm 2a^2x^2+b^4)^{-3/2}$, $0<a<b$	$2^{-1/2}(b^2 \mp a^2)^{-1/2}\, y^{3/2} \times J_1\left[(2^{-1}b^2 \mp 2^{-1}a^2)^{1/2}\, y\right] \times K_0\left[(2^{-1}b^2 \pm 2^{-1}a^2)^{1/2}\, y\right]$
(15)	$x^{-1/2}(a^2-x^2)^{-1/2} \times \{[x+(x^2-a^2)^{1/2}]^{2n} + [x-(x^2-a^2)^{1/2}]^{2n}\}$, $0<x<a$ 0, $a<x<\infty$	$(-1)^n\, \pi a^{2n+1/2} y^{1/2}\,[J_n(2^{-1}ay)]^2$
(16)	$x^{-1/2}(\alpha^2+x^2)^{-1/2}[(\alpha^2+x^2)^{1/2}+x]^{2\mu}$, $\operatorname{Re}\alpha>0$, $\operatorname{Re}\mu<3/4$	$y^{1/2}\alpha^{2\mu}\, K_\mu(2^{-1}\alpha y)\, I_{-\mu}(2^{-1}\alpha y)$
(17)	$x^{1/2}(x^2+\alpha^2)^{-1/2}(x^2+\beta^2)^{-1/2} \times [(x^2+\alpha^2)^{1/2}+(x^2+\beta^2)^{1/2}]^{\mu+1/2}$, $\operatorname{Re}\alpha>0$, $\operatorname{Re}\beta>0$ $\operatorname{Re}\mu<3/4$	$y^{1/2}(\alpha^2-\beta^2)^{\mu}\, K_\mu[2^{-1}y(\alpha+\beta)] \times I_{-\mu}[2^{-1}y(\alpha-\beta)]$
(18)	$x^{-1/2}e^{-\alpha x}$, $\operatorname{Re}\alpha>0$	$y^{1/2}(y^2+\alpha^2)^{-1/2}$
(19)	$x^{-3/2}(1-e^{-\alpha x})$, $\operatorname{Re}\alpha>0$	$y^{1/2}\operatorname{arsh}(\alpha/y)$
(20)	$x^{n-1/2}e^{-\alpha x}$, $\operatorname{Re}\alpha>0$	$\dfrac{n!\,y^{1/2}}{(\alpha^2+y^2)^{n/2+1/2}}\, P_n\left[\dfrac{\alpha}{(\alpha^2+y^2)^{1/2}}\right]$
(21)	$x^{2\mu-3/2}e^{-x^2/2}$, $\operatorname{Re}\mu>0$	$2^{\mu-1}\,\Gamma(\mu)\, y^{1/2}\, {}_1F_1(\mu;\, 1;\, -2^{-1}y^2)$
(22)	$x^{-1}\exp(-2\beta x^{1/2})$, $\operatorname{Re}\beta>0$	$\pi^{-1}\beta y^{-1/2} K_{1/4}(2^{-1}e^{i\pi/2}\beta^2 y^{-1}) \times K_{1/4}(2^{-1}e^{-i\pi/2}\beta^2 y^{-1})$
(23)	$x^{1/2}\exp[-\alpha(x^2+\beta^2)^{1/2}]$, $\operatorname{Re}\alpha>0$, $\operatorname{Re}\beta>0$	$\alpha y^{1/2}(y^2+\alpha^2)^{-3/2} \times \exp[-\beta(y^2+\alpha^2)^{1/2}] \times [1+\beta(y^2+\alpha^2)^{1/2}]$
(24)	$x^{1/2}(x^2+\beta^2)^{-1/2} \times \exp[-\alpha(x^2+\beta^2)^{1/2}]$, $\operatorname{Re}\alpha>0$, $\operatorname{Re}\beta>0$	$y^{1/2}(y^2+\alpha^2)^{-1/2}\exp[-\beta(y^2+\alpha^2)^{1/2}]$

	$f(x)$	$\int_0^\infty f(x) J_0(xy)(xy)^{1/2}\,dx,\ y>0$
(25)	$x^{1/2}(x^2+\beta^2)^{-1/2}\times$ $\times\exp[\pm ia(x^2+\beta^2)^{1/2}]$, $a>0,\quad \operatorname{Re}\beta>0$	$\pm iy^{1/2}(a^2-y^2)^{-1/2}\times$ $\times\exp[\pm i\beta(a^2-y^2)^{1/2}]$, $0<y<a$ $y^{1/2}(y^2-a^2)^{-1/2}\times$ $\times\exp[-\beta(y^2-a^2)^{1/2}]$, $a<y<\infty$
(26)	$\pm ix^{1/2}(b^2-x^2)^{-1/2}\times$ $\times\exp[\pm i\alpha(b^2-x^2)^{1/2}]$, $0<x<b$ $x^{1/2}(x^2-b^2)^{-1/2}\times$ $\times\exp[-\alpha(x^2-b^2)^{1/2}]$, $b<x<\infty$ $\operatorname{Re}\alpha>0$	$y^{1/2}(y^2+\alpha^2)^{-1/2}\times$ $\times\exp[\pm ib(y^2+\alpha^2)^{1/2}]$
(27)	$x^{-1/2}\ln x$	$-y^{-1/2}\ln(2\gamma y)$
(28)	$x^{-1/2}(x^2+a^2)^{-1/2}\times$ $\times\ln[x+(x^2+a^2)^{1/2}]$, $a>0$	$y^{1/2}[2^{-1}K_0^2(ay/2)+$ $+\ln a\, I_0(ay/2)K_0(ay/2)]$
(29)	$x^{-1/2}(x^2+\alpha^2)^{-1/2}\times$ $\times\ln\dfrac{(x^2+\alpha^2)^{1/2}+x}{(x^2+\alpha^2)^{1/2}-x}$, $\operatorname{Re}\alpha>0$	$y^{1/2}K_0^2(\alpha y/2)$
(30)	$x^{1/2}\ln(1+\alpha^2x^{-2})$, $\operatorname{Re}\alpha>0$	$2y^{-1/2}[y^{-1}-\alpha K_1(\alpha y)]$
(31)	$x^{1/2}\ln[\alpha x^{-1}+(1+\alpha^2x^{-2})^{1/2}]$, $\operatorname{Re}\alpha>0$	$y^{-3/2}(1-e^{-\alpha y})$
(32)	$x^{-1/2}\sin(ax)$, $a>0$	$y^{1/2}(a^2-y^2)^{-1/2}$, $0<y<a$ 0, $a<y<\infty$
(33)	$x^{-3/2}\sin(ax)$, $a>0$	$2^{-1}\pi y^{1/2}$, $0<y<a$ $y^{1/2}\arcsin(a/y)$, $a<y<\infty$

	$f(x)$	$\int\limits_0^\infty f(x)\, J_0(xy)\,(xy)^{1/2}\,dx,\ y>0$
(34)	$x^{-1/2}(1+x)^{-1}\sin(1+x)$	$2^{-1}\pi y^{1/2} J_0(y), \qquad 1\leqslant y<\infty$
(35)	$x^{-1/2}(\beta^2+x^2)^{-1}\sin(ax),$ $a>0,\ \operatorname{Re}\beta>0$	$y^{1/2}\beta^{-1}\operatorname{sh}(\beta a)\,K_0(\beta y),$ $a<y<\infty$
(36)	$x^{1/2}(\beta^2+x^2)^{-1}\sin(ax),$ $a>0,\ \operatorname{Re}\beta>0$	$2^{-1}\pi y^{1/2} e^{-a\beta} I_0(y\beta),$ $0<y<a$
(37)	$x^{-3/2}e^{-bx}\sin(ax)$	$y^{1/2}\arcsin\left(\frac{2a}{r_1+r_2}\right),$ $r_1^2=b^2+(a+y)^2,\ r_1>0$ $r_2^2=b^2+(a-y)^2,\ r_2>0$
(38)	$x^{1/2}\sin(2^{-1}a^2x^2), \qquad a>0$	$a^{-2}y^{1/2}\cos(2^{-1}a^{-2}y^2)$
(39)	$x^{-3/2}\sin(2^{-1}a^2x^2)$	$2^{-1}y^{1/2}\operatorname{si}(2^{-1}a^{-2}y^2)$
(40)	$x^{-1}\exp(-\alpha x^{1/2})\sin(\alpha x^{1/2}),$ $\lvert\arg\alpha\rvert<\pi/4$	$2^{-1}y^{-1/2}\alpha\, I_{1/4}(2^{-2}\alpha^2y^{-1})\times$ $\times K_{1/4}(2^{-2}\alpha^2y^{-1})$
(41)	$x^{1/2}(\beta^2+x^2)^{-1/2}\times$ $\times\sin\left[a(\beta^2+x^2)^{1/2}\right],$ $a>0,\ \operatorname{Re}\beta>0$	$y^{1/2}(a^2-y^2)^{-1/2}\cos\left[\beta(a^2-y^2)^{1/2}\right],$ $0<y<a$ $0, \qquad a<y<\infty$
(42)	$x^{-1/2}\cos(ax), \qquad a>0$	$0, \qquad 0<y<a$ $y^{1/2}(y^2-a^2)^{-1/2}, \qquad a<y<\infty$
(43)	$x^{-3/2}[1-\cos(ax)], \qquad a>0$	$y^{1/2}\operatorname{arch}(a/y), \qquad 0<y<a$ $0, \qquad a<y<\infty$
(44)	$x^{-1/2}(x^2+\beta^2)^{-1}\cos(ax),$ $a>0,\ \operatorname{Re}\beta>0$	$2^{-1}\beta^{-1}\pi y^{1/2}e^{-\alpha\beta} I_0(\beta y),$ $0<y<a$
(45)	$x^{1/2}(\beta^2+x^2)^{-1}\cos(ax),$ $a>0,\ \operatorname{Re}\beta>0$	$y^{1/2}\operatorname{ch}(\beta a)\,K_0(\beta y),$ $a<y<\infty$
(46)	$x^{-1/2}e^{-bx}\cos(ax)$	$y^{1/2}2^{-1/2}\left[(b^2+y^2-a^2)^2+4a^2b^2\right]^{-1/2}\times$ $\times\left\{\left[(b^2+y^2-a^2)^2+4a^2b^2\right]^{1/2}+\right.$ $\left.+b^2+y^2-a^2\right\}^{1/2}$

	$f(x)$	$\int\limits_0^\infty f(x)\, J_0(xy)\,(xy)^{1/2}\,dx, \quad y > 0$
(47)	$x^{1/2} \cos(2^{-1}a^2x^2), \qquad a > 0$	$a^{-2}y^{1/2} \sin(2^{-1}a^{-2}y^2)$
(48)	$x^{-3/2}[1 - \cos(2^{-1}a^2x^2)]$	$-2^{-1}y^{1/2}\,\mathrm{Ci}\,(2^{-1}a^{-2}y^2)$
(49)	$x^{-1} \exp(-\alpha x^{1/2}) \cos(\alpha x^{1/2}),$ $\lvert \arg \alpha \rvert < \pi/4$	$2^{-1}\alpha y^{-1/2} I_{-1/4}(2^{-2}\alpha^2 y^{-1}) \times$ $\times K_{1/4}(2^{-2}\alpha^2 y^{-1})$
(50)	$x^{1/2}(\beta^2 + x^2)^{-1/2} \times$ $\times \cos[a(\beta^2 + x^2)^{1/2}],$ $a > 0, \ \mathrm{Re}\,\beta > 0$	$-y^{1/2}(a^2 - y^2)^{-1/2} \times$ $\times \sin[\beta(a^2 - y^2)^{1/2}],$ $0 < y < a$ $y^{1/2}(y^2 - a^2)^{-1/2} \times$ $\times \exp[-\beta(y^2 - a^2)^{1/2}],$ $a < y < \infty$
(51)	$x^{-3/2} e^{-\alpha x}\,\mathrm{sh}\,(\alpha x), \qquad \mathrm{Re}\,\alpha > 0$	$2^{-1}y^{1/2} \ln\left[2\alpha y^{-1} + (1 + 4\alpha^2 y^{-2})^{1/2}\right]$
(52)	$x^{-1/2} e^{-\alpha x}\,\mathrm{sh}\,(\beta x),$ $\mathrm{Re}\,\alpha > \lvert \mathrm{Re}\,\beta \rvert$	$\dfrac{(\alpha\beta y)^{1/2}(r_2 - r_1)^{1/2}}{r_1 r_2 (r_2 + r_1)^{1/2}},$ $r_1 = [y^2 + (\beta - \alpha)^2]^{1/2},$ $r_2 = [y^2 + (\beta + \alpha)^2]^{1/2}$
(53)	$x^{1/2} \exp(-\alpha x^{-1})\,\mathrm{sh}\,(\alpha x^{-1}), \ \mathrm{Re}\,\alpha > 0$	$2\alpha y^{-1/2} J_1(2\alpha^{1/2}y^{1/2})\, K_1(2\alpha^{1/2}y^{1/2})$
(54)	$x^{-1/2} e^{-\alpha x}\,\mathrm{ch}\,(\beta x),$ $\mathrm{Re}\,\alpha > \lvert \mathrm{Re}\,\beta \rvert$	$\dfrac{(\alpha\beta y)^{1/2}(r_2 + r_1)^{1/2}}{r_1 r_2 (r_2 - r_1)^{1/2}},$ $r_1 = [y^2 + (\beta - \alpha)^2]^{1/2}$ $r_2 = [y^2 + (\beta + \alpha)^2]^{1/2}$
(55)	$x^{1/2}\,\mathrm{arsh}\,(\alpha x^{-1}), \qquad \mathrm{Re}\,\alpha > 0$	$y^{-3/2}(1 - e^{-\alpha y})$
(56)	$x^{-1/2}(1 + x^2)^{-1/2}\,\mathrm{sh}\,(2\mu\,\mathrm{arsh}\,x),$ $\lvert \mathrm{Re}\,\mu \rvert < 1/2$	$\pi^{-1} \sin(\pi\mu)\, y^{1/2}\,[K_\mu(y/2)]^2$
(57)	$x^{-1/2}(1 + x^2)^{-1/2}\,\mathrm{ch}\,(2\mu\,\mathrm{arsh}\,x),$ $\lvert \mathrm{Re}\,\mu \rvert < 1/2$	$2^{-1}y^{1/2} K_\mu(y/2) \times$ $\times [I_\mu(y/2) + I_{-\mu}(y/2)]$

8.3. Преобразования Ганкеля нулевого порядка; высшие трансцендентные функции

	$f(x)$	$\int\limits_0^\infty f(x)\, J_0(xy)\,(xy)^{1/2}\, dx,\quad y>0$
(1)	$x^{1/2}\, P_n(1-2x^2)$, $\quad 0<x<1$, $0 \quad 1<x<\infty$	$y^{-1/2} J_{2n+1}(y)$
(2)	$x^{5/2}\, P_n(1-2x^2)$, $\quad 0<x<1$ $0, \quad 1<x<\infty$	$y^{-1/2}(2n+1)^{-1}\times$ $\times\left[(n+1)\,J'_{2n+2}(y) - n\,J'_{2n}(y)\right]$
(3)	$x^{1/2}\exp\left(-2^{-1}x^2\right)L_n\left(x^2\right)$	$(-1)^n\exp\left(-2^{-1}y^2\right)y^{1/2}L_n\left(y^2\right)$
(4)	$x^{1/2}\exp\left(-2^{-1}\alpha x^2\right)L_n\left(2^{-1}\beta x^2\right)$, $\operatorname{Re}\alpha>0$	$y^{1/2}\dfrac{(\alpha-\beta)^n}{\alpha^{n+1}}\exp\left(-2^{-1}\alpha^{-1}y^2\right)\times$ $\times L_n\left[\dfrac{\beta y^2}{2\alpha(\beta-\alpha)}\right]$
(5)	$x^{1/2}\exp\left(-x^2\right)L_n\left(x^2\right)$	$[n!]^{-1}\,2^{-2n-1}y^{2n+1/2}\exp\left[-2^{-2}y^2\right]$
(6)	$x^{-1/2}\operatorname{si}(ax)$, $\quad a>0$	$-y^{-1/2}\arcsin(y/a)$, $\quad 0<y<a$ $0, \quad a<y<\infty$
(7)	$x^{1/2}\operatorname{si}\left(a^2x^2\right)$, $\quad a>0$	$-2y^{-3/2}\sin\left(2^{-2}a^{-2}y^2\right)$
(8)	$x^{1/2}\operatorname{Ci}\left(a^2x^2\right)$, $\quad a>0$	$2y^{-3/2}\left[1-\cos\left(2^{-2}a^{-2}y^2\right)\right]$
(9)	$x^{-1/2}\operatorname{Ci}\left(a^2x^2\right)$, $\quad a>0$	$y^{-1/2}\left[\operatorname{Ci}\left(2^{-2}a^{-2}y^2\right)+\right.$ $\left.+\ln\left(2^{-2}a^{-2}\gamma^2y^2\right)\right]$
(10)	$x^{1/2}\left(1+x^2\right)^{-\nu-1}\times$ $\times P_\nu\left[\left(1-x^2\right)\left(1+x^2\right)^{-1}\right]$, $\operatorname{Re}\nu>0$	$\left[2^\nu\,\Gamma(\nu+1)\right]^{-2}y^{2\nu+1/2}K_0(y)$
(11)	$x^{1/2}\left\{P_{\lambda-1/2}\left[\left(1+\alpha^2x^2\right)^{1/2}\right]\right\}^2$, $\operatorname{Re}\alpha>0,\ \lvert\operatorname{Re}\lambda\rvert<1/4$	$2\pi^{-2}\cos(\lambda\pi)\,\alpha^{-1}y^{-1/2}\times$ $\times\left[K_\lambda\left(2^{-1}\alpha^{-1}y\right)\right]^2$
(12)	$x^{1/2}P_{\sigma-1/2}\left[\left(1+\alpha^2x^2\right)^{1/2}\right]\times$ $\times Q_{\sigma-1/2}\left[\left(1+\alpha^2x^2\right)^{1/2}\right]$, $\operatorname{Re}\alpha>0,\ \operatorname{Re}\sigma>-1/4$	$\alpha^{-1}y^{-1/2}I_\sigma\left(2^{-1}\alpha^{-1}y\right)K_\sigma\left(2^{-1}\alpha^{-1}y\right)$

	$f(x)$	$\int_0^\infty f(x) J_0(xy)(xy)^{1/2}\,dx,\ y>0$
(13)	$x^{1/2}\left\{P^{\mu}_{\sigma-1/2}\left[(1+\alpha^2x^2)^{1/2}\right]\right\}^2$, $\operatorname{Re}\alpha>0$, $\lvert\operatorname{Re}\sigma\rvert<1/4$, $\operatorname{Re}\mu<1$	$-i\pi^{-1}y^{-3/2}W_{\mu,\sigma}(y/\alpha)\times$ $\times\left[W_{\mu,\sigma}(e^{\pi i}y/\alpha)-\right.$ $\left.-W_{\mu,\sigma}(e^{-\pi i}y/\alpha)\right]$
(14)	$x^{1/2}P^{\mu}_{\sigma-1/2}\left[(1+\alpha^2x^2)^{1/2}\right]\times$ $\times P^{-\mu}_{\sigma-1/2}\left[(1+\alpha^2x^2)^{1/2}\right]$, $\operatorname{Re}\alpha>0$, $\lvert\operatorname{Re}\sigma\rvert<1/4$	$2\pi^{-1}y^{-3/2}\cos(\sigma\pi)\times$ $\times W_{\mu,\sigma}(y/\alpha)W_{-\mu,\sigma}(y/\alpha)$
(15)	$x^{1/2}P^{\mu}_{\sigma-1/2}\left[(1+\alpha^2x^2)^{1/2}\right]\times$ $\times Q^{\mu}_{\sigma-1/2}\left[(1+\alpha^2x^2)^{1/2}\right]$, $\operatorname{Re}\alpha>0$, $\operatorname{Re}\sigma>-1/4$, $\operatorname{Re}\mu<1$	$e^{\mu\pi i}\dfrac{\Gamma(1/2+\sigma-\mu)}{\Gamma(1+2\sigma)}y^{-3/2}\times$ $\times W_{\mu,\sigma}(y/\alpha)M_{-\mu,\sigma}(y/\alpha)$
(16)	$x^{-3/2}[1-J_0(ax)]$, $a>0$	0, $y>a$ $y^{1/2}\ln(a/y)$, $y<a$
(17)	$x^{-1/2}J_0(\alpha x)e^{-\beta x}$, $\operatorname{Re}\beta>\lvert\operatorname{Im}\alpha\rvert$	$2\pi^{-1}y^{1/2}K(2\alpha^{1/2}y^{1/2}k^{-1})k^{-1/2}$, $k=\left[(\alpha+y)^2+\beta^2\right]^{1/2}$
(18)	$x^{-1/2}J_1(ax)$, $a>0$	$a^{-1}y^{1/2}$, $0<y<a$ 0, $a<y<\infty$
(19)	$x^{-3/2}J_1(ax)$, $a>0$	$2\pi^{-1}y^{1/2}E(y/a)$, $0<y<a$ $\dfrac{2y^{3/2}}{\pi a}\left[E\left(\dfrac{a}{y}\right)-\left(1-\dfrac{a^2}{y^2}\right)K\left(\dfrac{a}{y}\right)\right]$, $a<y<\infty$
(20)	$x^{1/2}J^2_{\nu}(2^{-1}ax)$, $\operatorname{Re}\nu>-1/2$	$2\pi^{-1}y^{-1/2}(a^2-y^2)^{-1/2}\times$ $\times\cos[2\nu\arcsin(y/a)]$, $0<y<a$ 0, $a<y<\infty$
(21)	$x^{1/2}J_0(ax)Y_0(ax)$, $a>0$	0, $0<y<2a$ $-2\pi^{-1}y^{-1/2}(y^2-4a^2)^{-1/2}$, $2a<y<\infty$

	$f(x)$	$\int_0^\infty f(x) J_0(xy)(xy)^{1/2}\,dx, \quad y>0$
(22)	$x^{-1/2}[\mathbf{H}_0(ax)-Y_0(ax)]$, $a>0$	$4\pi^{-1}(a+y)^{-1}y^{1/2}\times$ $\times K[\,\lvert a-y\rvert(a+y)^{-1}]$
(23)	$x^{-1/2}\operatorname{ch}(\beta x)K_0(\alpha x)$, $\operatorname{Re}\alpha>\lvert\operatorname{Re}\beta\rvert$	$y^{1/2}(u+v)^{-1/2}K(k)$, $u=2^{-1}\{[(\alpha^2+\beta^2+y^2)^2-4\alpha^2\beta^2]^{1/2}+$ $+\alpha^2-\beta^2-y^2\}$ $v=2^{-1}\{[(\alpha^2+\beta^2+y^2)^2-4\alpha^2\beta^2]^{1/2}-$ $-\alpha^2+\beta^2+y^2\}$ $k^2=v(u+v)^{-1}$
(24)	$x^{-1/2}\operatorname{sh}(\beta x)K_1(\alpha x)$, $\operatorname{Re}\alpha>\lvert\operatorname{Re}\beta\rvert$	$\alpha^{-1}y^{1/2}[u\,E(k)-K(k)\,E(u)+$ $+K(k)\operatorname{sn}u\operatorname{dn}u/(\operatorname{cn}u)]$, $\operatorname{cn}^2u=2y^2\{[(\alpha^2+\beta^2+y^2)^2-$ $-4\alpha^2\beta^2]^{1/2}-\alpha^2+\beta^2+y^2\}^{-1}$ $k^2=2^{-1}\{1-(\alpha^2-\beta^2-y^2)\times$ $\times[(\alpha^2+\beta^2+y^2)^2-4\alpha^2\beta^2]^{-1/2}\}$
(25)	$x^{1/2}J_0(\alpha x)K_0(\beta x)$, $\operatorname{Re}\beta>\lvert\operatorname{Im}\alpha\rvert$	$y^{1/2}(\beta^4+\alpha^4+y^4-2\alpha^2y^2+$ $+2\alpha^2\beta^2+2\beta^2y^2)^{-1/2}$
(26)	$x^{3/2}J_1(\alpha x)K_0(\beta x)$ $\operatorname{Re}\beta\geqslant\lvert\operatorname{Im}\alpha\rvert$, $\operatorname{Re}\alpha>0$	$2\alpha y^{1/2}(\alpha^2+\beta^2-y^2)\times$ $\times[(\alpha^2+\beta^2+y^2)^2-4y^2\alpha^2]^{-3/2}$
(27)	$x^{1/2}I_0(\alpha x)K_0(\beta x)$, $\operatorname{Re}\beta>\operatorname{Re}\alpha$	$y^{1/2}(\alpha^4+\beta^4+y^4-2\alpha^2\beta^2+$ $+2\alpha^2y^2+2\beta^2y^2)^{-1/2}$
(28)	$x^{3/2}I_0(\alpha x)K_1(\beta x)$, $\operatorname{Re}\beta>\lvert\operatorname{Re}\alpha\rvert$	$2y^{1/2}\beta(\beta^2+y^2-\alpha^2)\times$ $\times[(y^2+\alpha^2+\beta^2)^2-4\alpha^2\beta^2]^{-3/2}$
(29)	$x^{1/2}I_1(\alpha x)K_1(\beta x)$, $\operatorname{Re}\beta>\operatorname{Re}\alpha>0$	$2^{-1}y^{1/2}\alpha^{-1}\beta^{-1}\{(\alpha^2+\beta^2+y^2)\times$ $\times[(\alpha^2+\beta^2+y^2)^2-4\alpha^2\beta^2]^{-1/2}-1\}$
(30)	$x^{1/2}K_\mu(\alpha x)I_\mu(\beta x)$ $\operatorname{Re}\mu>-1$, $\operatorname{Re}\alpha>\lvert\operatorname{Re}\beta\rvert$	$y^{1/2}r_1^{-1}r_2^{-1}(r_2-r_1)^\mu(r_2+r_1)^{-\mu}$, $r_1=[y^2+(\beta-\alpha)^2]^{1/2}$ $r_2=[y^2+(\beta+\alpha)^2]^{1/2}$

	$f(x)$	$\int_0^\infty f(x) J_0(xy)(xy)^{1/2}\,dx, \quad y>0$
(31)	$x^{-1/2} I_\mu(2^{-1}ax) K_\mu(2^{-1}ax)$, $\operatorname{Re} a>0$, $\operatorname{Re}\mu>-1/2$	$a^{-1}y^{1/2}P_{\mu-1/2}[(1+y^2/a^2)^{1/2}] \times Q_{\mu-1/2}[(1+y^2/a^2)^{1/2}]$
(32)	$x^{1/2} K_0^2(\alpha x)$, $\operatorname{Re}\alpha>0$	$y^{-1/2}(y^2+4\alpha^2)^{-1/2}\ln\frac{(y^2+4\alpha^2)^{1/2}+y}{(y^2+4\alpha^2)^{1/2}-y}$
(33)	$x^{1/2} K_\mu^2(\alpha x)$, $\operatorname{Re}\alpha>0$, $\lvert\operatorname{Re}\mu\rvert<1$	$\pi 2^{-1-2\mu}\alpha^{-2\mu}\times(\sin\mu\pi)^{-1}y^{-1/2}(y^2+4\alpha^2)^{-1/2}\times\{[(y^2+4\alpha^2)^{1/2}+y]^{2\mu}-[(y^2+4\alpha^2)^{1/2}-y]^{2\mu}\}$
(34)	$x^{-1/2} J_\mu(a^2x^{-1}) J_{-\mu}(a^2x^{-1})$, $a>0$, $\lvert\operatorname{Re}\mu\rvert<1/4$	$-\frac{i}{\sin(2\mu\pi)\,y^{1/2}}\times[e^{2\pi i\mu}J_{2\mu}(2ay^{1/2}e^{-\pi i/4})\times J_{-2\mu}(2ay^{1/2}e^{\pi i/4})-e^{-2\pi i\mu}J_{2\mu}(2ay^{1/2}e^{\pi i/4})\times J_{-2\mu}(2ay^{1/2}e^{-\pi i/4})]$
(35)	$x^{-1/2}[J_\mu^2(a^2x^{-1})-J_{-\mu}^2(a^2x^{-1})]$, $a>0$, $\lvert\operatorname{Re}\mu\rvert<1/4$	$\frac{1}{y^{1/2}\cos(\mu\pi)}\times[J_{2\mu}(2ay^{1/2}e^{\pi i/4})\times J_{2\mu}(2ay^{1/2}e^{-\pi i/4})-J_{-2\mu}(2ay^{1/2}e^{\pi i/4})\times J_{-2\mu}(2ay^{1/2}e^{-\pi i/4})]$
(36)	$x^{-1/2} H_\mu^{(1)}(\alpha^2x^{-1}) H_\mu^{(2)}(\alpha^2x^{-1})$, $\lvert\arg\alpha\rvert<\pi/4$, $\lvert\operatorname{Re}\mu\rvert<1/4$	$16\pi^{-2}\cos(\mu\pi)\,y^{-1/2}\times K_{2\mu}(2\alpha y^{1/2}e^{\pi i/4})K_{2\mu}(2\alpha y^{1/2}e^{-\pi i/4})$
(37)	$x^{-1/2} I_\mu(\alpha^2x^{-1}) K_\mu(\alpha^2x^{-1})$, $\lvert\arg\alpha\rvert<\pi/4$, $\operatorname{Re}\mu>-1/4$	$2y^{-1/2}J_{2\mu}(2\alpha y^{1/2})K_{2\mu}(2\alpha y^{1/2})$
(38)	$x^{-1/2} J_\mu(\alpha x^{1/2}) K_\mu(\alpha x^{1/2})$, $\lvert\arg\alpha\rvert<\pi/4$, $\operatorname{Re}\mu>-1$	$2^{-1}y^{-1/2}I_{\mu/2}(2^{-2}\alpha^2y^{-1})\times K_{\mu/2}(2^{-2}\alpha^2y^{-1})$
(39)	$x^{-1/2} Y_0(\alpha x^{1/2}) K_0(\alpha x^{1/2})$, $\lvert\arg\alpha\rvert<\pi/4$	$-2^{-1}\pi^{-1}y^{-1/2}[K_0(2^{-2}\alpha^2y^{-1})]^2$

	$f(x)$	$\int_0^\infty f(x)\,J_0(xy)\,(xy)^{1/2}\,dx,\quad y>0$
(40)	$x^{-1/2}\,Y_\mu(\alpha x^{1/2})\,K_\mu(\alpha x^{1/2})$, $\lvert\arg\alpha\rvert<\pi/4$, $\lvert\operatorname{Re}\mu\rvert<1$	$-\dfrac{K_{\mu/2}(2^{-2}\alpha^2y^{-1})}{2y^{1/2}\cos(\mu\pi/2)}\times \times\left[\pi^{-1}K_{\mu/2}(2^{-2}\alpha^2y^{-1})+ +\sin(\mu\pi/2)\,I_{\mu/2}(2^{-2}\alpha^2y^{-1})\right]$
(41)	$x^{-1/2}\,K_\mu(\alpha e^{\pi i/4}x^{1/2})\times \times K_\mu(\alpha e^{-\pi i/4}x^{1/2})$, $\lvert\arg\alpha\rvert<\pi/4$, $\lvert\operatorname{Re}\mu\rvert<1$	$2^{-4}\pi^2\,[\cos(\mu\pi/2)]^{-1}\times \times H^{(1)}_{\mu/2}(2^{-2}\alpha^2y^{-1})\,H^{(2)}_{\mu/2}(2^{-2}\alpha^2y^{-1})$
(42)	$x^{-1/2}\,D_n(\alpha x)\,D_{n+1}(\alpha x)$, $\lvert\arg\alpha\rvert<\pi/4$	$(-1)^n\,y^{-1/2}\,D_n(y/\alpha)\,D_{n+1}(y/\alpha)$
(43)	$x^{-1}\,D_\nu(\alpha^{1/2}x^{1/2})\,D_{-\nu-1}(\alpha^{1/2}x^{1/2})$, $\operatorname{Re}\alpha>0$	$2^{-3/2}\pi\alpha^{-1/2}y^{1/2}\times \times P^{\nu/2+1/4}_{-1/4}\left[(1+4y^2/\alpha^2)^{1/2}\right]\times \times P^{-\nu/2-1/4}_{-1/4}\left[(1+4y^2/\alpha^2)^{1/2}\right]$
(44)	$x^{-3/2}\,W_{\varkappa,\mu}(\alpha x)\,M_{-\varkappa,\mu}(\alpha x)$, $\operatorname{Re}\alpha>0$, $\operatorname{Re}\mu>-1/2$, $\operatorname{Re}\varkappa<3/4$	$e^{-i\varkappa\pi}\dfrac{\Gamma(1+2\mu)}{\Gamma(1/2+\mu+\varkappa)}\,y^{1/2}\times \times P^{\varkappa}_{\mu-1/2}\left[(1+y^2/\alpha^2)^{1/2}\right]\times \times Q^{\varkappa}_{\mu-1/2}\left[(1+y^2/\alpha^2)^{1/2}\right]$
(45)	$x^{-3/2}\,W_{k,\mu}(\alpha x)\,W_{-k,\mu}(\alpha x)$, $\operatorname{Re}\alpha>0$, $-1/2<\operatorname{Re}\mu<1/2$	$2^{-1}\pi\cos(\mu\pi)\,y^{1/2}\times \times P^{k}_{\mu-1/2}\left[(1+y^2/\alpha^2)^{1/2}\right]\times \times P^{-k}_{\mu-1/2}\left[(1+y^2/\alpha^2)^{1/2}\right]$
(46)	$x^{1/2}\,{}_1F_1(\lambda;\,1;\,-x^2)$, $\operatorname{Re}\lambda>0$	$[2^{2\lambda-1}\,\Gamma(\lambda)]^{-1}\,y^{2\lambda-3/2}\exp(-2^{-2}y^2)$

8.4. Преобразования Ганкеля первого порядка

	$f(x)$	$\int_0^\infty f(x)\,J_1(xy)\,(xy)^{1/2}\,dx,\quad y>0$
(1)	$x^{-1/2}$, $0<x<a$; 0, $a<x<\infty$	$y^{-1/2}\,[1-J_0(ay)]$

	$f(x)$	$\int_0^\infty f(x) J_1(xy)(xy)^{1/2}\,dx, \quad y>0$
(2)	$0, \quad 0<x<a$ $x^{-1/2}, \quad a<x<\infty$	$y^{-1/2} J_0(ay)$
(3)	$x^{-1/2}(x^2+\alpha^2)^{-1/2}, \quad \operatorname{Re}\alpha>0$	$\alpha^{-1} y^{-1/2}(1-e^{-\alpha y})$
(4)	$x^{-1/2}(a^2-x^2)^{-1/2}, \quad 0<x<a$ $0, \quad a<x<\infty$	$a^{-1}y^{-1/2}[1-\cos(ay)]$
(5)	$0, \quad 0<x<a$ $x^{-1/2}(x^2-a^2)^{-1/2}, \quad a<x<\infty$	$a^{-1}y^{-1/2}\sin(ay)$
(6)	$x^{-1/2}e^{-\alpha x}, \quad \operatorname{Re}\alpha>0$	$y^{-1/2}\left[1-\alpha(\alpha^2+y^2)^{-1/2}\right]$
(7)	$x^{-1/2}\exp(-2^{-2}\alpha x^2), \quad \operatorname{Re}\alpha>0$	$y^{-1/2}[1-\exp(-y^2/\alpha)]$
(8)	$x^{3/2}\exp(-2^{-2}\alpha x^2), \quad \operatorname{Re}\alpha>0$	$4\alpha^{-2}y^{3/2}\exp(-y^2/\alpha)$
(9)	$x^{-1/2}\exp\left[-\alpha(x^2+\beta^2)^{1/2}\right],$ $\operatorname{Re}\alpha>0, \ \operatorname{Re}\beta>0$	$y^{-1/2}\left[e^{-\beta\alpha}-\alpha(\alpha^2+y^2)^{-1/2}\times\right.$ $\left.\times\exp\left[-\beta(\alpha^2+y^2)^{1/2}\right]\right]$
(10)	$x^{-1/2}(\beta^2+x^2)^{1/2}\times$ $\times\exp\left[-\alpha(\beta^2+x^2)^{1/2}\right],$ $\operatorname{Re}\alpha>0, \ \operatorname{Re}\beta>0$	$\dfrac{e^{-\beta\alpha}-\exp\left[-\beta(\alpha^2+y^2)^{1/2}\right]}{\beta y^{1/2}}$
(11)	$x^{-1/2}\ln x$	$-y^{-1/2}\ln(2^{-1}\gamma y)$
(12)	$x^{-1/2}\ln(a^2+x^2)$	$2y^{-1/2}[K_0(ay)+\ln a]$
(13)	$x^{-1/2}\ln(1+x^4)^{1/2}$	$2y^{-1/2}\operatorname{ker}_0 y$
(14)	$x^{-1/2}\sin(ax), \quad a>0$	$0, \quad 0<y<a$ $ay^{-1/2}(y^2-a^2)^{-1/2}, \quad a<y<\infty$
(15)	$x^{-3/2}e^{-ax}\sin(bx)$	$by^{-1/2}(1-r),$ $b^2=\dfrac{y^2}{1-r^2}-\dfrac{a^2}{r^2}$
(16)	$x^{-1/2}\sin(2^{-2}ax^2), \quad a>0$	$y^{-1/2}\sin(y^2/a)$

	$f(x)$	$\int_0^\infty f(x)\,J_1(xy)\,(xy)^{1/2}\,dx, \quad y>0$
(17)	$x^{-1/2}\sin^2(2^{-2}ax^2), \quad a>0$	$2^{-1}y^{-1/2}\cos(2^{-1}a^{-1}y^2)$
(18)	$x^{-1/2}(x^2+\alpha^2)^{-1/2}\times$ $\times\sin\left[b(x^2+\alpha^2)^{1/2}\right],$ $\operatorname{Re}\alpha>0,\ b>0$	$\dfrac{\sin(\alpha b)-\sin\left[\alpha(b^2-y^2)^{1/2}\right]}{\alpha y^{1/2}}, \quad 0<y<b$ $\alpha^{-1}y^{-1/2}\sin(\alpha b), \quad b<y<\infty$
(19)	$x^{-1/2}\cos(ax), \quad a>0$	$y^{-1/2}\left[1-a(a^2-y^2)^{-1/2}\right], \quad 0<y<a$ $y^{-1/2}, \quad a<y<\infty$
(20)	$x^{-1/2}\cos(2^{-2}ax^2), \quad a>0$	$2y^{-1/2}\sin^2(2^{-1}a^{-1}y^2)$
(21)	$x^{-1/2}(x^2+\alpha^2)^{-1/2}\times$ $\times\cos\left[b(x^2+\alpha^2)^{1/2}\right],$ $\operatorname{Re}\alpha>0,\ b>0$	$\alpha^{-1}y^{-1/2}\left\{-\cos\left[\alpha(b^2-y^2)^{1/2}\right]+\right.$ $\left.+\cos(\alpha b)\right\}, \quad 0<y<b$ $\alpha^{-1}y^{-1/2}\{\cos(\alpha b)-$ $-\exp\left[-\alpha(y^2-b^2)^{1/2}\right]\}, \quad b<y<\infty$
(22)	$x^{-1/2}\operatorname{arctg}(x^2)$	$-2y^{1/2}\operatorname{kei}_0 y$
(23)	$x^{3/2}P_n(1-2x^2), \quad 0<x<1$ $0, \quad 1<x<\infty$	$(2n+1)^{-1}y^{-1/2}[(n+1)J_{2n+2}(y)-$ $-nJ_{2n}(y)]$
(24)	$x^{-1/2}[D_n(\alpha x)]^2, \quad \lvert\arg\alpha\rvert<\pi/4$	$(-1)^{n-1}y^{-1/2}[D_n(y/\alpha)]^2$
(25)	$x^{-1/2}\operatorname{si}(a^2x^2), \quad a>0$	$y^{-1/2}[-\operatorname{si}(2^{-2}a^{-2}x^2)-\pi/2]$
(26)	$x^{-1/2}J_0(ax), \quad a>0$	$0, \quad 0<y<a$ $y^{-1/2}, \quad a<y<\infty$
(27)	$x^{-3/2}J_0(ax)$	$2a\pi^{-1}y^{-1/2}[E(y/a)-$ $-(1-y^2/a^2)K(y/a)], \quad 0<y<a$ $2\pi^{-1}y^{1/2}E(a/y), \quad a<y<\infty$
(28)	$x^{-5/2}[J_0(ax)-1], \quad a>0$	$-2^{-2}y^{3/2}[1+2\ln(a/y)], \quad 0<y<a$ $-2^{-2}y^{1/2}a^2, \quad a<y<\infty$

	$f(x)$	$\int\limits_0^\infty f(x)\, J_1(xy)\,(xy)^{1/2}\,dx, \quad y>0$
(29)	$x^{-1/2} J_0(ax) J_0(bx)$, $a, b > 0$	$0, \quad 0<y<\lvert a-b\rvert$ $\pi^{-1} y^{-1/2} \arccos \frac{a^2+b^2-y^2}{2ab}$, $\lvert a-b\rvert<y<a+b$ $y^{-1/2}, \quad a+b<y<\infty$
(30)	$x^{-5/2} J_1(ax)$, $a>0$	$\frac{y^{1/2}(y+a)}{\pi} \times$ $\times \left[E\left(\frac{2iy^{1/2}a^{1/2}}{\lvert y-a\rvert}\right) - K\left(\frac{2iy^{1/2}a^{1/2}}{\lvert y-a\rvert}\right)\right]$
(31)	$x^{-1/2} Y_0(ax)$, $a>0$	$-\pi^{-1} y^{-1/2} \ln(1-y^2/a^2), \; 0<y<a$
(32)	$x^{1/2} \operatorname{kei}_0 x$	$-2^{-1} y^{-1/2} \operatorname{arctg}(y^2)$
(33)	$x^{-1/2} \operatorname{ker}_0 x$	$2^{-1} y^{-1/2} \ln(1+y^4)^{1/2}$

ПРЕОБРАЗОВАНИЯ ГАНКЕЛЯ ПОРЯДКА ν

8.5. Алгебраические функции и степени с произвольным показателем

	$f(x)$	$\int\limits_0^\infty f(x)\, J_\nu(xy)\,(xy)^{1/2}\,dx, \quad y>0$
(1)	$1, \quad 0<x<1$ $0, \quad 1<x<\infty$ $\operatorname{Re}\nu > -3/2$	$2^{1/2} y^{-1} \frac{\Gamma(3/4+\nu/2)}{\Gamma(1/4+\nu/2)} +$ $+(\nu-1/2) J_\nu(y) S_{-1/2,\,\nu-1}(y) -$ $- J_{\nu-1}(y) S_{1/2,\,\nu}(y)$
(2)	$0, \quad 0<x<1$ $1, \quad 1<x<\infty$	$J_{\nu-1}(y) S_{1/2,\,\nu}(y) +$ $+(1/2-\nu) J_\nu(y) S_{-1/2,\,\nu-1}(y)$
(3)	$x^{-1/2}$, $\operatorname{Re}\nu > -1$	$y^{-1/2}$
(4)	$x^{1/2-\nu}, \quad 0<x<1$ $0, \quad 1<x<\infty$	$\frac{2^{1-\nu} y^{\nu-3/2}}{\Gamma(\nu)} - y^{-1/2} J_{\nu-1}(y)$

	$f(x)$	$\int_0^\infty f(x)\,J_\nu(xy)\,(xy)^{1/2}\,dx,\ y>0$
(5)	$x^{\nu-1/2}$, $0<x<1$; 0, $1<x<\infty$; $\operatorname{Re}\nu>-1/2$	$2^{\nu-1}y^{1/2-\nu}\pi^{1/2}\Gamma(\nu+1/2)\times[J_\nu(y)\mathbf{H}_{\nu-1}(y)-\mathbf{H}_\nu(y)J_{\nu-1}(y)]$
(6)	$x^{\nu+1/2}$, $0<x<1$; 0, $1<x<\infty$; $\operatorname{Re}\nu>-1$	$y^{-1/2}J_{\nu+1}(y)$
(7)	x^μ, $-\operatorname{Re}\nu-3/2<\operatorname{Re}\mu<-1/2$	$2^{\mu+1/2}y^{-\mu-1}\dfrac{\Gamma(\mu/2+\nu/2+3/4)}{\Gamma(\nu/2-\mu/2+1/4)}$
(8)	x^μ, $0<x<1$; 0, $1<x<\infty$; $\operatorname{Re}(\mu+\nu)>-3/2$	$y^{-\mu-1}\Big[(\nu+\mu-1/2)\,y\,J_\nu(y)\times S_{\mu-1/2,\,\nu-1}(y)-y\,J_{\nu-1}(y)\,S_{\mu+1/2,\,\nu}(y)+2^{\mu+1/2}\dfrac{\Gamma(\mu/2+\nu/2+3/4)}{\Gamma(\nu/2-\mu/2+1/4)}\Big]$
(9)	$x^{\nu-1/2}(x+\alpha)^{-1}$, $\lvert\arg\alpha\rvert<\pi$; $-1/2<\operatorname{Re}\nu<3/2$, $\nu\neq 1/2$	$\dfrac{\pi\alpha^\nu y^{1/2}[\mathbf{H}_{-\nu}(\alpha y)-Y_{-\nu}(\alpha y)]}{2\cos(\nu\pi)}$
(10)	$x^{\rho-3/2}(x+\alpha)^{-\mu-1}$, $\lvert\arg\alpha\rvert<\pi$; $\operatorname{Re}(\rho+\nu)>0$; $\operatorname{Re}(\rho-\mu)<5/2$	$\dfrac{y^{1/2}\pi\alpha^{\rho-\mu-1}}{\sin(\rho+\nu-\mu)\pi\,\Gamma(\mu+1)}\times\Big\{\sum_{m=0}^\infty\dfrac{(-1)^m(2^{-1}\alpha y)^{\nu+2m}\,\Gamma(\tau)}{m!\,\Gamma(\nu+m+1)\,\Gamma(\tau-\mu)}-\sum_{m=0}^\infty\dfrac{(2^{-1}\alpha y)^{\mu+1-\rho+m}\,\Gamma(\mu+m+1)}{m!\,\Gamma[2^{-1}(\mu+\nu-\rho+m+3)]}\times\dfrac{\sin 2^{-1}(\rho+\nu-\mu-m)\pi}{\Gamma[2^{-1}(\mu-\nu-\rho+m+3)]}\Big\}$, $\tau=\rho+\nu+2m$
(11)	$x^{-1/2}(x^2+\alpha^2)^{-1/2}$, $\operatorname{Re}\alpha>0$, $\operatorname{Re}\nu>-1$	$y^{1/2}I_{\nu/2}(2^{-1}\alpha y)K_{\nu/2}(2^{-1}\alpha y)$
(12)	$x^{\nu+1/2}(x^2+\alpha^2)^{-1}$, $\operatorname{Re}\alpha>0$, $-1<\operatorname{Re}\nu<3/2$	$\alpha^\nu y^{1/2}K_\nu(\alpha y)$
(13)	$x^{\nu-1/2}(x^2+\alpha^2)^{-1}$, $\operatorname{Re}\alpha>0$, $-1/2<\operatorname{Re}\nu<5/2$	$\dfrac{\pi\alpha^{\nu-1}y^{1/2}[I_\nu(\alpha y)-\mathbf{L}_{-\nu}(\alpha y)]}{2\cos(\nu\pi)}$

	$f(x)$	$\int_0^\infty f(x) J_\nu(xy)(xy)^{1/2}\,dx,\quad y>0$
(14)	$x^{-\nu-1/2}(x^2+\alpha^2)^{-1}$, $\operatorname{Re}\alpha>0$, $\operatorname{Re}\nu>-1/2$	$2^{-1}\pi\alpha^{-\nu-1}y^{1/2}[I_\nu(\alpha y)-\mathbf{L}_\nu(\alpha y)]$
(15)	$x^{\nu+1/2}(x^2+\alpha^2)^{-1/2}$, $\operatorname{Re}\alpha>0$, $-1<\operatorname{Re}\nu<1/2$	$2^{1/2}\pi^{-1/2}\alpha^{\nu+1/2}K_{\nu+1/2}(\alpha y)$
(16)	$x^{1/2-\nu}(x^2+\alpha^2)^{-1/2}$, $\operatorname{Re}\alpha>0$, $\operatorname{Re}\nu>-1/2$	$\pi^{1/2}2^{-1/2}\alpha^{1/2-\nu}[I_{\nu-1/2}(\alpha y)-$ $-\mathbf{L}_{\nu-1/2}(\alpha y)]$
(17)	$x^{-\nu-1/2}(x^2+\alpha^2)^{-\nu-1/2}$, $\operatorname{Re}\alpha>0$, $\operatorname{Re}\nu>-1/2$	$2^\nu\alpha^{-2\nu}y^{1/2+\nu}\dfrac{\Gamma(\nu+1)}{\Gamma(2\nu+1)}\times$ $\times I_\nu(2^{-1}\alpha y)K_\nu(2^{-1}\alpha y)$
(18)	$x^{\nu+1/2}(x^2+\alpha^2)^{-\nu-1/2}$, $\operatorname{Re}\alpha>0$, $\operatorname{Re}\nu>-1/2$	$\dfrac{\pi^{1/2}y^{\nu-1/2}}{2^\nu e^{\alpha y}\Gamma(\nu+1/2)}$
(19)	$x^{\nu+1/2}(x^2+\alpha^2)^{-\nu-3/2}$, $\operatorname{Re}\alpha>0$, $\operatorname{Re}\nu>-1$	$\dfrac{y^{\nu+1/2}\pi^{1/2}}{2^{\nu+1}\alpha\, e^{\alpha y}\Gamma(\nu+3/2)}$
(20)	$x^{\nu+1/2}(x^2+\alpha^2)^{-\mu-1}$, $\operatorname{Re}\alpha>0$, $-1<\operatorname{Re}\nu<2\operatorname{Re}\mu+3/2$	$\dfrac{\alpha^{\nu-\mu}y^{\mu+1/2}K_{\nu-\mu}(\alpha y)}{2^\mu\Gamma(\mu+1)}$
(21)	$x^{\lambda-3/2}(x^2+\alpha^2)^{-\mu-1}$, $\operatorname{Re}\alpha>0$, $-\operatorname{Re}\nu<\operatorname{Re}\lambda<2\operatorname{Re}\mu+7/2$	$\dfrac{y^{\nu+1/2}\Gamma(\lambda/2+\nu/2)\Gamma(\mu-\lambda/2-\nu/2+1)}{2^{\nu+1}\alpha^{2\mu-\lambda-\nu+2}\Gamma(\mu+1)\Gamma(\nu+1)}\times$ $\times{}_1F_2(\lambda/2+\nu/2;\ \lambda/2+\nu/2-\mu,$ $\nu+1;\ 2^{-2}y^2\alpha^2)+$ $+\dfrac{y^{2\mu-\lambda+5/2}\Gamma(\lambda/2+\nu/2-\mu-1)}{2^{2\mu-\lambda+3}\Gamma(\nu/2-\lambda/2+\mu+2)}\times$ $\times{}_1F_2(\mu+1;\ \mu+2+\nu/2-\lambda/2,$ $\mu+2-\lambda/2-\nu/2;\ 2^{-2}y^2\alpha^2)$
(22)	$x^{-1/2}(a^2-x^2)^{-1/2}$, $0<x<a$; 0, $a<x<\infty$; $\operatorname{Re}\nu>-1$	$2^{-1}\pi y^{1/2}[J_{\nu/2}(2^{-1}ay)]^2$
(23)	0, $0<x<a$; $x^{-1/2}(x^2-a^2)^{-1/2}$, $a<x<\infty$	$-2^{-1}\pi y^{1/2}J_{\nu/2}(2^{-1}ay)Y_{\nu/2}(2^{-1}ay)$

	$f(x)$	$\int\limits_0^\infty f(x)\, J_\nu(xy)\,(xy)^{1/2}\,dx, \quad y>0$
(24)	$x^{1/2-\nu}(a^2-x^2)^{-1/2}, \quad 0<x<a$ $0, \quad a<x<\infty$	$2^{-1/2}\pi^{1/2}a^{1/2-\nu}\mathbf{H}_{\nu-1/2}(ay)$
(25)	$x^{\nu-1/2}(a^2-x^2)^{\nu-1/2}, \quad 0<x<a$ $0, \quad a<x<\infty$ $\operatorname{Re}\nu>-1/2$	$2^{\nu-1}\pi^{1/2}\,\Gamma(\nu+1/2)\,a^{2\nu}\,y^{1/2-\nu}\times$ $\times[J_\nu(2^{-1}ay)]^2$
(26)	$0, \quad 0<x<a$ $x^{-\nu-1/2}(x^2-a^2)^{-\nu-1/2},$ $a<x<\infty$ $\lvert\operatorname{Re}\nu\rvert<1/2$	$-2^{-\nu-1}a^{-2\nu}\,\Gamma(1/2-\nu)\,y^{\nu+1/2}\pi^{1/2}\times$ $\times J_\nu(2^{-1}ay)\,Y_\nu(2^{-1}ay)$
(27)	$x^{\nu+1/2}(a^2-x^2)^{-\nu-1/2}, \quad 0<x<a$ $0, \quad a<x<\infty$ $\lvert\operatorname{Re}\nu\rvert<1/2$	$\pi^{-1/2}2^{-\nu}\,\Gamma(1/2-\nu)\,y^{\nu-1/2}\sin(ay)$
(28)	$0, \quad 0<x<a$ $x^{-\nu+1/2}(x^2-a^2)^{\nu-1/2}, \quad a<x<\infty$ $\lvert\operatorname{Re}\nu\rvert<1/2$	$\pi^{-1/2}2^{-\nu}\,\Gamma(1/2+\nu)\,y^{-\nu-1/2}\cos(ay)$
(29)	$x^{\nu+1/2}(a^2-x^2)^{-\nu-3/2},$ $0<x<a$ $0, \quad a<x<\infty$ $-1<\operatorname{Re}\nu<-1/2$	$2^{-1-\nu}\pi^{-1/2}\,\Gamma(-1/2-\nu)\,a^{-1}\times$ $\times\cos(ay)\,y^{\nu+1/2}$
(30)	$0, \quad 0<x<a$ $x^{1/2-\nu}(x^2-a^2)^{\nu-3/2}, \quad a<x<\infty$ $1/2<\operatorname{Re}\nu<5/2$	$2^{-\nu-1}\pi^{-1/2}a^{-1}\,\Gamma(\nu-1/2)\,y^{1/2-\nu}\times$ $\times\sin(ay)$
(31)	$x^{1/2-\nu}(a^2-x^2)^{\mu}, \quad 0<x<a$ $0, \quad a<x<\infty$ $\operatorname{Re}\mu>-1$	$\dfrac{2^{1-\nu}a^{\mu-\nu+1}\,s_{\nu+\mu,\,\mu-\nu+1}(ay)}{y^{\mu+1/2}\,\Gamma(\nu)}$
(32)	$0, \quad 0<x<a$ $x^{1/2-\nu}(x^2-a^2)^{\mu}, \quad a<x<\infty$ $\operatorname{Re}\mu>-1, \ \operatorname{Re}(\nu-2\mu)>1/2$	$2^{\mu}\,\Gamma(\mu+1)\,a^{1+\mu-\nu}y^{-\mu-1/2}\times$ $\times J_{\nu-\mu-1}(ay)$
(33)	$x^{\nu+1/2}(a^2-x^2)^{\mu}, \quad 0<x<a$ $0, \quad a<x<\infty$ $\operatorname{Re}\nu>-1, \ \operatorname{Re}\mu>-1$	$2^{\mu}\,\Gamma(\mu+1)\,y^{-\mu-1/2}a^{\nu+\mu+1}\times$ $\times J_{\nu+\mu+1}(ay)$

	$f(x)$	$\int\limits_0^\infty f(x)\,J_\nu(xy)\,(xy)^{1/2}\,dx,\ y>0$
(34)	$x^{\mu-1/2}(a^2-x^2)^{\lambda}$, $\quad 0<x<a$ 0, $\quad a<x<\infty$ $\operatorname{Re}\lambda>-1,\ \operatorname{Re}(\mu+\nu)>-1$	$\dfrac{a^{2\lambda+\mu+\nu+1}y^{\nu+1/2}\,\mathrm{B}(\lambda+1,\ \mu/2+\nu/2+1/2)}{2^{\nu+1}\,\Gamma(\nu+1)}\times$ $\times\,{}_1F_2\left(\dfrac{1+\mu+\nu}{2};\ \nu+1,\ \dfrac{3+\mu+\nu}{2}+\lambda;\ -\dfrac{a^2y^2}{4}\right)$
(35)	$x^{-\nu-1/2}(a^2+2x)^{-1/2}\times$ $\times\left[(a^2+2x)^{1/2}-a\right]^{2\nu}$, $\operatorname{Re}\nu>-1/2$	$2^{\nu}\,\Gamma(\nu+1/2)\,\pi^{-1/2}\times$ $\times\,D_{-\nu-1/2}\left(ae^{\pi i/4}y^{1/2}\right)\times$ $\times\,D_{-\nu-1/2}\left(ae^{-\pi i/4}y^{1/2}\right)$
(36)	$x^{-1/2}(x^2+\alpha^2)^{-1/2}\times$ $\times\left[(x^2+\alpha^2)^{1/2}+x\right]^{\nu-1}$, $\operatorname{Re}\alpha>0,\ -1<\operatorname{Re}\nu<5/2$	$2\pi^{-1/2}\alpha^{\nu-3/2}\operatorname{sh}(2^{-1}\alpha y)\times$ $\times\,K_{\nu-1/2}(2^{-1}\alpha y)$
(37)	$x^{-1/2}(x^2+\alpha^2)^{-1/2}\times$ $\times\left[(x^2+\alpha^2)^{1/2}+x\right]^{1-\nu}$, $\operatorname{Re}\alpha>0,\ \operatorname{Re}\nu>-1/2$	$\pi^{1/2}\alpha^{1/2-\nu}\times$ $\times\exp\left[-2^{-1}\alpha y\right]I_{\nu-1/2}(2^{-1}\alpha y)$
(38)	$x^{-1/2}(x^2+\alpha^2)^{-1/2}\times$ $\times\left[(x^2+\alpha^2)^{1/2}\pm x\right]^{\mu}$, $\operatorname{Re}\alpha>0$ $\operatorname{Re}\nu>-1,\ \operatorname{Re}\mu<3/2$	$y^{1/2}\alpha^{\mu}\,I_{(\nu\mp\mu)/2}(2^{-1}\alpha y)\times$ $\times\,K_{(\nu\pm\mu)/2}(2^{-1}\alpha y)$
(39)	$x^{-\nu+1/2}(x^2+\alpha^2)^{-1/2}\times$ $\times\left[(x^2+\alpha^2)^{1/2}-\alpha\right]^{\nu}$, $\operatorname{Re}\alpha>0,\ \operatorname{Re}\nu>-1$	$y^{-1/2}e^{-\alpha y}$
(40)	$x^{-\mu-1/2}(x^2+\alpha^2)^{-1/2}\times$ $\times\left[(x^2+\alpha^2)^{1/2}+\alpha\right]^{\mu}$, $\operatorname{Re}\alpha>0,\ \operatorname{Re}(\nu-\mu)>-1$	$\dfrac{\Gamma(1/2+\nu/2-\mu/2)}{\alpha y^{1/2}\,\Gamma(\nu+1)}\times$ $\times\,W_{\mu/2,\ \nu/2}(\alpha y)\,M_{-\mu/2,\ \nu/2}(\alpha y)$
(41)	0, $\quad 0<x<a$ $x^{-1/2}(x^2-a^2)^{-1/2}\times$ $\times\left\{\left[x+(x^2-a^2)^{1/2}\right]^{\nu+1}+\right.$ $\left.+\left[x-(x^2-a^2)^{1/2}\right]^{\nu+1}\right\}$, $a<x<\infty$ $\operatorname{Re}\nu<1/2$	$-\pi^{1/2}a^{\nu-3/2}\times$ $\times\left[\sin(2^{-1}ay)\,J_{\nu+1/2}(2^{-1}ay)+\right.$ $\left.+\cos(2^{-1}ay)\,Y_{\nu+1/2}(2^{-1}ay)\right]$

	$f(x)$	$\int_0^\infty f(x)\, J_\nu(xy)\,(xy)^{1/2}\,dx,\ y>0$
(42)	$0, \qquad 0<x<a$ $x^{-1/2}(x^2-a^2)^{-1/2}\times$ $\times\{[x+(x^2-a^2)^{1/2}]^{\nu-1}+$ $+[x-(x^2-a^2)^{1/2}]^{\nu-1}\},$ $a<x<\infty$ $\operatorname{Re}\nu<5/2$	$\pi^{1/2}a^{\nu-3/2}\times$ $\times[\cos(2^{-1}ay)\,J_{\nu-1/2}(2^{-1}ay)-$ $-\sin(2^{-1}ay)\,Y_{\nu-1/2}(2^{-1}ay)]$
(43)	$x^{-1/2}(a^2-x^2)^{-1/2}\times$ $\times\{[x+i(a^2-x^2)^{1/2}]^{\mu}+$ $+[x-i(a^2-x^2)^{1/2}]^{\mu}\},$ $0<x<a$ $0, \qquad a<x<\infty$ $\operatorname{Re}(\mu+\nu)>-1$	$\pi a^{\mu}y^{1/2}J_{(\nu+\mu)/2}(2^{-1}ay)\times$ $\times J_{(\nu-\mu)/2}(2^{-1}ay)$
(44)	$0, \qquad 0<x<a$ $x^{-1/2}(x^2-a^2)^{-1/2}\times$ $\times\{[x+(x^2-a^2)^{1/2}]^{\mu}+$ $+[x-(x^2-a^2)^{1/2}]^{\mu}\},$ $a<x<\infty$ $\operatorname{Re}\mu<3/2$	$-2^{-1}\pi y^{1/2}a^{\mu}[J_{(\nu+\mu)/2}(2^{-1}ay)\times$ $\times Y_{(\nu-\mu)/2}(2^{-1}ay)+$ $+J_{(\nu-\mu)/2}(2^{-1}ay)\,Y_{(\nu+\mu)/2}(2^{-1}ay)]$
(45)	$x^{-2\mu-1/2}(a^2-x^2)^{-1/2}\times$ $\times\{[a+(a^2-x^2)^{1/2}]^{2\mu}+$ $+[a-(a^2-x^2)^{1/2}]^{2\mu}\},$ $0<x<a$ $0, \qquad a<x<\infty$ $\operatorname{Re}(2\mu)<\operatorname{Re}\nu+1$	$\dfrac{a^{\nu}\,B(1/2+\nu/2+\mu,\ 1/2+\nu/2-\mu)\,y^{\nu+1/2}}{\Gamma(1+\nu)}\times$ $\times{}_1F_1(1/2+\nu/2-\mu;\ \nu+1;\ -iay)\times$ $\times{}_1F_1(1/2+\nu/2-\mu;\ \nu+1;\ iay)$
(46)	$x^{\mu-1/2}(1-2ax+a^2)^{-1/2},$ $0<x<1$ $0, \qquad 1<x<\infty$ $\operatorname{Re}(\nu+\mu+1/2)>0$	См. Bose S. K., 1946: Bull. Calcutta Math. Soc. 38, 177—180.
(47)	$x^{\nu+5/2}(x^4+4\alpha^4)^{-\nu-1/2},$ $\lvert\arg\alpha\rvert<\pi/4,\ \operatorname{Re}\nu>1/6$	$\dfrac{\pi^{1/2}y^{1/2+\nu}\,J_{\nu-1}(\alpha y)\,K_{\nu-1}(\alpha y)}{2^{3\nu-1}\alpha^{2\nu-2}\,\Gamma(\nu+1/2)}$

	$f(x)$	$\int\limits_0^\infty f(x)\, J_\nu(xy)\,(xy)^{1/2}\,dx, \quad y>0$
(48)	$x^{\nu+1/2}(x^4+4\alpha^4)^{-\nu-1/2}$, $\lvert\arg\alpha\rvert<\pi/4$, $\operatorname{Re}\nu>-1/2$	$\dfrac{y^{\nu+1/2}\pi^{1/2}J_\nu(\alpha y)\,K_\nu(\alpha y)}{\alpha^{2\nu}2^{3\nu}\,\Gamma(\nu+1/2)}$
(49)	$x^{\nu+1/2}(x^4\pm 2a^2x^2+b^4)^{-1/2}\times$ $\times[b^2+x^2+$ $+(x^4\pm 2a^2x^2+b^4)^{1/2}]^{-2\nu}$, $0<a<b$ $\operatorname{Re}\nu>-1/2$	$(b^2\mp a^2)^{-\nu}\,2^\nu y^{1/2}\times$ $\times K_0\left[(2^{-1}b^2\pm 2^{-1}a^2)^{1/2}y\right]\times$ $\times J_\nu\left[(2^{-1}b^2\mp 2^{-1}a^2)^{1/2}y\right]$
(50)	0, $0<x<a$ $x^{1/2-\nu}(x^2-a^2)^{-1/2}\times$ $\times\left\{[a+(a^2-x^2)^{1/2}]^\nu+\right.$ $\left.+[a-(a^2-x^2)^{1/2}]^\nu\right\}$, $a<x<\infty$ $\operatorname{Re}\nu>-1$	$2y^{-1/2}\cos(ay-2^{-1}\nu\pi)$

8.6. Показательные и логарифмические функции

(1)	$x^{-1/2}e^{-\alpha x}$, $\operatorname{Re}\alpha>0$, $\operatorname{Re}\nu>-1$	$y^{1/2-\nu}(\alpha^2+y^2)^{-1/2}\left[(\alpha^2+y^2)^{1/2}-\alpha\right]^\nu$
(2)	$x^{-3/2}e^{-\alpha x}$, $\operatorname{Re}\alpha>0$, $\operatorname{Re}\nu>0$	$\nu^{-1}y^{1/2-\nu}\left[(\alpha^2+y^2)^{1/2}-\alpha\right]^\nu$
(3)	$x^{m+1/2}e^{-ax}$, $\operatorname{Re}\nu>-m-2$	$(-1)^{m+1}y^{1/2-\nu}\times$ $\times\dfrac{d^{m+1}}{da^{m+1}}\left\{\dfrac{[(a^2+y^2)^{1/2}-a]^\nu}{(a^2+y^2)^{1/2}}\right\}$
(4)	$x^{\nu+1/2}e^{-\alpha x}$, $\operatorname{Re}\alpha>0$, $\operatorname{Re}\nu>-1$	$\pi^{-1/2}2^{\nu+1}\,\Gamma(\nu+3/2)\,\alpha y^{\nu+1/2}\times$ $\times(\alpha^2+y^2)^{-\nu-3/2}$
(5)	$x^{\nu-1/2}e^{-\alpha x}$, $\operatorname{Re}\alpha>0$, $\operatorname{Re}\nu>-1/2$	$2^\nu\pi^{-1/2}\times$ $\times\Gamma(\nu+1/2)\,y^{\nu+1/2}(\alpha^2+y^2)^{-\nu-1/2}$
(6)	$x^{\mu-3/2}e^{-ax}$, $a>0$, $\operatorname{Re}(\mu+\nu)>0$	$y^{1/2}(a^2+y^2)^{-\mu/2}\,\Gamma(\mu+\nu)\times$ $\times P_{\mu-1}^{-\nu}\left[a(a^2+y^2)^{-1/2}\right]$

	$f(x)$	$\int_0^\infty f(x)\, J_\nu(xy)\,(xy)^{1/2}\,dx,\ y>0$
(7)	$x^{\mu-3/2}e^{-\alpha x}$, $\operatorname{Re}\alpha>0,\ \operatorname{Re}(\mu+\nu)>0$	$\frac{y^{\nu+1/2}\,\Gamma(\mu+\nu)}{2^\nu\alpha^{\mu+\nu}\,\Gamma(\nu+1)}\times {}_2F_1\left(\frac{\mu+\nu}{2},\ \frac{\mu+\nu+1}{2};\ \nu+1;\ -\frac{y^2}{\alpha^2}\right)=$ $=\frac{y^{\nu+1/2}\,\Gamma(\mu+\nu)}{2^\nu(\alpha^2+y^2)^{(\mu+\nu)/2}\,\Gamma(\nu+1)}\times {}_2F_1\left[\frac{\mu+\nu}{2},\ \frac{1-\mu+\nu}{2};\ \nu+1;\ \frac{y^2}{(\alpha^2+y^2)}\right]$
(8)	$x^{-1/2}\exp(-\alpha x^2)$, $\operatorname{Re}\alpha>0,\quad \operatorname{Re}\nu>-1$	$\frac{\pi^{1/2}y^{1/2}}{2\alpha^{1/2}}\exp\left(-\frac{y^2}{8\alpha}\right)I_{\nu/2}\left(\frac{y^2}{8\alpha}\right)$
(9)	$x^{1/2}\exp(-\alpha x^2)$, $\operatorname{Re}\alpha>0,\ \operatorname{Re}\nu>-2$	$\frac{\pi^{1/2}y^{3/2}}{8\alpha^{3/2}}\exp\left(-\frac{y^2}{8\alpha}\right)\times\left[I_{\nu/2-1/2}\left(\frac{y^2}{8\alpha}\right)-I_{\nu/2+1/2}\left(\frac{y^2}{8\alpha}\right)\right]$
(10)	$x^{\nu+1/2}\exp(-\alpha x^2)$, $\operatorname{Re}\alpha>0,\ \operatorname{Re}\nu>-1$	$\frac{y^{\nu+1/2}}{(2\alpha)^{\nu+1}}\exp\left(-\frac{y^2}{4\alpha}\right)$
(11)	$x^{\nu-3/2}\exp(-\alpha x^2)$, $\operatorname{Re}\alpha>0,\ \operatorname{Re}\nu>0$	$2^{\nu-1}y^{1/2-\nu}\gamma\left(\nu,\ \frac{y^2}{4\alpha}\right)$
(12)	$x^{\nu+1/2}\exp(\pm iax^2)$, $a>0,\ \ -1<\operatorname{Re}\nu<1/2$	$\frac{y^{\nu+1/2}}{(2a)^{\nu+1}}\exp\left[\pm i\left(\frac{\nu+1}{2}\pi-\frac{y^2}{4a}\right)\right]$
(13)	$x^{2n+\nu+1/2}\exp\left(-2^{-2}x^2\right)$, $\operatorname{Re}\nu>-1-2n$	$2^{2n+\nu+1}n!\,y^{\nu+1/2}\exp\left(-y^2\right)L_n^\nu\left(y^2\right)$
(14)	$x^{\mu-1/2}\exp(-\alpha x^2)$, $\operatorname{Re}\alpha>0,\ \operatorname{Re}(\mu+\nu)>-1$	$\frac{y^{\nu+1/2}\,\Gamma(\nu/2+\mu/2+1/2)}{2^{\nu+1}\alpha^{(\mu+\nu+1)/2}\,\Gamma(\nu+1)}\times {}_1F_1\left(\frac{\nu+\mu+1}{2};\ \nu+1;\ -\frac{y^2}{4\alpha}\right)=$ $=\frac{\Gamma(\nu/2+\mu/2+1/2)}{y^{1/2}\alpha^{\mu/2}\,\Gamma(\nu+1)}\times\exp\left(-\frac{y^2}{8\alpha}\right)M_{\mu/2,\ \nu/2}\left(\frac{y^2}{4\alpha}\right)$
(15)	$x^{-3/2}e^{-\alpha/x}$, $\operatorname{Re}\alpha>0$	$2y^{1/2}J_\nu\left[(2\alpha y)^{1/2}\right]K_\nu\left[(2\alpha y)^{1/2}\right]$

	$f(x)$	$\int_0^\infty f(x)\,J_\nu(xy)\,(xy)^{1/2}\,dx,\ y>0$
(16)	$x^{-3/2}e^{-\alpha/x-\beta x}$, $\operatorname{Re}\alpha>0,\ \operatorname{Re}\beta>0$	$2y^{1/2}J_\nu\left\{(2\alpha)^{1/2}\left[(\beta^2+y^2)^{1/2}-\beta\right]^{1/2}\right\}\times$ $\times K_\nu\left\{(2\alpha)^{1/2}\left[(\beta^2+y^2)^{1/2}+\beta\right]^{1/2}\right\}$
(17)	$x^{-1}\exp(-\alpha x^{1/2})$, $\operatorname{Re}\alpha>0,\ \operatorname{Re}\nu>-1/2$	$\pi^{-1/2}2^{1/2}\,\Gamma(\nu+1/2)\times$ $\times D_{-\nu-1/2}(2^{-1/2}\alpha e^{\pi i/4}y^{-1/2})\times$ $\times D_{-\nu-1/2}(2^{-1/2}\alpha e^{-\pi i/4}y^{-1/2})$
(18)	$x^{\nu+1/2}\exp[\alpha(1-x^2)],\ 0<x<1$ $0,\ 1<x<\infty$ $\operatorname{Re}\nu>-1/2$	$(2i\alpha)^{-\nu-1}y^{1/2+\nu}[U_{\nu+1}(2i\alpha,\ y)-$ $-iU_{\nu+2}(2i\alpha,\ y)]$
(19)	$x^{\nu+1/2}\exp\left[-\alpha(x^2+\beta^2)^{1/2}\right]$, $\operatorname{Re}\alpha>0,\ \operatorname{Re}\beta>0,\ \operatorname{Re}\nu>-1$	$(\pi/2)^{-1/2}\alpha\beta^{\nu+3/2}\times$ $\times y^{\nu+1/2}(y^2+\alpha^2)^{-\nu-3/4}\times$ $\times K_{\nu+3/2}\left[\beta(y^2+\alpha^2)^{1/2}\right]$
(20)	$x^{-1/2}(\beta^2+x^2)^{-1/2}\times$ $\times\exp\left[-\alpha(\beta^2+x^2)^{1/2}\right]$, $\operatorname{Re}\alpha>0,\ \operatorname{Re}\beta>0\ \operatorname{Re}\nu>-1$	$y^{1/2}I_{\nu/2}\{2^{-1}\beta\left[(\alpha^2+y^2)^{1/2}-\alpha\right]\}\times$ $\times K_{\nu/2}\{2^{-1}\beta\left[(\alpha^2+y^2)^{1/2}+\alpha\right]\}$
(21)	$x^{\nu+1/2}(\beta^2+x^2)^{-1/2}\times$ $\times\exp\left[ia(\beta^2+x^2)^{1/2}\right]$, $a>0$ $\operatorname{Re}\beta>0,\ -1<\operatorname{Re}\nu<1/2$	$i2^{-1/2}\pi^{1/2}\beta^{1/2+\nu}\times$ $\times(a^2-y^2)^{-1/4-\nu/2}y^{1/2+\nu}\times$ $\times H^{(1)}_{-\nu-1/2}\left[\beta(\alpha^2-y^2)^{1/2}\right]$, $0<y<a$ $2^{1/2}\pi^{-1/2}\beta^{1/2+\nu}\times$ $\times y^{1/2+\nu}(y^2-a^2)^{-1/4-\nu/2}\times$ $\times K_{\nu+1/2}\left[\beta(y^2-a^2)^{1/2}\right]$, $a<y<\infty$
(22)	$x^{\nu+1/2}(\beta^2+x^2)^{-1/2}\times$ $\times\exp\left[-\alpha(\beta^2+x^2)^{1/2}\right]$, $\operatorname{Re}\alpha>0,\ \operatorname{Re}\beta>0,\ \operatorname{Re}\nu>-1$	$(\pi/2)^{-1/2}\beta^{\nu+1/2}\times$ $\times y^{\nu+1/2}(\alpha^2+y^2)^{-\nu/2-1/4}\times$ $\times K_{\nu+1/2}\left[\beta(\alpha^2+y^2)^{1/2}\right]$
(23)	$x^{-\nu+1/2}(x^2+\beta^2)^{-1/2}\times$ $\times\left[(x^2+\beta^2)^{1/2}-\beta\right]^\nu\times$ $\times\exp\left[-\alpha(x^2+\beta^2)^{1/2}\right]$, $\operatorname{Re}\alpha>0,\ \operatorname{Re}\beta>0,\ \operatorname{Re}\nu>-1$	$y^{\nu+1/2}\left[\alpha+(y^2+\alpha^2)^{1/2}\right]^{-\nu}\times$ $\times(y^2+\alpha^2)^{-1/2}\times$ $\times\exp\left[-\beta(y^2+\alpha^2)^{1/2}\right]$

	$f(x)$	$\int_0^\infty f(x) J_\nu(xy)(xy)^{1/2}\,dx,\ y>0$
(24)	$x^{\sigma-1/2}(x^2+\beta^2)^{-1/2}\times$ $\times\left[(x^2+\beta^2)^{1/2}+\beta\right]^{-\sigma}\times$ $\times\exp\left[-\alpha(x^2+\beta^2)^{1/2}\right]$, $\operatorname{Re}\alpha>0,\ \operatorname{Re}\beta>0$ $\operatorname{Re}(\nu+\sigma)>-1$	$\frac{\Gamma(\nu/2+\sigma/2+1/2)}{\beta\,\Gamma(\nu+1)\,y^{1/2}}\times$ $\times M_{\sigma/2,\,\nu/2}\{\beta[(y^2+\alpha^2)^{1/2}-\alpha]\}\times$ $\times W_{-\sigma/2,\,\nu/2}\{\beta[(y^2+\alpha^2)^{1/2}+\alpha]\}$
Относительно преобразований Ганкеля других функций, содержащих показательную функцию, см. таблицы преобразований Лапласа.		
(25)	$x^\mu\ln x$, $-\operatorname{Re}\nu-3/2<\operatorname{Re}\mu<0$	$\frac{2^{\mu-1/2}\,\Gamma(\mu/2+\nu/2+3/4)}{\Gamma(\nu/2-\mu/2+1/4)\,y^{\mu+1}}\times$ $\times[\psi(\mu/2+\nu/2+3/4)+$ $+\psi(\nu/2-\mu/2+1/4)-\ln(y^2/4)]$

8.7. Тригонометрические и обратные тригонометрические функции

(1)	$x^{-1/2}\sin(ax)$, $a>0,\ \operatorname{Re}\nu>-2$	$\cos(2^{-1}\pi\nu)\,y^{\nu+1/2}(a^2-y^2)^{-1/2}\times$ $\times[a+(a^2-y^2)^{1/2}]^{-\nu},\quad 0<y<a$ $y^{1/2}(y^2-a^2)^{-1/2}\sin[\nu\arcsin(a/y)]$, $a<y<\infty$
(2)	$x^{-3/2}\sin(ax)$, $a>0,\ \operatorname{Re}\nu>-1$	$\nu^{-1}\sin(2^{-1}\pi\nu)\,y^{\nu+1/2}\times$ $\times[a+(a^2-y^2)^{1/2}]^{-\nu},\quad 0<y\leqslant a$ $\nu^{-1}y^{1/2}\sin[\nu\arcsin(a/y)]$, $a<y<\infty$
(3)	$x^{\nu+1/2}\sin(ax)$, $a>0,\ -3/2<\operatorname{Re}\nu<-1/2$	$-2^{1+\nu}\pi^{-1/2}\sin(\nu\pi)\,\Gamma(\nu+3/2)\,a\times$ $\times y^{\nu+1/2}(a^2-y^2)^{-\nu-3/2}$, $0<y<a$ $-2^{1+\nu}\pi^{-1/2}\Gamma(\nu+3/2)\times$ $\times a y^{\nu+1/2}(y^2-a^2)^{-\nu-3/2}$, $a<y<\infty$
(4)	$x^{\nu-1/2}\sin(ax)$, $a>0,\ -1<\operatorname{Re}\nu<1/2$	$[\Gamma(1/2-\nu)]^{-1}\pi^{1/2}2^\nu y^{\nu+1/2}\times$ $\times(a^2-y^2)^{-\nu-1/2},\quad 0<y<a$ $0,\quad a<y<\infty$

	$f(x)$	$\int\limits_0^\infty f(x)\, J_\nu(xy)\,(xy)^{1/2}\,dx,\quad y>0$
(5)	$x^{1/2-\nu}\sin(ax)$, $a>0,\ \operatorname{Re}\nu>1/2$	$0,\quad 0<y<a$ $2^{1-\nu}\pi^{1/2}a\,[\Gamma(\nu-1/2)]^{-1}y^{1/2+\nu}\times$ $\times(y^2-a^2)^{\nu-3/2},\quad a<y<\infty$
(6)	$x^{-\nu+2n+1/2}\sin(ax)$, $a>0,\quad \operatorname{Re}\nu>2n+1/2$	$0,\quad 0<y<a$ $(-1)^n 2^{\nu-2n-2}y^{2n-\nu+3/2}(2n+1)!\times$ $\times\Gamma(\nu-2n-1)[\Gamma(2\nu-2n-1)]^{-1}\times$ $\times(y^2-a^2)^{\nu-2n-3/2}C_{2n+1}^{\nu-2n-1}(ay^{-1}),$ $a<y<\infty$
(7)	$x^{\mu-3/2}\sin(ax)$, $a>0,\quad -\operatorname{Re}\nu<\operatorname{Re}\mu<3/2$	$\dfrac{y^{\nu+1/2}\,\Gamma(\nu+\mu)\sin[2^{-1}\pi(\nu+\mu)]}{2^\nu a^{\nu+\mu}\,\Gamma(\nu+1)}\times$ $\times{}_2F_1\left(\dfrac{1+\nu+\mu}{2},\ \dfrac{\nu+\mu}{2};\ \nu+1;\ \dfrac{y^2}{a^2}\right),$ $0<y<a$ $\dfrac{2^\mu a\,\Gamma(1/2+\nu/2+\mu/2)}{y^{\mu+1/2}\,\Gamma(1/2+\nu/2-\mu/2)}\times$ $\times{}_2F_1\left(\dfrac{1+\nu+\mu}{2},\ \dfrac{1+\mu-\nu}{2};\ \dfrac{3}{2};\ \dfrac{a^2}{y^2}\right),$ $a<y<\infty$
(8)	$x^{\nu-1/2}(x^2+\beta^2)^{-1}\sin(ax)$, $a>0$ $\operatorname{Re}\beta>0,\quad -1<\operatorname{Re}\nu<3/2$	$\beta^{\nu-1}\operatorname{sh}(a\beta)\,y^{1/2}K_\nu(\beta y),$ $y\geqslant a$
(9)	$x^{1/2-\nu}(x^2+\beta^2)^{-1}\sin(ax)$, $a>0$ $\operatorname{Re}\beta>0,\quad \operatorname{Re}\nu>-1/2$	$2^{-1}\pi\beta^{-\nu}e^{-a\beta}y^{1/2}I_\nu(\beta y),$ $0<y\leqslant a$
(10)	$x^{-3/2}e^{-xa\cos\varphi\cos\psi}\sin(xa\sin\psi)$, $\operatorname{Re}\nu>-1,\ a>0,\ 0<\varphi,\ \psi<\pi/2$	$a^{1/2}\nu^{-1}(\sin\varphi)^{1/2}(\operatorname{tg}\varphi/2)^\nu\sin(\nu\psi),$ $y=a\sin\varphi$
(11)	$x^{\nu+1/2}e^{-xa\cos\varphi\cos\psi}\sin(xa\sin\psi)$, $a>0,\quad 0<\varphi,\ \psi<\pi/2$ $\operatorname{Re}\nu>-3/2$	$2^{\nu+1}\pi^{-1/2}\Gamma(\nu+3/2)\times$ $\times a^{-\nu-3/2}(\sin\varphi)^{\nu+1/2}\times$ $\times(\cos^2\psi+\sin^2\psi\cos^2\varphi)^{-\nu-3/2}\times$ $\times\sin[(\nu+3/2)\alpha],$ $y=a\sin\varphi$ $\operatorname{tg}(\alpha/2)=\operatorname{tg}\psi\cos\varphi$

	$f(x)$	$\int_0^\infty f(x) J_\nu(xy)(xy)^{1/2}\,dx, \quad y>0$
(12)	$x^{\nu-1/2} e^{-xa\cos\varphi\cos\psi}\sin(xa\sin\psi)$, $a>0, \quad 0<\varphi, \psi<\pi/2$, $\operatorname{Re}\nu>-1$	$2^\nu \pi^{-1/2} a^{-\nu-1/2} \times \Gamma(\nu+1/2)(\sin\varphi)^{\nu+1/2} \times (\cos^2\psi+\sin^2\psi\cos^2\varphi)^{-\nu-1/2} \times \sin[(\nu+3/2)\alpha], \quad y=a\sin\varphi$, $\operatorname{tg}(\alpha/2)=\operatorname{tg}\psi\cos\varphi$
(13)	$x^{-1/2}\sin(ax^2)$, $a>0, \quad \operatorname{Re}\nu>-3$	$-\frac{\pi^{1/2}y^{1/2}}{2a^{1/2}}\sin\left(\frac{y^2}{8a}-\frac{\nu+1}{4}\pi\right)\times J_{\nu/2}\left(\frac{y^2}{8a}\right)$
(14)	$x^{1/2}\sin(ax^2)$, $a>0, \quad \operatorname{Re}\nu>-4$	$\frac{\pi^{1/2}y^{3/2}}{8a^{3/2}}\left[\cos\left(\frac{y^2}{8a}-\frac{\nu\pi}{4}\right)J_{\nu/2-1/2}\left(\frac{y^2}{8a}\right)-\sin\left(\frac{y^2}{8a}-\frac{\nu\pi}{4}\right)J_{\nu/2+1/2}\left(\frac{y^2}{8a}\right)\right]$
(15)	$x^{\nu+1/2}\sin(ax^2)$, $a>0, \quad -2<\operatorname{Re}\nu<1/2$	$\frac{y^{\nu+1/2}}{2^{\nu+1}a^{\nu+1}}\cos\left(\frac{y^2}{4a}-\frac{\nu\pi}{2}\right)$
(16)	$x^{\nu+1/2}\sin(ax^2), \quad 0<x<b$; $0, \quad b<x<\infty$; $\operatorname{Re}\nu>-2$	$(2a)^{-\nu-1}y^{\nu+1/2}\times[\sin(ab^2)U_{\nu+1}(2ab^2, by)-\cos(ab^2)U_{\nu+2}(2ab^2, by)]$
(17)	$x^{-1}\exp(-ax^{1/2})\sin(ax^{1/2})$, $a>0, \operatorname{Re}\nu>-1$	$i2^{-1/2}\pi^{-1/2}\times\Gamma(\nu+1/2)D_{-\nu-1/2}(ay^{-1/2})\times\left[D_{-\nu-1/2}(iay^{-1/2})-D_{-\nu-1/2}(-iay^{-1/2})\right]$
(18)	$x^{\nu+1/2}\sin\left[a(x^2+\beta^2)^{1/2}\right], \quad a>0$, $\operatorname{Re}\beta>0, \quad -1<\operatorname{Re}\nu<-1/2$	$(\pi/2)^{1/2}a\beta^{\nu+3/2}\times y^{\nu+1/2}(a^2-y^2)^{-\nu/2-3/4}\times\left\{\sin(\nu\pi)J_{\nu+3/2}[\beta(a^2-y^2)^{1/2}]+\cos(\nu\pi)Y_{\nu+3/2}[\beta(a^2-y^2)^{1/2}]\right\}, \quad 0<y<a$; $-(\pi/2)^{-1/2}a\beta^{\nu+3/2}\times y^{\nu+1/2}(y^2-a^2)^{-\nu/2-3/4}\times K_{\nu+3/2}[\beta(y^2-a^2)^{1/2}], \quad a<y<\infty$

	$f(x)$	$\int_0^\infty f(x)\, J_\nu(xy)\,(xy)^{1/2}\,dx, \quad y>0$
(19)	$x^{-1/2}(x^2+\beta^2)^{-1/2}\times$ $\times\sin[a(x^2+\beta^2)^{1/2}]$, $a>0$ $\operatorname{Re}\beta>0, \quad \operatorname{Re}\nu>-1$	$2^{-1}\pi y^{1/2}\times$ $\times J_{\nu/2}\{2^{-1}\beta[a-(a^2-y^2)^{1/2}]\}\times$ $\times J_{-\nu/2}\{2^{-1}\beta[a+(a^2-y^2)^{1/2}]\}$, $0<y<a$
(20)	$x^{\nu+1/2}(\beta^2+x^2)^{-1/2}\times$ $\times\sin[a(\beta^2+x^2)^{1/2}], \quad a>0$ $\operatorname{Re}\beta>0, \quad -1<\operatorname{Re}\nu<1/2$	$2^{-1/2}\pi^{1/2}\beta^{1/2+\nu}\times$ $\times y^{1/2+\nu}(a^2-y^2)^{-1/4-\nu/2}\times$ $\times J_{-\nu-1/2}[\beta(a^2-y^2)^{1/2}]$, $0<y<a$ $0, \quad a<y<\infty$
(21)	$x^{\nu+1/2}(b^2+x^2)^{-2}\times$ $\times\sin[a(x^2+b^2)^{1/2}], \quad a>0$ $b>0, \quad -1<\operatorname{Re}\nu<7/2$	$ay^{1/2}b^{\nu}K_\nu(yb), \quad y>a$
(22)	$x^{-1/2}(a^2-x^2)^{-1/2}\times$ $\times\sin[b(a^2-x^2)^{1/2}]$, $0<x<a$ $-x^{-1/2}(x^2-a^2)^{-1/2}\times$ $\times\exp[-b(x^2-a^2)^{1/2}]$, $a<x<\infty$ $b>0, \quad \operatorname{Re}\nu>-1$	$2^{-1}\pi y^{1/2}\times$ $\times J_{\nu/2}\{2^{-1}a[(b^2+y^2)^{1/2}-b]\}\times$ $\times Y_{\nu/2}\{2^{-1}a[(b^2+y^2)^{1/2}+b]\}$
(23)	$x^{\nu+1/2}(a^2-x^2)^{-1/2}\times$ $\times\sin[b(a^2-x^2)^{1/2}], \quad 0<x<a$ $-x^{\nu+1/2}(x^2-a^2)^{-1/2}\times$ $\times\exp[-b(x^2-a^2)^{1/2}]$, $a<x<\infty$ $b>0, \quad \operatorname{Re}\nu>-1$	$2^{-1/2}\pi^{1/2}a^{\nu+1/2}\times$ $\times(b^2+y^2)^{-\nu/2-1/4}y^{\nu+1/2}$ $\times Y_{\nu+1/2}[a(b^2+y^2)^{1/2}]$
(24)	$0, \quad 0<x<a$ $x^{1/2-\nu}\sin[b(x^2-a^2)^{1/2}]$, $a<x<\infty$ $b>0, \quad \operatorname{Re}\nu>1/2$	$0, \quad 0<y<b$ $2^{-1/2}\pi^{1/2}a^{3/2-\nu}by^{1/2-\nu}\times$ $\times(y^2-b^2)^{\nu/2-3/4}\times$ $\times J_{\nu-3/2}[a(y^2-b^2)^{1/2}]$, $b<y<\infty$

	$f(x)$	$\int_0^\infty f(x)\, J_\nu(xy)\, (xy)^{1/2}\, dx,\ y>0$
(25)	$\dfrac{x^{1/2-\nu}\left[(x^2+\beta^2)^{1/2}-\beta\right]^\nu}{(x^2+\beta^2)^{1/2}} \times$ $\times \sin\left[a(x^2+\beta^2)^{1/2}\right]$, $a>0$, $\operatorname{Re}\beta>0,\ \operatorname{Re}\nu>-1$	$\dfrac{y^{\nu+1/2}}{\left[a+(a^2-y^2)^{1/2}\right]^\nu (a^2-y^2)^{1/2}} \times$ $\times \cos\left[\beta(a^2-y^2)^{1/2}+2^{-1}\pi\nu\right]$, $0<y<a$ $y^{-1/2}(y^2-a^2)^{-1/2} \times$ $\times \exp\left[-\beta(y^2-a^2)^{1/2}\right] \times$ $\times \sin\left[\nu \arcsin(a/y)\right]$, $a<y<\infty$
(26)	0, $0<x<c$ $x^{1/2-\nu}(x^2+b^2)^{-1} \times$ $\times \sin\left[a(x^2-c^2)^{1/2}\right]$, $c<x<\infty$ $\operatorname{Re}\nu>-3/2$	$2^{-1}\pi y^{1/2} b^{-\nu} \exp\left[-a(c^2+b^2)^{1/2}\right] \times$ $\times I_\nu(by)$, $0<y<a$

Относительно преобразований Ганкеля других функций, содержащих синус, см. таблицы синус-преобразований Фурье.

(27)	$x^{-3/2}\cos(ax)$, $a>0$, $\operatorname{Re}\nu>0$	$\nu^{-1}\cos(2^{-1}\nu\pi)\, y^{\nu+1/2} \times$ $\times \left[a+(a^2-y^2)^{1/2}\right]^{-\nu}$, $0<y\leqslant a$ $\nu^{-1} y^{1/2}\cos\left[\nu \arcsin(a/y)\right]$, $a<y<\infty$
(28)	$x^{\nu+1/2}\cos(ax)$, $a>0,\ -1<\operatorname{Re}\nu<-1/2$.	$2^{1+\nu}\pi^{1/2} a\left[\Gamma(-1/2-\nu)\right]^{-1} y^{\nu+1/2} \times$ $\times (a^2-y^2)^{-\nu-3/2}$, $0<y<a$ 0, $a<y<\infty$
(29)	$x^{\nu-1/2}\cos(ax)$, $a>0,\ \lvert\operatorname{Re}\nu\rvert<1/2$	$-2^\nu \pi^{-1/2}\sin(\nu\pi)\,\Gamma(1/2+\nu) \times$ $\times y^{\nu+1/2}(a^2-y^2)^{-\nu-1/2}$, $0<y<a$ $2^\nu\pi^{-1/2}\Gamma(1/2+\nu)\, y^{\nu+1/2} \times$ $\times (y^2-a^2)^{-\nu-1/2}$, $a<y<\infty$
(30)	$x^{-\nu-1/2}\cos(ax)$, $a>0,\ \operatorname{Re}\nu>-1/2$	0, $0<y<a$ $\dfrac{\pi^{1/2}(y^2-a^2)^{\nu-1/2}}{2^\nu y^{\nu-1/2}\,\Gamma(\nu+1/2)}$, $a<y<\infty$

	$f(x)$	$\int\limits_0^\infty f(x)\,J_\nu(xy)\,(xy)^{1/2}\,dx,\ y>0$
(31)	$x^{-\nu-2n-1/2}\cos(ax)$, $a>0,\ \operatorname{Re}\nu>2n-1/2$	$0,\qquad 0<y<a$ $(-1)^n y^{-\nu+2n+1/2}2^{\nu-2n-1}\times$ $\times\Gamma(\nu-2n)\times$ $\times[\Gamma(2\nu-2n)]^{-1}(2n)!\times$ $\times(y^2-a^2)^{\nu-2n-1/2}C_{2n}^{\nu-2n}(ay^{-1})$, $a<y<\infty$
(32)	$x^{\mu-3/2}\cos(ax)$, $a>0,\ -\operatorname{Re}\nu<\operatorname{Re}\mu<3/2$	$\dfrac{y^{\nu+1/2}\Gamma(\nu+\mu)\cos[2^{-1}\pi(\nu+\mu)]}{2^\nu a^{\nu+\mu}\Gamma(\nu+1)}\times$ $\times{}_2F_1\left(\frac{\nu+\mu}{2},\ \frac{\nu+\mu+1}{2};\ \nu+1;\ \frac{y^2}{a^2}\right)$, $0<y<a$ $\dfrac{2^{\mu-1}y^{1/2-\mu}\Gamma(\nu/2+\mu/2)}{\Gamma(1+\nu/2-\mu/2)}\times$ $\times{}_2F_1\left(\frac{\nu+\mu}{2},\ \frac{\mu-\nu}{2};\ \frac{1}{2},\ \frac{a^2}{y^2}\right)$, $a<y<\infty$
(33)	$x^{\nu+1/2}(x^2+\beta^2)^{-1}\cos(ax)$, $a>0,\ \operatorname{Re}\beta>0,\ -1<\operatorname{Re}\nu<1/2$	$\beta^\nu\operatorname{ch}(a\beta)\,y^{1/2}K_\nu(\beta y)$, $y\geqslant a$
(34)	$x^{-\nu-1/2}(x^2+\beta^2)^{-1}\cos(ax)$, $a>0,\ \operatorname{Re}\beta>0,\ \operatorname{Re}\nu>-3/2$	$2^{-1}\pi\beta^{-\nu-1}e^{-a\beta}y^{1/2}I_\nu(\beta y)$, $0<y\leqslant a$
(35)	$x^{-3/2}e^{-xa\cos\varphi\cos\psi}\times$ $\times\cos(xa\sin\psi),\quad a>0$ $0<\varphi,\ \psi<\pi/2,\ \operatorname{Re}\nu>0$	$a^{1/2}\nu^{-1}(\sin\varphi)^{1/2}(\operatorname{tg}\varphi/2)^\nu\cos(\nu\psi)$, $y=a\sin\varphi$
(36)	$x^{\nu+1/2}e^{-xa\cos\varphi\cos\psi}\times$ $\times\cos(xa\sin\psi),\quad a>0$ $0<\varphi,\ \psi<\pi/2,\ \operatorname{Re}\nu>-1$	$2^{\nu+1}\pi^{-1/2}\Gamma(\nu+3/2)\times$ $\times a^{-\nu-3/2}(\sin\varphi)^{\nu+1/2}\times$ $\times(\cos^2\psi+\sin^2\psi\cos^2\varphi)^{-\nu-3/2}\times$ $\times\cos[(\nu+3/2)\alpha],\quad y=a\sin\varphi$ $\operatorname{tg}(\alpha/2)=\operatorname{tg}\psi\cos\varphi$
(37)	$x^{\nu-1/2}e^{-xa\cos\varphi\cos\psi}\times$ $\times\cos(xa\sin\psi),\quad a>0$ $0<\varphi,\ \psi<\pi/2,\ \operatorname{Re}\nu>-1/2$	$2^\nu\pi^{-1/2}a^{-\nu-1/2}\Gamma(\nu+1/2)\times$ $\times(\sin\varphi)^{\nu+1/2}\times$ $\times(\cos^2\psi+\sin^2\psi\cos^2\varphi)^{-\nu-1/2}\times$ $\times\cos[(\nu+1/2)\alpha],\quad y=a\sin\varphi$ $\operatorname{tg}(\alpha/2)=\operatorname{tg}\psi\cos\varphi$

	$f(x)$	$\int\limits_0^\infty f(x)\, J_\nu(xy)\,(xy)^{1/2}\,dx, \quad y>0$
(38)	$x^{-1/2}\cos(ax^2)$, $a>0,\ \operatorname{Re}\nu>-1$	$\frac{\pi^{1/2}y^{1/2}}{2a^{1/2}}\cos\left(\frac{y^2}{8a}-\frac{\nu+1}{4}\pi\right)\times J_{\nu/2}\left(\frac{y^2}{8a}\right)$
(39)	$x^{1/2}\cos(ax^2)$, $a>0,\ \operatorname{Re}\nu>-2$	$\frac{\pi^{1/2}y^{3/2}}{8a^{3/2}}\left[\cos\left(\frac{y^2}{8a}-\frac{\nu\pi}{4}\right)\times J_{\nu/2+1/2}\left(\frac{y^2}{8a}\right)+\sin\left(\frac{y^2}{8a}-\frac{\nu\pi}{4}\right)J_{\nu/2-1/2}\left(\frac{y^2}{8a}\right)\right]$
(40)	$x^{\nu+1/2}\cos(ax^2)$, $-1<\operatorname{Re}\nu<1/2$	$\frac{y^{\nu+1/2}}{2^{\nu+1}a^{\nu+1}}\sin\left(\frac{y^2}{4a}-\frac{\nu\pi}{2}\right)$
(41)	$x^{\nu+1/2}\cos(ax^2)$, $0<x<b$ 0, $b<x<\infty$ $\operatorname{Re}\nu>-1$	$(2a)^{-\nu-1}y^{\nu+1/2}\times[\sin(ab^2)\,U_{\nu+2}(2ab^2, by)+\cos(ab^2)\,U_{\nu+1}(2ab^2, by)]$
(42)	$x^{-1}\exp[-ax^{1/2}]\cos(ax^{1/2})$, $a>0,\ \operatorname{Re}\nu>-1/2$	$\frac{\Gamma(\nu+1/2)\,D_{-\nu-1/2}(ay^{-1/2})}{2^{1/2}\pi^{1/2}}\times\left[D_{-\nu-1/2}\left(\frac{ia}{y^{1/2}}\right)+D_{-\nu-1/2}\left(-\frac{ia}{y^{1/2}}\right)\right]$
(43)	$x^{\nu+1/2}\cos[a(x^2+\beta^2)^{1/2}]$, $a>0$ $\operatorname{Re}\beta>0,\ -1<\operatorname{Re}\nu<-1/2$	$(\pi/2)^{1/2}a\beta^{\nu+3/2}y^{\nu+1/2}\times(a^2-y^2)^{-\nu/2-3/4}\times\{\cos(\pi\nu)\,J_{\nu+3/2}[\beta(a^2-y^2)^{1/2}]-\sin(\pi\nu)\,Y_{\nu+3/2}[\beta(a^2-y^2)^{1/2}]\}$, $0<y<a$ 0, $a<y<\infty$
(44)	$x^{-1/2}(x^2+\beta^2)^{-1/2}\times\cos[a(x^2+\beta^2)^{1/2}]$, $a>0$ $\operatorname{Re}\beta>0,\ \operatorname{Re}\nu>-1$	$2^{-1}\pi y^{1/2}\times J_{\nu/2}\{2^{-1}\beta[a-(a^2-y^2)^{1/2}]\}\times Y_{-\nu/2}\{2^{-1}\beta[a+(a^2-y^2)^{1/2}]\}$, $0<y<a$

	$f(x)$	$\int\limits_0^\infty f(x)\, J_\nu(xy)\,(xy)^{1/2}\,dx, \quad y>0$
(45)	$x^{\nu+1/2}(x^2+\beta^2)^{-1/2}\times$ $\times\cos[a(x^2+\beta^2)^{1/2}], \quad a>0$ $\operatorname{Re}\beta>0, \ -1<\operatorname{Re}\nu<1/2$	$-\dfrac{\pi^{1/2}\beta^{1/2+\nu}y^{1/2+\nu}}{2^{1/2}(a^2-y^2)^{1/4+\nu/2}}\times$ $\times Y_{-\nu-1/2}[\beta(a^2-y^2)^{1/2}],$ $0<y<a$ $2^{1/2}\pi^{-1/2}\beta^{1/2+\nu}y^{1/2+\nu}(y^2-a^2)^{-1/4-\nu/2}\times$ $\times K_{\nu+1/2}[\beta(y^2-a^2)^{1/2}],$ $a<y<\infty$
(46)	$x^{\nu+1/2}(x^2+b^2)^{-3/2}\times$ $\times\cos[a(x^2+b^2)^{1/2}], \quad a>0$ $b>0, \ -1<\operatorname{Re}\nu<5/2$	$y^{1/2}b^{\nu}K_\nu(by), \qquad y>a$
(47)	$x^{-1/2}(a^2-x^2)^{-1/2}\cos[b(a^2-x^2)^{1/2}],$ $0<x<a$ $0, \qquad a<x<\infty$ $\operatorname{Re}\nu>-1$	$2^{-1}\pi y^{1/2}\times$ $\times J_{\nu/2}\{2^{-1}a[(b^2+y^2)^{1/2}-b]\}\times$ $\times J_{\nu/2}\{2^{-1}a[(b^2+y^2)^{1/2}+b]\}$
(48)	$x^{\nu+1/2}(a^2-x^2)^{-1/2}\times$ $\times\cos[b(a^2-x^2)^{1/2}], \quad 0<x<a$ $0, \qquad a<x<\infty$ $\operatorname{Re}\nu>-1$	$2^{-1/2}\pi^{1/2}a^{\nu+1/2}\times$ $\times y^{\nu+1/2}(b^2+y^2)^{-\nu/2-1/4}\times$ $\times J_{\nu+1/2}[a(b^2+y^2)^{1/2}]$
(49)	$0, \qquad 0<x<a$ $x^{1/2-\nu}(x^2-a^2)^{-1/2}\times$ $\times\cos[b(x^2-a^2)^{1/2}], \quad a<x<\infty$ $b>0, \ \operatorname{Re}\nu>-1/2$	$0, \qquad 0<y<b$ $2^{-1/2}\pi^{1/2}a^{1/2-\nu}\times$ $\times y^{1/2-\nu}(y^2-b^2)^{\nu/2-1/4}\times$ $\times J_{\nu-1/2}[a(y^2-b^2)^{1/2}], \ b<y<\infty$
(50)	$x^{1/2-\nu}(x^2+\beta^2)^{-1/2}\times$ $\times[(x^2+\beta^2)^{1/2}-\beta]^{\nu}\times$ $\times\cos[a(x^2+\beta^2)^{1/2}],$ $a>0$ $\operatorname{Re}\beta>0, \ \operatorname{Re}\nu>-1$	$-y^{\nu+1/2}[a+(a^2-y^2)^{1/2}]^{-\nu}\times$ $\times(a^2-y^2)^{-1/2}\times$ $\times\sin[\beta(a^2-y^2)^{1/2}+2^{-1}\pi\nu],$ $0<y<a$ $y^{-1/2}(y^2-a^2)^{-1/2}\times$ $\times\exp[-\beta(y^2-a^2)^{1/2}]\times$ $\times\cos[\nu\arcsin(a/y)],$ $a<y<\infty$

	$f(x)$	$\int\limits_0^\infty f(x)\,J_\nu(xy)\,(xy)^{1/2}\,dx, \quad y>0$
(51)	$0, \quad 0<x<c$ $x^{1/2-\nu}(x^2+b^2)^{-1}(x^2-c^2)^{-1/2}\times$ $\times\cos\left[a(x^2-c^2)^{1/2}\right], \quad c<x<\infty$ $\operatorname{Re}\nu>-5/2$	$2^{-1}\pi y^{1/2}b^{-\nu}(c^2+b^2)^{-1/2}\times$ $\times\exp\left[-a(c^2+b^2)^{1/2}\right]I_\nu(by),$ $0<y<a$

Относительно преобразований Ганкеля других функций, содержащих косинус, см. таблицы косинус-преобразований Фурье.

(52)	$x^{-1/2}(1-x^2)^{-1/2}\times$ $\times\cos[(\nu-1)\ \mathrm{rccos}\,x],$ $0<x<1$ $0, \quad 1<x<\infty$ $\operatorname{Re}\nu>0$	$\pi^{1/2}\sin(y/2)\,J_{\nu-1/2}(y/2)$
(53)	$x^{-1/2}(1-x^2)^{-1/2}\times$ $\times\cos[(\nu+1)\arccos x],$ $0<x<1$ $0, \quad 1<x<\infty$ $\operatorname{Re}\nu>-1$	$\pi^{1/2}\cos(y/2)\,J_{\nu+1/2}(y/2)$
(54)	$x^{-1/2}(1-x^2)^{-1/2}\cos(\mu\arccos x),$ $0<x<1$ $0, \quad 1<x<\infty$ $\operatorname{Re}(\mu+\nu)>-1$	$2^{-1}\pi y^{1/2}J_{(\mu+\nu)/2}(y/2)\,J_{(\nu-\mu)/2}(y/2)$
(55)	$0, \quad 0<x<1$ $x^{1/2}(x^2-1)^{-1/2}\cos(\nu\arccos x^{-1}),$ $1<x<\infty$ $\operatorname{Re}\nu>-1$	$y^{-1/2}\cos(y-2^{-1}\nu\pi)$

8.8. Гиперболические и обратные гиперболические функции

(1)	$\dfrac{2x^{\nu-1/2}}{e^{\pi x}-1},$ $\operatorname{Re}\nu>-1/2$	$\pi^{-1/2}2^{\nu+1}\Gamma(\nu+1/2)\,y^{\nu+1/2}\times$ $\times\sum\limits_{n=1}^{\infty}(n^2\pi^2+y^2)^{-\nu-1/2}$
(2)	$x^{1/2}\dfrac{x\operatorname{ch}x+\operatorname{sh}x}{\operatorname{sh}(2x)+2x}$	Относительно этого и подобных интегралов см. Boit M. A., 1935: J. Appl. Phys. 6, 337—375.

	$f(x)$	$\int\limits_0^\infty f(x)\,J_\nu(xy)\,(xy)^{1/2}\,dx, \quad y>0$
(3)	$\dfrac{x^{\nu+1/2}\,\mathrm{sh}\,(\alpha x)}{\mathrm{sh}\,(\pi x)}$, $\lvert\mathrm{Re}\,\alpha\rvert<\pi, \quad \mathrm{Re}\,\nu>-1$	$2\pi^{-1}y^{1/2}\sum\limits_{n=1}^{\infty}(-1)^{n-1}n^{\nu+1}\sin(n\alpha)\times$ $\times K_\nu(ny)$
Относительно других подобных интегралов см. Weber H., 1873: J. of Math. 75, 75—105.		
(4)	$x^{-1/2}(1+x^2)^{-1/2}\,\mathrm{sh}\,(2\mu\,\mathrm{arsh}\,x)$, $\mathrm{Re}\,\nu>-1, \quad \lvert\mathrm{Re}\,\mu\rvert<3/4$	$2^{-1}y^{1/2}\left[I_{\nu/2-\mu}(y/2)\,K_{\nu/2+\mu}(y/2)-\right.$ $\left.-I_{\nu/2+\mu}(y/2)\,K_{\nu/2-\mu}(y/2)\right]$
(5)	$x^{-1/2}(1+x^2)^{-1/2}\,\mathrm{ch}\,(2\mu\,\mathrm{arsh}\,x)$, $\mathrm{Re}\,\nu>-1, \quad \lvert\mathrm{Re}\,\mu\rvert<3/4$	$2^{-1}y^{1/2}\left[I_{\nu/2-\mu}(y/2)\,K_{\nu/2+\mu}(y/2)+\right.$ $\left.+I_{\nu/2+\mu}(y/2)\,K_{\nu/2-\mu}(y/2)\right]$
(6)	$0, \quad 0<x<1$ $x^{-1/2}(x^2-1)^{-1/2}\times$ $\times\,\mathrm{ch}\,[(\nu-1)\,\mathrm{arch}\,x], \quad 1<x<\infty$ $-1/2<\mathrm{Re}\,\nu<5/2$	$2^{-1}\pi^{1/2}\left[\cos(y/2)\,J_{\nu-1/2}(y/2)-\right.$ $\left.-\sin(y/2)\,Y_{\nu-1/2}(y/2)\right]$
(7)	$0, \quad 0<x<1$ $x^{-1/2}(x^2-1)^{-1/2}\times$ $\times\,\mathrm{ch}\,[(\nu+1)\,\mathrm{arch}\,x], \quad 1<x<\infty$ $-5/2<\mathrm{Re}\,\nu<1/2$	$-2^{-1}\pi^{1/2}\left[\sin(y/2)\,J_{\nu+1/2}(y/2)+\right.$ $\left.+\cos(y/2)\,Y_{\nu+1/2}(y/2)\right]$
(8)	$0, \quad 0<x<1$ $x^{-1/2}(x^2-1)^{-1/2}\,\mathrm{ch}\,(\mu\,\mathrm{arch}\,x), \quad 1<x<\infty$ $\lvert\mathrm{Re}\,\mu\rvert<3/2$	$-2^{-2}\pi y^{1/2}\times$ $\times\left[J_{(\nu+\mu)/2}(y/2)\,Y_{(\nu-\mu)/2}(y/2)+\right.$ $\left.+J_{(\nu-\mu)/2}(y/2)\,Y_{(\nu+\mu)/2}(y/2)\right]$

8.9. Ортогональные многочлены

(1)	$x^{-1/2}(1-x^2)^{-1/2}\,T_n(x), \quad 0<x<1$ $0, \quad 1<x<\infty$ $\mathrm{Re}\,\nu>-n-1$	$2^{-1}\pi y^{1/2}\times$ $\times J_{(\nu+n)/2}(y/2)\,J_{(\nu-n)/2}(y/2)$
(2)	$x^{\nu+1/2}\exp(-x^2)\,L_n^\nu(x^2)$, $\mathrm{Re}\,\nu>-1$	$2^{-2n-\nu-1}(n!)^{-1}y^{2n+\nu+1/2}\times$ $\times\exp(-y^2/4)$

	$f(x)$	$\int_0^\infty f(x) J_\nu(xy)(xy)^{1/2}\,dx, \quad y>0$
(3)	$x^{\nu+1/2}\exp(-x^2/2)\,L_n^\nu(x^2)$, $\operatorname{Re}\nu>-1$	$(-1)^n\exp(-y^2/2)\,y^{\nu+1/2}L_n^\nu(y^2)$
(4)	$x^{2n+\nu+1/2}\exp(-x^2/2)\times$ $\times L_n^{\nu+n}(x^2/2)$, $\operatorname{Re}\nu>-1$	$y^{2n+\nu+1/2}\exp(-y^2/2)\,L_n^{\nu+n}(y^2/2)$
(5)	$x^{\nu+1/2}\exp(-\beta x^2)\,L_n^\nu(\alpha x^2)$, $\operatorname{Re}\beta>0,\ \operatorname{Re}\nu>0$	$2^{-\nu-1}\beta^{-\nu-n-1}(\beta-\alpha)^n\,y^{\nu+1/2}\times$ $\times\exp\left(-\frac{y^2}{4\beta}\right)L_n^\nu\left[\frac{\alpha y^2}{4\beta(\alpha-\beta)}\right]$
(6)	$x^{\nu+1/2}\exp(-\alpha x^2)\left[L_n^{\nu/2}(\alpha x^2)\right]^2$, $\operatorname{Re}\alpha>0,\ \operatorname{Re}\nu>-1$	$(2\alpha)^{-\nu-1}y^{\nu+1/2}\times$ $\times\exp\left(-\frac{y^2}{4\alpha}\right)\left[L_n^{\nu/2}\left(\frac{y^2}{4\alpha}\right)\right]^2$
(7)	$x^{\nu+1/2}\exp(-\beta x^2)\left[L_n^{\nu/2}(\alpha x^2)\right]^2$, $\operatorname{Re}\beta>0,\ \operatorname{Re}\nu>-1$	$\frac{y^{1/2+\nu}}{\pi n!}\Gamma(n+1+\nu/2)(2\beta)^{-\nu-1}\times$ $\times\exp\left(-\frac{y^2}{4\beta}\right)\times$ $\times\sum_{l=0}^{n}\frac{(-1)^l\,\Gamma(n-l+1/2)\,\Gamma(l+1/2)}{\Gamma(l+1+\nu/2)\,(n-l)!}\times$ $\times\left(\frac{2\alpha-\beta}{\beta}\right)^{2l}L_{2l}^\nu\left[\frac{\alpha y^2}{2\beta(2\alpha-\beta)}\right]$
(8)	$x^{\nu+1/2}\exp(-\alpha x^2)\,L_m^{\nu-\sigma}(\alpha x^2)\times$ $\times L_n^\sigma(\alpha x^2)$, $\operatorname{Re}\alpha>0,\ \operatorname{Re}\nu>-1$	$(-1)^{m+n}(2\alpha)^{-\nu-1}y^{\nu+1/2}\times$ $\times\exp\left(-\frac{y^2}{4\alpha}\right)L_n^{\sigma-m+n}\left(\frac{y^2}{4\alpha}\right)\times$ $\times L_m^{\nu-\sigma+m-n}\left(\frac{y^2}{4\alpha}\right)$
(9)	$x^{\nu+1/2}\exp(-x^2)\,L_n^\sigma(x^2)\,L_n^{\nu-\sigma}(x^2)$, $\operatorname{Re}\nu>-1$	$2^{-\nu-1}y^{\nu+1/2}\exp(-y^2/4)\times$ $\times L_n^\sigma(y^2/4)\,L_n^{\nu-\sigma}(y^2/4)$
(10)	$0,\quad 0<x<a$ $x^{2n+1/2-\nu}(x^2-a^2)^{\nu-2n-1/2}\times$ $\times C_{2n}^{\nu-2n}(a/x),\quad a<x<\infty$ $2n-1/2<\operatorname{Re}\nu<2n+1/2$	$(-1)^n 2^{2n-\nu+1}\Gamma(2\nu-2n)\times$ $\times[(2n)!\,\Gamma(\nu-2n)]^{-1}\times$ $\times y^{-\nu+2n-1/2}\cos(ay)$

	$f(x)$	$\int\limits_0^\infty f(x)\, J_\nu(xy)\,(xy)^{1/2}\,dx, \quad y>0$
(11)	$0, \qquad 0<x<a$ $x^{2n-\nu+3/2}\left(x^2-a^2\right)^{\nu-2n-3/2} \times$ $\times\, C_{2n+1}^{\nu-2n-1}(a/x), \quad a<x<\infty$ $2n+1/2<\operatorname{Re}\nu<2n+3/2$	$(-1)^n\, 2^{2n-\nu+2}\,\Gamma(2\nu-2n-1)\times$ $\times\,[(2n+1)!\,\Gamma(\nu-2n-1)]^{-1}\times$ $\times\, y^{-\nu+2n+1/2}\sin(ay)$
(12)	$x^{\nu+1/2}\left(1-x^2\right)^{-1/2}\times$ $\times\sin\left[\alpha\left(1-x^2\right)^{1/2}\right]\times$ $\times\, C_{2n+1}^{\nu+1/2}\left[\left(1-x^2\right)^{1/2}\right], \quad 0<x<1$ $0, \qquad 1<x<\infty$ $\operatorname{Re}\alpha>0, \qquad \operatorname{Re}\nu>-1/2$	$(-1)^n\, 2^{-1/2}\pi^{1/2}\times$ $\times\, y^{\nu+1/2}\left(\alpha^2+y^2\right)^{-\nu/2-1/4}\times$ $\times\, C_{2n+1}^{\nu+1/2}\left[\alpha\left(y^2+\alpha^2\right)^{-1/2}\right]\times$ $\times\, J_{\nu+3/2+2n}\left[\left(\alpha^2+y^2\right)^{1/2}\right]$
(13)	$x^{\nu+1/2}\left(1-x^2\right)^{-1/2}\times$ $\times\cos\left[\alpha\left(1-x^2\right)^{1/2}\right]\times$ $\times\, C_{2n}^{\nu+1/2}\left[\left(1-x^2\right)^{1/2}\right], \quad 0<x<1$ $0, \qquad 1<x<\infty$ $\operatorname{Re}\alpha>0, \qquad \operatorname{Re}\nu>-1/2$	$(-1)^n\, 2^{-1/2}\pi^{1/2}\times$ $\times\, y^{\nu+1/2}\left(\alpha^2+y^2\right)^{-\nu/2-1/4}\times$ $\times\, C_{2n}^{\nu+1/2}\left[\alpha\left(y^2+\alpha^2\right)^{-1/2}\right]\times$ $\times\, J_{\nu+1/2+2n}\left[\left(\alpha^2+y^2\right)^{1/2}\right]$

8.10. Функции Лежандра

(1)	$\left(x^2+2\right)^{-\nu/2-1/4} P_\mu^{-\nu-1/2}\left(x^2+1\right)$, $\operatorname{Re}\nu>-1$ $-3/2-\operatorname{Re}\nu<\operatorname{Re}\mu<\operatorname{Re}\nu+1/2$	$\dfrac{2^{1/2-\nu}\pi^{-1/2}\left[K_{\mu+1/2}\left(2^{-1/2}y\right)\right]^2}{\Gamma(\nu+\mu+3/2)\,\Gamma(\nu-\mu+1/2)}$
(2)	$0, \qquad 0<x<a$ $\left(x^2-a^2\right)^{\mu/2-1/4}\, \mathrm{P}_{-1/2+\nu}^{1/2-\mu}\left(ax^{-1}\right)$, $a<x<\infty$ $\lvert\operatorname{Re}\mu\rvert<1/2, \quad \operatorname{Re}\nu>-1$	$2^{1/2}\pi^{-1/2}y^{-\mu-1/2}\times$ $\times\cos\left[ay+2^{-1}(\nu-\mu)\pi\right]$
(3)	$x^{\nu-1/2}\left(1-x^2\right)^{\nu/2+1/4}\times$ $\times\, P_\mu^{-\nu-1/2}\left(2x^{-2}-1\right), \quad 0<x<1$ $0, \qquad 1<x<\infty$ $-3/2-\operatorname{Re}\nu<\operatorname{Re}\mu<\operatorname{Re}\nu+1/2$	$\dfrac{\Gamma(3/2+\mu+\nu)\,\Gamma(1/2+\nu-\mu)\,(2y)^{\nu+1/2}}{(2\pi)^{1/2}\left[\Gamma(3/2+\nu)\right]^2}\times$ $\times\, {}_1F_1(\nu+\mu+3/2;\ 2\nu+2;\ iy)\times$ $\times\, {}_1F_1(\nu+\mu+3/2;\ 2\nu+2;\ -iy)$
(4)	$x^{1/2}\left(\alpha^2+x^2\right)^{-\mu/2}\times$ $\times\, \mathrm{P}_{\mu-1}^{-\nu}\left[\alpha\left(\alpha^2+x^2\right)^{-1/2}\right], \quad \operatorname{Re}\alpha>0$ $\operatorname{Re}\nu>-1, \quad \operatorname{Re}\mu>1/2$	$\dfrac{y^{\mu-3/2}e^{-\alpha y}}{\Gamma(\mu+\nu)}$

	$f(x)$	$\int\limits_0^\infty f(x)\, J_\nu(xy)\,(xy)^{1/2}\,dx,\quad y>0$
(5)	$x^{\nu+1/2}(x^2+\alpha^2)^{\nu/2}\times$ $\times P_\nu\left[\frac{x^2+2\alpha^2}{2\alpha\,(x^2+\alpha^2)^{1/2}}\right]$, $\operatorname{Re}\alpha>0,\ -1<\operatorname{Re}\nu<0$	$\frac{(2\alpha)^{\nu+1}y^{-\nu-1/2}}{\pi\,\Gamma(-\nu)}\left[K_{\nu+1/2}(2^{-1}\alpha y)\right]^2$
(6)	$x^{1/2-\nu}(x^2+\alpha^2)^{-\nu/2}\times$ $\times P_{\nu-1}\left[\frac{x^2+2\alpha^2}{2\alpha\,(x^2+\alpha^2)^{1/2}}\right]$, $\operatorname{Re}\alpha>0,\ 0<\operatorname{Re}\nu<1$	$\frac{(2\alpha)^{1-\nu}}{\Gamma(\nu)}\,y^{\nu-1/2}\,I_{\nu-1/2}(2^{-1}\alpha y)\times$ $\times K_{\nu-1/2}(2^{-1}\alpha y)$
(7)	$x^{1/2}\left\{P_\mu^{-\nu/2}\left[(1+\alpha^2x^2)^{1/2}\right]\right\}^2$, $\operatorname{Re}\alpha>0$ $-3/4<\operatorname{Re}\mu<-1/4,\ \operatorname{Re}\nu>-1$	$\frac{2\left[K_{\mu+1/2}(2^{-1}\alpha^{-1}y)\right]^2}{\pi\alpha\,\Gamma(1+\mu+\nu/2)\,\Gamma(\nu/2-\mu)\,y^{1/2}}$
(8)	$x^{1/2}(1+\alpha^2x^2)^{-1/2}\times$ $\times P_\mu^{-\nu/2}\left[(1+\alpha^2x^2)^{1/2}\right]\times$ $\times P_{\mu+1}^{-\nu/2}\left[(1+\alpha^2x^2)^{1/2}\right]$, $\operatorname{Re}\nu>-1,\ \operatorname{Re}\alpha>0$ $-7/4<\operatorname{Re}\mu<-1/4$	$\frac{y^{1/2}K_{\mu+1/2}(2^{-1}\alpha^{-1}y)\,K_{\mu+3/2}(2^{-1}\alpha^{-1}y)}{\pi\alpha^2\,\Gamma(2+\nu/2+\mu)\,\Gamma(\nu/2-\mu)}$
(9)	$x^{1/2}(1+\alpha^2x^2)^{-1/2}\times$ $\times P_\mu^{-1/2-\nu/2}\left[(1+\alpha^2x^2)^{1/2}\right]\times$ $\times P_\mu^{1/2-\nu/2}\left[(1+\alpha^2x^2)^{1/2}\right]$, $\operatorname{Re}\nu>-1,\ \operatorname{Re}\alpha>0$ $-5/4<\operatorname{Re}\mu<1/4$	$\frac{y^{1/2}\left[K_{\mu+1/2}(2^{-1}\alpha^{-1}y)\right]^2}{\pi\alpha^2\,\Gamma(\nu/2+\mu+3/2)\,\Gamma(\nu/2-\mu+1/2)}$
(10)	$Q_{\nu-1/2}\left[(a^2+x^2)\,x^{-1}\right]$, $\operatorname{Re}\nu>-1/2$	$2^{-1/2}\pi y^{-1/2}\exp\left[-(a^2-1/4)^{1/2}y\right]\times$ $\times J_\nu(y/2)$
(11)	$x^{1/2-\mu}(1+\alpha^2x^2)^{-\mu/2-1/4}\times$ $\times Q_{\nu-1/2}^{\mu+1/2}(\pm i\alpha x),\quad \operatorname{Re}\alpha>0$ $-3/4-2^{-1}\operatorname{Re}\nu<\operatorname{Re}\mu<$ $<1+\operatorname{Re}\nu$	$i\,(2\pi)^{1/2}\,e^{i\pi(\mu\mp\nu/2\mp1/4)}\,\alpha^{-1}y^{\mu-1/2}\times$ $\times I_\nu(2^{-1}\alpha^{-1}y)\,K_\mu(2^{-1}\alpha^{-1}y)$
(12)	$(x^2+2)^{-\nu/2-1/4}Q_\mu^{\nu+1/2}(x^2+1)$, $\operatorname{Re}\nu>-1$, $\operatorname{Re}(2\mu+\nu)>-5/2$	$2^{-\nu-1/2}\pi^{1/2}e^{(\nu+1/2)\pi i}y^{\nu+1/2}\times$ $\times K_{\mu+1/2}(2^{-1/2}y)\,I_{\mu+1/2}(2^{-1/2}y)$

	$f(x)$	$\int_0^\infty f(x)\, J_\nu(xy)\,(xy)^{1/2}\,dx, \quad y>0$
(13)	$x^{-\nu-1/2}\,Q_{-1/2}^{\nu-1/2}(1+2\alpha^2/x^2),$ $\operatorname{Re}\alpha>0,\ 0<\operatorname{Re}\nu<3/2$	$-ie^{i\pi\nu}\pi^{1/2}2^{-\nu}\,(y/\alpha)^{\nu-1/2}\times$ $\times I_{\nu-1/2}(2^{-1}\alpha y)\,K_{\nu-1/2}(2^{-1}\alpha y)$
(14)	$x^{\nu-1/2}\,(\alpha^2+x^2)^{1/4+\nu/2}\times$ $\times Q_\mu^{\nu+1/2}(1+2\alpha^2/x^2),$ $\operatorname{Re}\alpha>0$ $\operatorname{Re}(\mu+\nu)>-3/2$ $\operatorname{Re}(\nu-\mu)>-1/2$	$-ie^{i\pi\nu}\pi^{-1/2}2^{\nu}\,[\Gamma(3/2+\mu+\nu)]^2\times$ $\times\Gamma(1/2+\nu-\mu)\,\alpha^{\nu-1/2}y^{-\nu-3/2}\times$ $\times W_{-\mu-1/2,\,\nu+1/2}(\alpha y)\times$ $\times\Big[\frac{\cos(\mu\pi)}{\Gamma(2+2\nu)}\,M_{\mu+1/2,\,\nu+1/2}(\alpha y)+$ $+\frac{\sin(\pi\nu)}{\Gamma(\nu+\mu+3/2)}\,W_{\mu+1/2,\,\nu+1/2}(\alpha y)\Big]$
(15)	$x^{-\nu-1/2}\,(x^2+\alpha^2)^{1/4-\nu/2}\times$ $\times Q_\mu^{1/2-\nu}(1+2\alpha^2/x^2),$ $\operatorname{Re}\alpha>0$ $0<\operatorname{Re}\nu<\operatorname{Re}\mu+3/2$	$\frac{ie^{-i\nu\pi}\pi^{1/2}\,\Gamma(3/2+\mu-\nu)}{2^{\nu}\alpha^{\nu+1/2}\,\Gamma(2\nu)}\,y^{-\nu-3/2}\times$ $\times M_{\mu+1/2,\,\nu-1/2}(\alpha y)\times$ $\times W_{-\mu-1/2,\,\nu-1/2}(\alpha y)$
(16)	$x^{1/2}\,P_\mu^{-\nu/2}\big[(1+\alpha^2x^2)^{1/2}\big]\times$ $\times Q_\mu^{-\nu/2}\big[(1+\alpha^2x^2)^{1/2}\big],$ $\operatorname{Re}\alpha>0$ $\operatorname{Re}\mu>-3/4,\ \operatorname{Re}\nu>-1$	$\frac{\exp(-2^{-1}\nu\pi i)\,\Gamma(1+\mu+\nu/2)}{\alpha\,\Gamma(1+\mu-\nu/2)\,y^{1/2}}\times$ $\times I_{\mu+1/2}\left(\frac{y}{2\alpha}\right)K_{\mu+1/2}\left(\frac{y}{2\alpha}\right)$

8.11. Функции Бесселя аргумента kx

(1)	$x^{-1/2}\,J_{\nu-1}(ax),$ $a>0,\ \operatorname{Re}\nu>-1$	$0, \qquad 0<y<a$ $a^{\nu-1}y^{-\nu+1/2}, \qquad a<y<\infty$
(2)	$x^{-3/2}\,J_\nu(ax),$ $a>0,\ \operatorname{Re}\nu>0$	$2^{-1}\nu^{-1}a^{-\nu}y^{\nu+1/2}, \qquad 0<y\leqslant a$ $2^{-1}\nu^{-1}a^{\nu}y^{-\nu+1/2}, \qquad a\leqslant y<\infty$
(3)	$x^{-1/2}\,J_{\nu+1}(ax),$ $a>0,\ \operatorname{Re}\nu>-3/2$	$a^{-\nu-1}y^{\nu+1/2}, \qquad 0<y<a$ $0, \qquad a<y<\infty$
(4)	$x^{-2\lambda-1/2}\,J_\nu(ax), \quad a>0$ $\operatorname{Re}\nu+1/2>\operatorname{Re}\lambda>-1/2$	$\frac{a^{\nu}y^{\nu+1/2}\,\Gamma(\nu-\lambda+1/2)}{2^{2\lambda}\,(a+y)^{2\nu-2\lambda+1}\,\Gamma(\nu+1)\,\Gamma(\lambda+1/2)}\times$ $\times {}_2F_1\Big[\nu-\lambda+\frac{1}{2},\ \nu+\frac{1}{2};$ $2\nu+1;\ \frac{4ay}{(a+y)^2}\Big]$

	$f(x)$	$\int_0^\infty f(x) J_\nu(xy)(xy)^{1/2}\,dx, \quad y>0$
(5)	$x^{-1/2} J_{\nu+2n+1}(ax)$ $a>0,\ \operatorname{Re}\nu>-1-n$	$y^{\nu+1/2} a^{-\nu-1} P_n^{(\nu,0)}(1-2y^2/a^2)$, $\quad 0<y<a$ $0, \quad a<y<\infty$
(6)	$x^{-1/2} J_\mu(ax)$, $a>0,\ \operatorname{Re}(\mu+\nu)>-1$	$y^{\nu+1/2} a^{-\nu-1} \dfrac{\Gamma(\mu/2+\nu/2+1/2)}{\Gamma(\nu+1)\,\Gamma(\mu/2-\nu/2+1/2)} \times$ $\times {}_2F_1\left(\dfrac{\mu+\nu+1}{2}, \dfrac{\nu-\mu+1}{2}; \nu+1; \dfrac{y^2}{a^2}\right)$, $\quad 0<y<a$ При $y>a$ переставить местами μ и ν.
(7)	$x^{\nu-\mu+1/2} J_\mu(ax)$, $a>0,\ -1<\operatorname{Re}\nu<\operatorname{Re}\mu$	$\dfrac{2^{\nu-\mu+1} y^{\nu+1/2}}{\Gamma(\mu-\nu)\,a^\mu}(a^2-y^2)^{\mu-\nu-1}$, $\quad 0<y<a$ $0, \quad a<y<\infty$
(8)	$x^{\mu-\nu+1/2} J_\mu(ax)$, $a>0,\ \operatorname{Re}\nu>\operatorname{Re}\mu>-1$	$0, \quad 0<y<a$ $\dfrac{2^{\mu-\nu+1} a^\mu}{\Gamma(\nu-\mu)\,y^{\nu-1/2}}(y^2-a^2)^{\nu-\mu-1}$, $\quad a<y<\infty$
(9)	$x^{-\lambda-1/2} J_\mu(ax)$, $\quad a>0$ $\operatorname{Re}(\mu+\nu)+1>\operatorname{Re}\lambda>-1$	$\dfrac{\Gamma[2^{-1}(\mu+\nu-\lambda+1)]\,a^{\lambda-\nu-1}\,y^{\nu+1/2}}{2^\lambda\,\Gamma(\nu+1)\,\Gamma[2^{-1}(\lambda+\mu-\nu+1)]} \times$ $\times {}_2F_1\left(\dfrac{\mu+\nu-\lambda+1}{2}, \dfrac{\nu-\lambda-\mu+1}{2}; \nu+1; \dfrac{y^2}{a^2}\right)$, $\quad 0<y<a$ $\dfrac{\Gamma[2^{-1}(\mu+\nu-\lambda+1)]\,a^\mu y^{\lambda-\mu-1/2}}{2^\lambda\,\Gamma(\mu+1)\,\Gamma[2^{-1}(\lambda+\nu-\mu+1)]} \times$ $\times {}_2F_1\left(\dfrac{\mu+\nu-\lambda+1}{2}, \dfrac{\mu-\lambda-\nu+1}{2}; \mu+1; \dfrac{a^2}{y^2}\right)$, $\quad a<y<\infty$
(10)	$x^{1/2}(x^2+\beta^2)^{-1} J_\nu(ax)$, $a>0$ $\operatorname{Re}\beta>0,\ \operatorname{Re}\nu>-1$	$y^{1/2} I_\nu(y\beta) K_\nu(a\beta), \quad 0<y<a$ $y^{1/2} I_\nu(a\beta) K_\nu(y\beta), \quad a<y<\infty$

	$f(x)$	$\int_0^\infty f(x)\, J_\nu(xy)\,(xy)^{1/2}\,dx, \quad y>0$
(11)	$x^{1/2-2n}(\beta^2+x^2)^{-1} J_\nu(ax)$, $a>0$, $\operatorname{Re}\beta>0$, $\operatorname{Re}\nu>n-1$, $n=0, 1, 2, \ldots$	$(-1)^n \beta^{-2n} y^{1/2} I_\nu(y\beta) K_\nu(a\beta)$, $0<y<a$; $(-1)^n \beta^{-2n} y^{1/2} I_\nu(a\beta) K_\nu(y\beta)$, $a<y<\infty$
(12)	$x^{\nu-\mu+1/2}(\beta^2+x^2)^{-1} J_\mu(ax)$, $a>0$, $\operatorname{Re}\beta>0$, $1+\operatorname{Re}\mu>\operatorname{Re}\nu>-1$	$\beta^{\nu-\mu} y^{1/2} I_\mu(a\beta) K_\nu(y\beta)$, $a<y<\infty$
(13)	$x^{\nu-\mu+2n+1/2}(\beta^2+x^2)^{-1} J_\mu(ax)$, $a>0$, $\operatorname{Re}\beta>0$, $\operatorname{Re}\mu-2n+1>\operatorname{Re}\nu>-n-1$, $n=0, 1, 2, \ldots$	$(-1)^n \beta^{\nu-\mu+2n} y^{1/2} I_\mu(a\beta) K_\nu(y\beta)$, $a<y<\infty$
(14)	$x^{\mu-\nu+1/2}(\beta^2+x^2)^{-1} J_\mu(ax)$, $a>0$, $\operatorname{Re}\beta>0$, $1+\operatorname{Re}\nu>\operatorname{Re}\mu>-1$	$y^{1/2}\beta^{\mu-\nu} I_\nu(y\beta) K_\mu(a\beta)$, $0<y<a$
(15)	$x^{\mu-\nu+2n+1/2}(\beta^2+x^2)^{-1} J_\mu(ax)$, $a>0$, $\operatorname{Re}\beta>0$, $\operatorname{Re}\nu-2n+1>\operatorname{Re}\mu>-n-1$, $n=0, 1, 2, \ldots$	$(-1)^n \beta^{\mu-\nu+2n} y^{1/2} I_\nu(y\beta) K_\mu(a\beta)$, $0<y<a$
(16)	$x^{1/2}(x^2+\beta^2)^{-1} J_{\nu-2n}(ax)$, $a>0$, $\operatorname{Re}\beta>0$, $\operatorname{Re}\nu>n-1$	$(-1)^n y^{1/2} I_\nu(y\beta) K_{\nu-2n}(a\beta)$, $0<y<a$
(17)	$x^{-1/2} e^{-\alpha x} J_\nu(\beta x)$, $\operatorname{Re}\alpha>\operatorname{Im}\beta>0$, $\operatorname{Re}\nu>-1/2$	$\pi^{-1}\beta^{-1/2} Q_{\nu-1/2}\left(\frac{\alpha^2+\beta^2+y^2}{2\beta y}\right)$
(18)	$x^{\mu-3/2} e^{-\alpha x} J_\nu(\beta x)$, $\operatorname{Re}\alpha>\lvert\operatorname{Im}\beta\rvert$, $\operatorname{Re}(\mu+2\nu)>0$	$\frac{\beta^\nu y^{\nu+1/2}\,\Gamma(\mu+2\nu)}{\pi\alpha^{\mu+2\nu}\,\Gamma(2\nu+1)} \times$ $\times \int_0^\pi {}_2F_1\left(\frac{\mu}{2}+\nu, \frac{\mu+1}{2}+\nu; \nu+1; -\frac{u^2}{\alpha^2}\right)(\sin\varphi)^{2\nu}\,d\varphi$, $u^2=\beta^2+y^2-2\beta y\cos\varphi$

	$f(x)$	$\int_0^\infty f(x) J_\nu(xy)(xy)^{1/2}\,dx, \quad y>0$
(19)	$x^{-1} e^{-xa\cos\varphi\cos\psi} J_\mu(xa\sin\varphi)$, $a>0,\ 0<\varphi,\psi<\pi/2$ $\operatorname{Re}(\mu+\nu)>-1/2$	$\Gamma(\mu+\nu+1/2)(\sin\psi)^{1/2}\times$ $\times\, \mathrm{P}_{\nu-1/2}^{-\mu}(\cos\varphi)\,\mathrm{P}_{\mu-1/2}^{-\nu}(\cos\psi)$, $y=a\sin\psi$
(20)	$x^{-1/2} e^{-\beta x} J_\mu(\alpha x)$, $\operatorname{Re}\beta>\lvert\operatorname{Im}\alpha\rvert$ $\operatorname{Re}(\mu+\nu+1)>0$	$2\pi^{-1}\alpha^\mu\beta y^{\nu+1/2}\int_0^{\pi/2}\left(\frac{2\beta}{\cos\theta}\right)^{\mu+\nu}\times$ $\times\left(\frac{\beta^2}{\cos^2\theta}+y^2-\alpha^2+u\right)^{-\mu}\times$ $\times\left(\frac{\beta^2}{\cos^2\theta}+\alpha^2-y^2+u\right)^{-\nu}\times$ $\times\frac{\cos[(\mu-\nu)\theta]}{u\cos^2\theta}\,d\theta$, $u^2=\left(\frac{b^2}{\cos^2\theta}+\alpha^2+y^2\right)^2-4\alpha^2y^2$
(21)	$x^{\mu-\nu-1/2} e^{-\alpha x} J_\mu(\beta x)$, $\operatorname{Re}\alpha>\lvert\operatorname{Im}\beta\rvert$, $\operatorname{Re}\mu>-1/2$	$\frac{\beta^\mu y^{\nu+1/2}\Gamma(\mu+1/2)}{2^{\nu-\mu}\pi\Gamma(\nu+1/2)}\int_0^\pi(\sin\varphi)^{2\nu}\times$ $\times[(\alpha+iy\cos\varphi)^2+\beta^2]^{-\mu-1/2}\,d\varphi$
(22)	$x^{\lambda-3/2} e^{-\alpha x} J_\mu(\beta x)$, $\operatorname{Re}\alpha>\operatorname{Im}\beta>0$ $\operatorname{Re}(\lambda+\mu+\nu)>0$	$\frac{\beta^\mu y^{\nu+1/2}}{2^{\mu+\nu}\Gamma(\nu+1)\alpha^{\mu+\nu+\lambda}}\times$ $\times\sum_{m=0}^\infty\frac{\Gamma(\lambda+\mu+\nu+2m)}{m!\,\Gamma(\mu+m+1)}\left(-\frac{\beta^2}{4\alpha^2}\right)^m\times$ $\times\,{}_2F_1(-m,\ -\mu-m;\ \nu+1;\ y^2\beta^{-2})$
(23)	$x^{1/2}\exp(-\beta x^2) J_\nu(\alpha x)$, $\operatorname{Re}\beta>0$, $\operatorname{Re}\nu>-1$	$\frac{y^{1/2}}{2\beta}\exp\left(-\frac{\alpha^2+y^2}{4\beta}\right) I_\nu\left(\frac{\alpha y}{2\beta}\right)$
(24)	$x^{\lambda+1/2}\exp(-\alpha x^2) J_\mu(\beta x)$, $\operatorname{Re}\alpha>0$ $\operatorname{Re}(\mu+\nu+\lambda)>-2$	$y^{1/2}\sum_{m=0}^\infty\frac{\Gamma(m+\nu/2+\mu/2+\lambda/2)}{m!\,\Gamma(m+\mu+1)}\times$ $\times\left(-\frac{\beta^2}{4\alpha}\right)^m{}_2F_1(-m,\ -\mu-m;$ $\nu+1,\ y^2\beta^{-2})$
(25)	$x^\lambda J_\mu(ax)\begin{matrix}\cos(bx)\\ \sin(bx)\end{matrix}$	См. в главах I и II.

	$f(x)$	$\int\limits_0^\infty f(x)\, J_\nu(xy)\,(xy)^{1/2}\,dx, \quad y>0$
(26)	$x^{1/2}\sin(ax^2)\,J_\nu(bx)$, $a>0,\ b>0,\ \operatorname{Re}\nu>-2$	$\frac{y^{1/2}}{2a}\cos\left(\frac{y^2+b^2}{4a}-\frac{\nu\pi}{2}\right) J_\nu\left(\frac{by}{2a}\right)$
(27)	$x^{1/2}\cos(ax^2)\,J_\nu(bx)$, $a>0,\ b>0,\ \operatorname{Re}\nu>-1$	$\frac{y^{1/2}}{2a}\sin\left(\frac{b^2+y^2}{4a}-\frac{\nu\pi}{2}\right) J_\nu\left(\frac{by}{2a}\right)$
(28)	$x^{-1/2}\left[J_0(2^{-1}ax)\right]^2$, $a>0,\ \operatorname{Re}\nu>-1$	$y^{-1/2}\left\{P_{\nu/2-1/2}\left[(1-a^2/y^2)^{1/2}\right]\right\}^2$, $a<y<\infty$
(29)	$x^{1/2}\left[J_{\nu/2}(2^{-1}ax)\right]^2$, $a>0,\ \operatorname{Re}\nu>-1$	$2\pi^{-1}y^{-1/2}(a^2-y^2)^{-1/2}$, $0<y<a$ 0, $a<y<\infty$
(30)	$x^{1/2-\nu}\left[J_\nu(2^{-1}ax)\right]^2$, $a>0,\ \operatorname{Re}\nu>-1/2$	$\frac{2^{1-\nu}y^{\nu-1/2}(a^2-y^2)^{\nu-1/2}}{\pi^{1/2}a^{2\nu}\,\Gamma(\nu+1/2)}$, $0<y<a$ 0, $a<y<\infty$
(31)	$x^{1/2-\nu}J_\nu(ax)\,J_\nu(bx)$, $a,b>0,\ \operatorname{Re}\nu>-1/2$	$\frac{[y^2-(a-b)^2]^{\nu-1/2}\,[(a+b)^2-y^2]^{\nu-1/2}}{y^{\nu-1/2}2^{3\nu-1}\pi^{1/2}(ab)^\nu\,\Gamma(\nu+1/2)}$, $\lvert a-b\rvert<y<a+b$ 0, $0<y<\lvert a-b\rvert$ или $a+b<y<\infty$
(32)	$x^{1/2}J_{(\nu+n)/2}(ax/2)\,J_{(\nu-n)/2}(ax/2)$, $a>0,\ \operatorname{Re}\nu>-1$	$2\pi^{-1}y^{-1/2}(a^2-y^2)^{-1/2}\,T_n(a^{-1}y)$, $0<y<a$ 0, $a<y<\infty$
(33)	$x^{-1/2}J_\mu^2(ax/2)$, $a>0$ $\operatorname{Re}\nu+\operatorname{Re}2\mu>-1$	$(a/2)^{2\mu}\,y^{-2\mu-1/2}\times$ $\times\frac{\Gamma(1/2+\nu/2+\mu)}{[\Gamma(\mu+1)]^2\,\Gamma(1/2+\nu/2-\mu)}\times$ $\times\left\{{}_2F_1\left[1/2-\nu/2+\mu,\ 1/2+\nu/2+\mu;\ \mu+1;\ 2^{-1}\left(1-\left(1-a^2y^{-2}\right)^{1/2}\right)\right]\right\}^2$, $a<y<\infty$
(34)	$x^{1/2-\mu}J_\mu(ax)\,J_\nu(bx)$, $a,\ b>0$ $\operatorname{Re}\nu>-1,\ \operatorname{Re}\mu>-1/2$	$\frac{y^{\mu-1/2}(\operatorname{sh}u)^{\mu-1/2}}{(2^{-1}\pi^3)^{1/2}\,a^\mu b^{1-\mu}}\,e^{(\mu-1/2)\pi i}\times$ $\times\sin[(\nu-\mu)\pi]\,Q_{\nu-1/2}^{1/2-\mu}(\operatorname{ch}u)$, $0<y<a-b$ $\frac{b^{\mu-1}y^{\mu-1/2}}{(2\pi)^{1/2}a^\mu}(\sin v)^{\mu-1/2}\,\mathrm{P}_{\nu-1/2}^{1/2-\mu}(\cos v)$, $\lvert a-b\rvert<y<a+b$ 0, $0<y<b-a$ или $a+b<y<\infty$ $2by\operatorname{ch}u=a^2-b^2-y^2$ $2by\cos v=b^2+y^2-a^2$

	$f(x)$	$\int_0^\infty f(x)\, J_\nu(xy)\,(xy)^{1/2}\,dx, \quad y>0$
(35)	$x^{1/2-\nu} J_\mu(ax)\, J_\mu(bx)$, $a,\ b>0$, $\operatorname{Re}\mu>-1,\ \operatorname{Re}\nu>-1/2$	$0, \quad 0<y<\|a-b\|$; $\dfrac{(ab)^{\nu-1}}{(2\pi)^{1/2}\, y^{\nu-1/2}} (\sin u)^{\nu-1/2}\, \mathrm{P}^{1/2-\nu}_{\mu-1/2}(\cos u)$, $\|a-b\|<y<a+b$; $\dfrac{(ab)^{\nu-1}\,(\operatorname{sh} v)^{\nu-1/2}}{(2^{-1}\pi^3)^{1/2}\, y^{\nu-1/2}}\, e^{(\nu-1/2)\pi i}\, \times$ $\times \sin[(\mu-\nu)\pi]\, Q^{1/2-\nu}_{\mu-1/2}(\operatorname{ch} v)$, $a+b<y<\infty$; $2ab\cos u = a^2+b^2-y^2$; $2ab\operatorname{ch} v = y^2-a^2-b^2$
(36)	$x^{\rho-\mu-\nu+1/2} J_\mu(ax)\, J_\rho(bx)$, $b>a>0$, $\operatorname{Re}\rho>-1,\ \operatorname{Re}(\rho-\mu-\nu)<1/2$	$0, \quad 0<y<b-a$
(37)	$x^{\rho-\mu-\nu-3/2} J_\mu(ax)\, J_\rho(bx)$, $b>a>0$, $\operatorname{Re}\rho>0,\ \operatorname{Re}(\rho-\mu-\nu)>5/2$	$\dfrac{2^{\rho-\mu-\nu-1} y^{\nu+1/2} a^{\mu}\, \Gamma(\rho)}{b^{\rho}\, \Gamma(\mu+1)\, \Gamma(\nu+1)}$, $0<y<b-a$
(38)	$x^{-1/2} J_\mu(xa\sin\varphi\cos\psi)\, J_\rho(ax)$, $a>0,\ 0<\varphi,\ \psi<\pi/2$, $\operatorname{Re}(\mu+\nu+\rho)>-1$	$\dfrac{a^{-1/2}\,\Gamma[2^{-1}(1+\sigma+\rho)]}{\Gamma(\mu+1)\,\Gamma(\nu+1)\,\Gamma[2^{-1}(1-\sigma+\rho)]}\, \times$ $\times (\sin\varphi\cos\psi)^{\mu}(\sin\psi\cos\varphi)^{\nu+1/2} \times$ $\times\, {}_2F_1\left(\dfrac{1+\sigma-\rho}{2}, \dfrac{1+\sigma+\rho}{2};\ \mu+1;\ \sin^2\varphi\right) {}_2F_1\left(\dfrac{1+\sigma-\rho}{2}, \dfrac{1+\sigma+\rho}{2};\ \nu+1;\ \sin^2\psi\right)$, $\sigma=\mu+\nu,\ y=a\cos\varphi\sin\psi$
(39)	$x^{1/2} J_\mu(xa\sin\varphi\cos\psi)\, J_{\nu-\mu}(ax)$, $a>0,\ 0<\varphi,\ \psi<\pi/2$, $\operatorname{Re}\nu>-1$	$2\pi^{-1} a^{-3/2} \times$ $\times \sin(\mu\pi)(\sin\varphi)^{\mu}(\sin\psi)^{\nu+1/2} \times$ $\times (\cos\varphi)^{1/2-\nu}(\cos\psi)^{-\mu} \times$ $\times [\cos(\varphi+\psi)\cos(\varphi-\psi)]^{-1}$, $y=a\cos\varphi\sin\psi$
(40)	$x^{\lambda} J_\mu(ax)\, J_\rho(bx)$	См. Bailey W. N., 1936: Proc. London Math. Soc. (2), 40, 37—48.

	$f(x)$	$\int_0^\infty f(x)\, J_\nu(xy)\,(xy)^{1/2}\,dx, \quad y>0$
(4)	$x^{(1/2-\nu)/3}\sin(2^{-2}ax^2)\times J_{(\nu-1/2)/3}(2^{-2}ax^2)$, $a>0$, $\operatorname{Re}\nu>-5/2$	$a^{(\nu-2)/3}y^{(1/2-\nu)/3}\times\sin\left(\frac{\nu+1}{6}\pi-\frac{y^2}{4a}\right)\times J_{(\nu-1/2)/3}\left(\frac{y^2}{4a}\right)$
(5)	$x^{(1/2-\nu)/3}\cos(2^{-2}ax^2)\times J_{(\nu-1/2)/3}(2^{-2}ax^2)$, $a>0$, $\operatorname{Re}\nu>-1$	$a^{(\nu-2)/3}y^{(1/2-\nu)/3}\times\cos\left(\frac{\nu+1}{6}\pi-\frac{y^2}{4a}\right)\times J_{(\nu-1/2)/3}\left(\frac{y^2}{4a}\right)$
(6)	$x^{1/2}\left[J_{\nu/4}(2^{-2}ax^2)\right]^2$, $a>0$, $\operatorname{Re}\nu>-1$	$-\frac{y^{1/2}}{a}J_{\nu/4}\left(\frac{y^2}{4a}\right)Y_{\nu/4}\left(\frac{y^2}{4a}\right)$
(7)	$x^{1/2}J_{\nu/4}(2^{-2}ax^2)J_{-\nu/4}(2^{-2}ax^2)$, $a>0$, $\operatorname{Re}\nu>-2$	$\frac{y^{1/2}}{a}J_{\nu/4}\left(\frac{y^2}{4a}\right)\left[J_{\nu/4}\left(\frac{y^2}{4a}\right)\sin\left(\frac{\pi\nu}{4}\right)-Y_{\nu/4}\left(\frac{y^2}{4a}\right)\cos\left(\frac{\pi\nu}{4}\right)\right]$
(8)	$x^{1/2}J_{\nu/4-\mu}(ax^2)J_{\nu/4+\mu}(ax^2)$, $a>0$, $\operatorname{Re}\nu>-1/2$	$\frac{2}{\pi y^{3/2}}\left[\exp(2^{-2}\nu\pi i)\,W_{\mu,\nu/4}(u)\times W_{-\mu,\nu/4}(u)+\exp(-2^{-2}\nu\pi i)\times W_{\mu,\nu/4}(v)\,W_{-\mu,\nu/4}(v)\right]$, $u=\frac{y^2}{8a}e^{\pi i/2}$, $v=\frac{y^2}{8a}e^{-\pi i/2}$
(9)	$x^{-1/2}J_\nu(ax^{-1})$, $a>0$, $\operatorname{Re}\nu>-1/2$	$y^{-1/2}J_{2\nu}(2a^{1/2}y^{1/2})$
(10)	$x^{-5/2}J_\nu(ax^{-1})$, $a>0$, $\operatorname{Re}\nu>-1/2$	$a^{-1}y^{1/2}J_{2\nu}(2a^{1/2}y^{1/2})$
(11)	$x^{-3/2}J_{\nu-1}(ax^{-1})$, $a>0$, $\operatorname{Re}\nu>-1/2$	$a^{-1/2}J_{2\nu-1}(2a^{1/2}y^{1/2})$
(12)	$x^{-2\nu}J_{1/2-\nu}(ax^{-1})$, $a>0$, $-1/2<\operatorname{Re}\nu<3$	$-\frac{i(a^{-1}y)^{\nu-1/2}}{2\sin(2\nu\pi)}\times\left[e^{2\nu\pi i}J_{1-2\nu}(u)\,J_{2\nu-1}(v)-e^{-2\nu\pi i}J_{2\nu-1}(u)\,J_{1-2\nu}(v)\right]$, $u=(ay/2)^{1/2}\exp(2^{-2}\pi i)$, $v=(ay/2)^{1/2}\exp(-2^{-2}\pi i)$

	$f(x)$	$\int\limits_0^\infty f(x)\,J_\nu(xy)\,(xy)^{1/2}\,dx, \quad y>0$
(13)	$x^{\rho-3/2} J_\mu(x^{-1})$, $-3/2-\operatorname{Re}\nu<\operatorname{Re}\rho<\operatorname{Re}\mu+3/2$	$\frac{\pi y^{\nu+1/2}}{2\sin[2^{-1}(\mu-\nu-\rho)\pi]}\times$ $\times\left[A\,{}_0F_3\left(1+\nu,\ 1+\frac{\rho-\mu+\nu}{2},\ 1+\frac{\mu+\nu+\rho}{2};\ \frac{y^2}{16}\right)-\right.$ $\left.-y^\mu B\,{}_0F_3\left(1+\mu,\ 1+\frac{\mu+\nu-\rho}{2},\ 1+\frac{\mu-\nu-\rho}{2};\ \frac{y^2}{16}\right)\right]$, $A^{-1}=2^{\nu+\rho}\Gamma(1+\nu)\times\Gamma[1+2^{-1}(\rho-\mu+\nu)]\times\Gamma[1+2^{-1}(\rho+\mu+\nu)]$ $B^{-1}=2^{2\mu-\rho}\Gamma(1+\mu)\times\Gamma[1+2^{-1}(\mu+\nu-\rho)]\times\Gamma[1+2^{-1}(\mu-\nu-\rho)]$
(14)	$x^{1/2}(\beta^2+x^2)^{-1/2}\exp\left(-\frac{\alpha^2\beta}{\beta^2+x^2}\right)\times J_\nu\left(\frac{\alpha^2 x}{\beta^2+x^2}\right)$, $\operatorname{Re}\beta>0,\ \operatorname{Re}\nu>-1/2$	$y^{-1/2}e^{-\beta y}J_{2\nu}(2\alpha y^{1/2})$
(15)	$J_{2\nu-1}(\alpha x^{1/2})$, $\operatorname{Re}\nu>-1/2$	$2^{-1}\alpha y^{-3/2}J_{\nu-1}(2^{-2}\alpha^2 y^{-1})$
(16)	$x^{-1/2}J_{2\nu}(\alpha x^{1/2})$, $\operatorname{Re}\nu>-1/2$	$y^{-1/2}J_\nu(2^{-2}\alpha^2 y^{-1})$
(17)	$x^{-1/2}e^{-\beta x}J_{2\nu}(2\alpha x^{1/2})$, $\operatorname{Re}\beta>0,\ \operatorname{Re}\nu>-1/2$	$y^{1/2}(y^2+\beta^2)^{-1/2}\exp\left(-\frac{\alpha^2\beta}{\beta^2+y^2}\right)\times J_\nu\left(\frac{\alpha^2 y}{\beta^2+y^2}\right)$
(18)	$x^{\nu+1/2}(x^2+\beta^2)^{-\mu/2}\times J_\mu[a(x^2+\beta^2)^{1/2}]$, $a>0$ $\operatorname{Re}\beta>0,\ \operatorname{Re}\mu>\operatorname{Re}\nu>-1$	$a^{-\mu}y^{\nu+1/2}\beta^{-\mu+\nu+1}\times(a^2-y^2)^{\mu/2-\nu/2-1/2}\times J_{\mu-\nu-1}[\beta(a^2-y^2)^{1/2}]$, $0<y<a$; 0, $a<y<\infty$
(19)	$x^{\nu+1/2}(x^2+\beta^2)^{-\mu/2-1}\times J_{\mu-1}[a(x^2+\beta^2)^{1/2}]$, $a>0$ $\operatorname{Re}\beta>0,\ \operatorname{Re}(\mu+2)>\operatorname{Re}\nu>-1$	$(a/2)^{\mu-1}\beta^\nu[\Gamma(\mu)]^{-1}y^{1/2}K_\nu(\beta y)$, $a<y<\infty$

	$f(x)$	$\int_0^\infty f(x)\, J_\nu(xy)\,(xy)^{1/2}\,dx, \quad y>0$
(20)	$x^{\nu-3/2}(x^2+\beta^2)^{-\mu/2}\times$ $\times J_\mu\left[a(x^2+\beta^2)^{1/2}\right], \quad a>0$ $\operatorname{Re}\beta>0,\ \operatorname{Re}(\mu+2)>\operatorname{Re}\nu>0$	$\beta^{-\mu}2^{\nu-1}\Gamma(\nu)\,y^{1/2-\nu}J_\mu(a\beta),$ $a<y<\infty$
(21)	$x^{\nu+1/2}(x^2+\alpha^2)^{-1}(x^2+\beta^2)^{-\mu/2}\times$ $\times J_\mu\left[c(x^2+\beta^2)^{1/2}\right],$ $\operatorname{Re}\alpha>0,\ \operatorname{Re}\beta>0,\ c>0$ $-1<\operatorname{Re}\nu<2+\operatorname{Re}\mu$	$\alpha^\nu y^{1/2}(\beta^2-\alpha^2)^{-\mu/2}J_\mu\left[c(\beta^2-\alpha^2)^{1/2}\right]\times$ $\times K_\nu(\alpha y), \quad c\leqslant y<\infty$
(22)	$x^{\nu+2n-3/2}(x^2+\alpha^2)^{-1}(x^2+\beta^2)^{-\mu/2}\times$ $\times J_\mu\left[c(x^2+\beta^2)^{1/2}\right],$ $\operatorname{Re}\alpha>0,\ \operatorname{Re}\beta>0,\ c>0$ $-n<\operatorname{Re}\nu<4-2n+\operatorname{Re}\mu$	$(-1)^{n+1}y^{1/2}\alpha^{\nu+2n-2}(\beta^2-\alpha^2)^{-\mu/2}\times$ $\times J_\mu\left[c(\beta^2-\alpha^2)^{1/2}\right]K_\nu(\alpha y),$ $c<y<\infty$
(23)	$x^{\nu+1/2}(x^2+\beta^2)^{-\nu/2-1/4}\times$ $\times C_{2n+1}^{\nu+1/2}\left[\beta(x^2+\beta^2)^{-1/2}\right]\times$ $\times J_{\nu+3/2+2n}\left[a(x^2+\beta^2)^{1/2}\right],$ $a>0,\ \operatorname{Re}\beta>0,\ \operatorname{Re}\nu>-1$	$(-1)^n\,2^{1/2}\pi^{-1/2}a^{-1/2-\nu}y^{\nu+1/2}\times$ $\times(a^2-y^2)^{-1/2}\sin\left[\beta(a^2-y^2)^{1/2}\right]\times$ $\times C_{2n+1}^{\nu+1/2}\left[(1-y^2/a^2)^{1/2}\right],$ $0<y<a$ $0, \quad a<y<\infty$
(24)	$x^{\nu+1/2}(x^2+\beta^2)^{-\nu/2-1/4}\times$ $\times C_{2n}^{\nu+1/2}\left[\beta(x^2+\beta^2)^{-1/2}\right]\times$ $\times J_{\nu+1/2+2n}\left[a(x^2+\beta^2)^{1/2}\right],$ $a>0,\ \operatorname{Re}\beta>0,\ \operatorname{Re}\nu>-1$	$(-1)^n\,2^{1/2}\pi^{-1/2}a^{-1/2-\nu}\times$ $\times y^{\nu+1/2}(a^2-y^2)^{-1/2}\times$ $\times\cos\left[\beta(a^2-y^2)^{1/2}\right]\times$ $\times C_{2n}^{\nu+1/2}\left[(1-y^2/a^2)^{1/2}\right], \quad 0<y<a$ $0, \quad a<y<\infty$
(25)	$x^{\nu-3/2}(x^2+\beta^2)^{-n\mu/2}\times$ $\times\prod_{i=1}^{n}J_\mu\left[a_i(x^2+\beta^2)^{1/2}\right],$ $a_i>0,\ \operatorname{Re}\beta>0$ $\operatorname{Re}(n\mu+n/2+1/2)>\operatorname{Re}\nu>0$	$2^{\nu-1}\beta^{-n\mu}\Gamma(\nu)\,y^{1/2-\nu}\prod_{i=1}^{n}J_\mu(a_i\beta),$ $\sum_{i=1}^{n}a_i<y<\infty$
(26)	$x^{\nu+1/2}\prod_{i=1}^{n}z_i^{-\mu_i}J_{\mu_i}(a_iz_i),$ $a_i>0,\ \operatorname{Re}\beta_i>0,$ $z_i=\left(x^2+\beta_i^2\right)^{1/2}$ $n/2+\sum_{i=1}^{n}\mu_i-1/2>\operatorname{Re}\nu>-1$	$0, \quad \sum_{i-1}^{n}a_i<y<\infty$

	$f(x)$	$\int_0^\infty f(x)\, J_\nu(xy)\,(xy)^{1/2}\,dx, \quad y>0$
(27)	$x^{\nu-3/2}\prod_{i=1}^{n} z_i^{-\mu_i} J_{\mu_i}(a_i z_i)$, $a_i>0$, $\operatorname{Re}\beta_i>0$, $z_i=(x^2+\beta_i^2)^{1/2}$, $n/2+\sum_{i=1}^{n}\mu_i+3/2>\operatorname{Re}\nu>0$	$2^{\nu-1}\Gamma(\nu)\,y^{1/2-\nu}\times\prod_{i=1}^{n}\left[\beta_i^{-\mu_i} J_{\mu_i}(a_i\beta_i)\right]$, $\sum_{i=1}^{n} a_i<y<\infty$
(28)	$x^{\nu+1/2}(1-x^2)^{\mu/2} J_\mu[\alpha(1-x^2)^{1/2}]$, $0<x<1$; 0, $1<x<\infty$; $\operatorname{Re}\mu>-1$, $\operatorname{Re}\nu>-1$	$\alpha^\mu y^{\nu+1/2}(\alpha^2+y^2)^{-(\mu+\nu+1)/2}\times J_{\mu+\nu+1}[(\alpha^2+y^2)^{1/2}]$
(29)	0, $0<x<c$; $x^{1/2-\nu}(x^2-c^2)^{\mu/2} J_\mu[a(x^2-c^2)^{1/2}]$, $c<x<\infty$; $a>0$, $\operatorname{Re}\nu>\operatorname{Re}\mu>-1$	0, $0<y<a$; $a^\mu c^{1+\mu-\nu} y^{-\nu+1/2}\times(y^2-a^2)^{\nu/2-\mu/2-1/2}\times J_{\nu-\mu-1}[c(y^2-a^2)^{1/2}]$, $a<y<\infty$
(30)	0, $0<x<c$; $x^{1/2-\nu}(x^2+\beta^2)^{-1}(x^2-c^2)^{\mu/2}\times J_\mu[a(x^2-c^2)^{1/2}]$, $c<x<\infty$; $a>0$, $\operatorname{Re}\beta>0$; $-1<\operatorname{Re}\mu<2+\operatorname{Re}\nu$	$\beta^{-\nu}(c^2+\beta^2)^{\mu/2} y^{1/2}\times K_\mu[a(\beta^2+c^2)^{1/2}]\, I_\nu(\beta y)$, $0<y<a$
(31)	0, $0<x<c$; $x^{1/2-\nu}(x^2+\beta^2)^{-1}(x^2-c^2)^{\mu/2+n-1}\times J_\mu[a(x^2-c^2)^{1/2}]$, $c<x<\infty$; $a>0$, $\operatorname{Re}\beta>0$; $-n<\operatorname{Re}\mu<4-2n+\operatorname{Re}\nu$	$(-1)^{n+1}\beta^{-\nu}(\beta^2+c^2)^{\mu/2+n-1} y^{1/2}\times K_\mu[a(\beta^2+c^2)^{1/2}]\, I_\nu(\beta y)$, $0<y<a$
(32)	$x^{\nu+2n+1/2}(1-x^2)^{\lambda/2+m}\times J_\lambda[a(1-x^2)^{1/2}]$, $0<x<1$; 0, $1<x<\infty$; $a>0$, $\operatorname{Re}\lambda>-1$, $\operatorname{Re}\nu>-1$	$a^{-\lambda} y^{-\nu+1/2}\left(\frac{d}{a\,da}\right)^m\left(\frac{d}{y\,dy}\right)^n z$, где $z=a^{2\lambda+2m} y^{2\nu+2n}\times(a^2+y^2)^{-(\lambda+\nu+m+n+1)/2}\times J_{\lambda+\nu+m+n+1}[(a^2+y^2)^{1/2}]$

	$f(x)$	$\int\limits_0^\infty f(x)\, J_\nu(xy)\,(xy)^{1/2}\,dx, \quad y>0$
(33)	$x^{\rho}(1-x^2)^{\mu} J_\lambda\left[a(1-x^2)^{1/2}\right], \quad 0<x<1$ $0, \quad 1<x<\infty$	См. Bailey W. N., 1938: Quart. J. Math. Oxford Series 9, 141–147.
(34)	$x^{1/2} J_{\nu/2}\{2^{-1}a[(x^2+\beta^2)^{1/2}-\beta]\} \times$ $\times J_{\nu/2}\{2^{-1}a[(x^2+\beta^2)^{1/2}+\beta]\},$ $a>0,\ \operatorname{Re}\nu>-1$	$2\pi^{-1}y^{-1/2}(a^2-y^2)^{-1/2}\times$ $\times\cos[\beta(a^2-y^2)^{1/2}], \quad 0<y<a$ $0, \quad a<y<\infty$
(35)	$x^{1/2} Y_{\nu/2}(2^{-2}ax^2),$ $a>0,\ \operatorname{Re}\nu>-1$	$-2a^{-1}y^{1/2}\mathbf{H}_{\nu/2}(y^2/a)$
(36)	$x^{1/2} J_{\nu/4}(2^{-2}ax^2)\, Y_{\nu/4}(2^{-2}ax^2),$ $a>0,\ \operatorname{Re}\nu>-1$	$-2a^{-1}y^{1/2}\left[J_{\nu/4}\left(\frac{y^2}{4a}\right)\right]^2$
(37)	$x^{-1/2} Y_\nu(ax^{-1}),$ $a>0,\ \lvert\operatorname{Re}\nu\rvert<1/2$	$-2\pi^{-1}y^{-1/2}[K_{2\nu}(2a^{1/2}y^{1/2})-$ $-2^{-1}\pi Y_{2\mu}(2a^{1/2}y^{1/2})]$
(38)	$x^{-5/2} Y_\nu(ax^{-1}),$ $a>0,\ \lvert\operatorname{Re}\nu\rvert<1/2$	$2y^{1/2}a^{-1}\pi^{-1}[K_{2\nu}(2a^{1/2}y^{1/2})+$ $+2^{-1}\pi Y_{2\nu}(2a^{1/2}y^{1/2})]$
(39)	$x^{-1/2} Y_{2\nu}(2ax^{1/2}),$ $a>0,\ \operatorname{Re}\nu>-1/2$	$\frac{2}{y^{1/2}\cos(\nu\pi)}[2^{-1}\cos(\nu\pi)\, Y_\nu(a^2/y)-$ $-Y_{-\nu}(a^2/y)+\mathbf{H}_{-\nu}(a^2/y)]$
(40)	$0, \quad 0<x<c$ $x^{1/2-\nu}(x^2+\beta^2)^{-1}\times$ $\times(x^2-c^2)^{\mu/2+n-1/2}\times$ $\times Y_\mu[a(x^2-c^2)^{1/2}], \quad c<x<\infty$ $a>0,\ \operatorname{Re}\beta>0$ $-1/2-n<\operatorname{Re}\mu<3-2n+\operatorname{Re}\nu$	$(-1)^{n+1}\beta^{-\nu}y^{1/2}(\beta^2+c^2)^{\mu/2+n-1/2}\times$ $\times K_\mu[a(\beta^2+c^2)^{1/2}]\, I_\nu(\beta y),$ $0<y<a$
(41)	$x^{1/2} J_{\nu/2}\{2^{-1}a[(x^2+\beta^2)^{1/2}-\beta]\}\times$ $\times Y_{\nu/2}\{2^{-1}a[(x^2+\beta^2)^{1/2}+\beta]\},$ $a>0,\ \operatorname{Re}\nu>-1$	$2\pi^{-1}y^{-1/2}(a^2-y^2)^{-1/2}\times$ $\times\sin[\beta(a^2-y^2)^{1/2}], \quad 0<y<a$ $-2\pi^{-1}y^{-1/2}(y^2-a^2)^{-1/2}\times$ $\times\exp[-\beta(y^2-a^2)^{1/2}], \quad a<y<\infty$

	$f(x)$	$\int_0^\infty f(x) J_\nu(xy)(xy)^{1/2}\,dx, \quad y>0$
(42)	$x^{1/2}\left[H^{(1)}_{\nu/4+\mu}(ax^2)\,H^{(1)}_{\nu/4-\mu}(ax^2) - - H^{(2)}_{\nu/4+\mu}(ax^2)\,H^{(2)}_{\nu/4-\mu}(ax^2)\right]$, $\operatorname{Re}\nu > -1/2$, $\operatorname{Re}(1/2 \pm \mu + \nu/2) > 0$	$\frac{8\,\Gamma(1/2-\mu+\nu/4)\,\Gamma(1/2+\mu+\nu/4)}{i\pi\,[\Gamma(\nu/2+1)]^2\,y^{3/2}} \times \times M_{\mu,\,\nu/4}\left(\frac{y^2}{8a}e^{\pi i/2}\right) \times \times M_{\mu,\,\nu/4}\left(\frac{y^2}{8a}e^{-\pi i/2}\right)$

8.13. Модифицированные функции Бесселя аргумента kx

(1)	$x^{1/2}\exp(-\beta x^2)\,I_\nu(\alpha x)$, $\operatorname{Re}\beta>0,\ \operatorname{Re}\nu>-1$	$\frac{y^{1/2}}{2\beta}\exp\left(\frac{\alpha^2-y^2}{4\beta}\right) J_\nu\left(\frac{\alpha y}{2\beta}\right)$
(2)	$x^{1/2}K_\nu(\alpha x)$, $\operatorname{Re}\alpha>0,\ \operatorname{Re}\nu>-1$	$\frac{y^{\nu+1/2}}{\alpha^\nu(y^2+\alpha^2)}$
(3)	$x^{\mu+\nu+1/2}K_\mu(\alpha x)$, $\operatorname{Re}\alpha>0,\ \operatorname{Re}(\nu+1)>\lvert\operatorname{Re}\mu\rvert$	$\frac{2^{\nu+\mu}\,\Gamma(\mu+\nu+1)\,\alpha^\mu y^{\nu+1/2}}{(y^2+\alpha^2)^{\mu+\nu+1}}$
(4)	$x^{-\lambda-1/2}K_\mu(\alpha x)$, $\operatorname{Re}\alpha>0,\ \operatorname{Re}(\nu-\lambda+1)>\lvert\operatorname{Re}\mu\rvert$	$\frac{\Gamma(\tau+\mu)\,\Gamma(\tau)}{2^{\lambda+1}\alpha^{\nu-\lambda+1}\,\Gamma(\nu+1)\,y^{-\nu-1/2}} \times \times\ {}_2F_1\left(\tau+\mu,\ \tau;\ \nu+1;\ -\frac{y^2}{\alpha^2}\right)$, $\tau=\frac{\nu-\lambda-\mu+1}{2}$
(5)	$x^\lambda K_\mu(\alpha x)\ \begin{matrix}\cos(\beta x)\\ \sin(\beta x)\end{matrix}$	См. в главах I и II.
(6)	$x^{1/2}K_0(\alpha x)\,J_\nu(\beta x)$, $\operatorname{Re}\nu>-1,\ \operatorname{Re}\alpha>\lvert\operatorname{Im}\beta\rvert$	$y^{1/2}r_1^{-1}r_2^{-1}(r_2-r_1)^\nu(r_2+r_1)^{-\nu}$, $r_1=[\alpha^2+(\beta-y)^2]^{1/2}$, $r_2=[\alpha^2+(\beta+y)^2]^{1/2}$
(7)	$x^{\nu+1/2}J_\nu(2^{-1}\alpha x)\,K_\nu(2^{-1}\alpha x)$, $\lvert\arg\alpha\rvert<\pi/4,\ \operatorname{Re}\nu>-1/2$	$\frac{\alpha^{2\nu}2^\nu\,\Gamma(\nu+1/2)\,y^{\nu+1/2}}{\pi^{1/2}(y^4+\alpha^4)^{\nu+1/2}}$
(8)	$x^{\nu+1/2}J_\nu(\alpha x)\,K_\nu(\beta x)$, $\operatorname{Re}\beta>\lvert\operatorname{Im}\alpha\rvert,\ \operatorname{Re}\nu>-1/2$	$\frac{2^{3\nu}(\alpha\beta)^\nu y^{\nu+1/2}\,\Gamma(\nu+1/2)}{\pi^{1/2}\left[(\alpha^2+\beta^2+y^2)^2-4\alpha^2y^2\right]^{\nu+1/2}}$
(9)	$x^{\nu+1/2}J_{\nu-1}(\alpha x)\,K_{\nu-1}(\alpha x)$ $\lvert\arg\alpha\rvert<\pi/4,\ 0<\operatorname{Re}\nu<1/2$	$\frac{2^{3\nu-1}\alpha^{2\nu-2}\,\Gamma(\nu+1/2)\,y^{\nu+5/2}}{\pi^{1/2}(y^4+\alpha^4)^{\nu+1/2}}$

	$f(x)$	$\int\limits_0^\infty f(x)\,J_\nu(xy)\,(xy)^{1/2}\,dx,\quad y>0$
(10)	$x^{1/2} J_\mu(xa\sin\varphi)\times$ $\times K_{\nu-\mu}(xa\cos\varphi\cos\psi)$, $a>0,\ 0<\varphi,\ \psi<\pi/2$ $\operatorname{Re}\mu>-1,\ \operatorname{Re}\nu>-1$	$(\sin\varphi)^{\mu}(\sin\psi)^{\nu+1/2}\times$ $\times\dfrac{(\cos\varphi)^{\nu-\mu}(\cos\psi)^{\mu-\nu}}{a^{3/2}(1-\sin^2\varphi\sin^2\psi)}$, $y=a\sin\psi$
(11)	$x^{\nu+1/2} J_\mu(xa\sin\psi)\times$ $\times K_\mu(xa\cos\varphi\cos\psi)$, $a>0,\ 0<\varphi,\ \psi<\pi/2$ $\operatorname{Re}\nu>-1,\ \operatorname{Re}(\mu+\nu)>-1$	$\dfrac{2^{\nu}\Gamma(\mu+\nu+1)\,[\sin\varphi\cos^2(\alpha/2)]^{\nu+1/2}}{a^{\nu+3/2}(\cos\psi)^{2\nu+2}}\times$ $\times P_\nu^{-\mu}(\cos\alpha),\quad y=a\sin\varphi$, $\operatorname{tg}(\alpha/2)=\operatorname{tg}\psi\cos\varphi$
(12)	$x^{\mu+1/2} J_\nu(\beta x)\,K_\mu(\alpha x)$, $\operatorname{Re}\alpha>\lvert\operatorname{Im}\beta\rvert$ $\operatorname{Re}\nu>-1,\ \operatorname{Re}(\mu+\nu)>-1$	$(2\pi)^{-1/2}\times$ $\times\alpha^{\mu}\beta^{-\mu-1}y^{-\mu-1/2}e^{-(\mu+1/2)\pi i}\times$ $\times(u^2-1)^{-\mu/2-1/4}Q_{\nu-1/2}^{\mu+1/2}(u)$, $2\beta y u=\alpha^2+\beta^2+y^2$
(13)	$x^{-1/2} J_\mu(xa\sin\varphi)\times$ $\times K_\rho(xa\cos\varphi\cos\psi)$, $a>0,\ 0<\varphi,\ \psi<\pi/2$ $\operatorname{Re}(\mu+\nu+1)>\operatorname{Re}\rho$	$\dfrac{(\sin\varphi)^{\mu}(\sin\psi)^{\nu+1/2}}{2a^{1/2}(\cos\varphi\cos\psi)^{\rho}}\times$ $\times\dfrac{\Gamma(\tau)\,\Gamma(\tau+\rho)}{\Gamma(1+\mu)\,\Gamma(1+\nu)}\times$ $\times{}_2F_1(\tau,\ \tau-\nu;\ \mu+1;\ \sin^2\varphi)\times$ $\times{}_2F_1(\tau,\ \tau-\mu;\ \nu+1;\ \sin^2\psi)$, $\tau=\dfrac{1+\mu+\nu-\rho}{2}\quad y=a\sin\psi$
(14)	$x^{\rho+\nu-\mu+1/2} J_\mu(\alpha x)\,K_\rho(\beta x)$, $\operatorname{Re}\beta>\lvert\operatorname{Im}\alpha\rvert,\ \operatorname{Re}\nu>-1$ $\operatorname{Re}\rho>-1,\ \operatorname{Re}\mu>-1$ $\operatorname{Re}(\rho+\nu)>-1$	$2^{\rho+\nu-\mu-1}[\Gamma(\mu+1)]^{-1}\times$ $\times\Gamma(\rho+\nu+1)\,\Gamma(\rho+1)\times$ $\times\Gamma(\nu+1)\,\alpha^{\mu-\rho-\nu-2}y^{1/2}\times$ $\times(\operatorname{ch}\sigma-\cos\theta)\,P_{\rho+\nu-\mu}^{-\rho}(\cos\theta)\times$ $\times P_{\rho+\nu-\mu}^{-\nu}(\operatorname{ch}\sigma)$, $y+i\beta=i\alpha\operatorname{ctg}[2^{-1}(\theta+i\sigma)]$
(15)	$x^{\lambda} J_\mu(\alpha x)\,K_\rho(\beta x)$	См. Bailey W. N., 1936: Proc. London Math. Soc. (2), 40, 37—48.

	$f(x)$	$\int_0^\infty f(x)\, J_\nu(xy)\, (xy)^{1/2}\, dx, \quad y>0$
(16)	$x^{1/2} I_{\nu/2}(\alpha x)\, K_{\nu/2}(\alpha x)$, $\operatorname{Re}\alpha>0$, $\operatorname{Re}\nu>-1$	$y^{-1/2}(y^2+4\alpha^2)^{-1/2}$
(17)	$x^{\nu+1/2} I_\nu(2^{-1}\alpha x)\, K_\nu(2^{-1}\alpha x)$, $\operatorname{Re}\alpha>0$, $\lvert\operatorname{Re}\nu\rvert<1/2$	$\dfrac{2^\nu\alpha^{2\nu}\,\Gamma(\nu+1/2)}{\pi^{1/2}(y^3+\alpha^2 y)^{\nu+1/2}}$
(18)	$x^{\nu+1/2} I_\nu(\alpha x)\, K_\nu(\beta x)$, $\operatorname{Re}\beta>\operatorname{Re}\alpha$, $\operatorname{Re}\nu>-1/2$	$\dfrac{2^{3\nu}(\alpha\beta)^\nu y^{\nu+1/2}\,\Gamma(\nu+1/2)}{\pi^{1/2}[(\beta^2-\alpha^2-y^2)^2+4\alpha^2y^2]^{\nu+1/2}}$
(19)	$x^{\nu-1/2} I_{\nu-1/2}(2^{-1}\alpha x)\, K_{\nu-1/2}(2^{-1}\alpha x)$, $\operatorname{Re}\alpha>0$, $0<\operatorname{Re}\nu<3/2$	$\dfrac{\Gamma(\nu)\,(2\alpha)^{\nu-1}}{y^{\nu-1/2}}\, P_{-\nu}\left[\dfrac{2\alpha^2+y^2}{2\alpha(\alpha^2+y^2)^{1/2}}\right]$
(20)	$x^{-1/2} I_\mu(2^{-1}\alpha x)\, K_\mu(2^{-1}\alpha x)$, $\operatorname{Re}\alpha>0$, $\operatorname{Re}\nu>-1$, $\operatorname{Re}(\nu+2\mu)>-1$	$\dfrac{e^{\mu\pi i}\,\Gamma(\nu/2+\mu+1/2)}{\Gamma(\nu/2-\mu+1/2)\, y^{1/2}}\times$ $\times P^{-\mu}_{\nu/2-1/2}[(1+\alpha^2/y^2)^{1/2}]\times$ $\times Q^{-\mu}_{\nu/2-1/2}[(1+\alpha^2/y^2)^{1/2}]$
(21)	$x^{\mu+1/2} I_\nu(2^{-1}\alpha x)\, K_\mu(2^{-1}\alpha x)$, $\operatorname{Re}\alpha>0$, $\operatorname{Re}\nu>-1$, $-\operatorname{Re}\nu-1<\operatorname{Re}\mu<1/2$	$(\pi/2)^{-1/2}\alpha^{-1}y^{-\mu-1/2}\times$ $\times e^{-(\mu-\nu/2+1/4)\pi i}\times$ $\times(1+y^2/\alpha^2)^{-\mu/2-1/4}\, Q^{\mu+1/2}_{\nu-1/2}(iy/\alpha)$
(22)	$x^{\mu+1/2} I_\nu(\alpha x)\, K_\mu(\beta x)$, $\operatorname{Re}\beta>\lvert\operatorname{Re}\alpha\rvert$, $\operatorname{Re}\nu>-1$, $\operatorname{Re}(\mu+\nu)>-1$	$(2\pi)^{-1/2}\alpha^{-\mu-1}\beta^\mu y^{-\mu-1/2}\times$ $\times e^{-(\mu-\nu/2+1/4)\pi i}\times$ $\times(v^2+1)^{-\mu/2-1/4}\, Q^{\mu+1/2}_{\nu-1/2}(iv)$, $2\alpha yv=\beta^2-\alpha^2+y^2$
(23)	$x^{1/2} I_{(\nu-\mu)/2}(2^{-1}\alpha x)\times$ $\times K_{(\nu+\mu)/2}(2^{-1}\alpha x)$, $\operatorname{Re}\alpha>0$, $\operatorname{Re}\nu>-1$, $\operatorname{Re}(\nu-\mu)>-2$	$\alpha^{-\mu}y^{-1/2}(y^2+\alpha^2)^{-1/2}\times$ $\times[y+(y^2+\alpha^2)^{1/2}]^\mu$
(24)	$x^{\nu+1/2} I_\mu(\beta x)\, K_\mu(\alpha x)$, $\operatorname{Re}\alpha>\lvert\operatorname{Re}\beta\rvert$, $\operatorname{Re}\nu>-1$, $\operatorname{Re}(\mu+\nu)>-1$	$(2\pi)^{-1/2}(\alpha\beta)^{-\nu-1}y^{\nu+1/2}\times$ $\times e^{-(\nu+1/2)\pi i}\times$ $\times(u^2-1)^{-\nu/2-1/4}\, Q^{\nu+1/2}_{\mu-1/2}(u)$, $2\alpha\beta u=\alpha^2+\beta^2+y^2$

	$f(x)$	$\int\limits_0^\infty f(x)\,J_\nu(xy)\,(xy)^{1/2}\,dx,\quad y>0$
(25)	$x^\lambda I_\mu(\alpha x) K_\rho(\beta x)$	См. Bailey W. N., 1936: Proc. London Math. Soc. (2), 40, 37—48.
(26)	$x^{-\nu-1/2}\left[K_{\nu+1/2}(2^{-1}\alpha x)\right]^2$, $\operatorname{Re}\alpha>0,\ -1<\operatorname{Re}\nu<0$	$\pi^{1/2}(2\alpha)^{-\nu-1}\times$ $\times\Gamma(-\nu)\,y^{\nu+1/2}(\alpha^2+y^2)^{\nu/2}\times$ $\times P_\nu\left[\frac{2\alpha^2+y^2}{2\alpha(\alpha^2+y^2)^{1/2}}\right]$
(27)	$x^{1/2}\left[K_\mu(2^{-1}\alpha x)\right]^2$, $\operatorname{Re}\alpha>0$, $\operatorname{Re}(\nu/2\pm\mu)>-1$	$\frac{e^{2\mu\pi i}y^{1/2}\,\Gamma(1+\nu/2+\mu)}{(y^2+\alpha^2)^{1/2}\,\Gamma(\nu/2-\mu)}\times$ $\times Q_{\nu/2}^{-\mu}\left[(1+\alpha^2/y^2)^{1/2}\right]\times$ $\times Q_{\nu/2-1}^{-\mu}\left[(1+\alpha^2/y^2)^{1/2}\right]$
(28)	$x^{-1/2}\left[K_\mu(2^{-1}\alpha x)\right]^2$, $\operatorname{Re}\alpha>0$, $\operatorname{Re}(\nu/2\pm\mu)>-1/2$	$\frac{e^{2\mu\pi i}\,\Gamma(1/2+\nu/2+\mu)}{\Gamma(1/2+\nu/2-\mu)\,y^{1/2}}\times$ $\times\left\{Q_{\nu/2-1/2}^{-\mu}\left[(1+\alpha^2/y^2)^{1/2}\right]\right\}^2$
(29)	$x^{1/2}K_{\mu-1/2}(2^{-1}\alpha x)\,K_{\mu+1/2}(2^{-1}\alpha x)$, $\operatorname{Re}\alpha>0,\ \operatorname{Re}\nu>-1$, $\lvert\operatorname{Re}\mu\rvert<1+2^{-1}\operatorname{Re}\nu$	$\frac{e^{2\mu\pi i}\,\Gamma(\nu/2+\mu+1)\,y^{1/2}}{\Gamma(\nu/2-\mu)\,(y^2+\alpha^2)^{1/2}}\times$ $\times Q_{\nu/2-1/2}^{-\mu+1/2}\left[(1+\alpha^2/y^2)^{1/2}\right]\times$ $\times Q_{\nu/2-1/2}^{-\mu-1/2}\left[(1+\alpha^2/y^2)^{1/2}\right]$
(30)	$x^{\nu+1/2}K_\mu(\alpha x)\,K_\mu(\beta x)$, $\operatorname{Re}\alpha>0,\ \operatorname{Re}\beta>0$, $\operatorname{Re}(\nu\pm\mu)>-1,\ \operatorname{Re}\nu>-1$	$\frac{\pi^{1/2}y^{\nu+1/2}\,\Gamma(\nu+\mu+1)\,\Gamma(\nu-\mu+1)}{2^{3/2}(\alpha\beta)^{\nu+1}(u^2-1)^{\nu/2+1/4}}\times$ $\times P_{\mu-1/2}^{-\nu-1/2}(u)$, $2\alpha\beta u=y^2+\beta^2+\alpha^2$
(31)	$x^\lambda K_\mu(\alpha x)\,K_\rho(\beta x)$	См. Bailey W. N., 1936: Proc. London Math. Soc. (2), 41, 215—220.

8.14. Модифицированные функции Бесселя других аргументов

(1)	$x^{1/2-\nu}\exp(-2^{-2}\alpha^2x^2)\times$ $\times I_\nu(2^{-2}\alpha^2x^2)$, $\lvert\arg\alpha\rvert<\pi/4,\ \operatorname{Re}\nu>-1/2$	$(\pi/2)^{-1/2}\alpha^{-1}y^{\nu-1/2}\times$ $\times\exp\left(-\frac{y^2}{4\alpha^2}\right)D_{-2\nu}\left(\frac{y}{\alpha}\right)$

	$f(x)$	$\int_0^\infty f(x)\, I_\nu(xy)\,(xy)^{1/2}\,dx, \quad y>0$
(2)	$x^{-\nu-3/2}\exp(-2^{-2}\alpha^2x^2)\times$ $\times I_{\nu+1}(2^{-2}\alpha^2x^2)$, $\lvert\arg\alpha\rvert<\pi/4$, $\operatorname{Re}\nu>-1$	$(\pi/2)^{-1/2}y^{\nu+1/2}\times$ $\times\exp\left(-\frac{y^2}{4\alpha^2}\right)D_{-2\nu-3}\left(\frac{y}{\alpha}\right)$
(3)	$x^{1/2}\exp(-2^{-2}\alpha x^2)\,I_{\nu/2}(2^{-2}\alpha x^2)$, $\operatorname{Re}\alpha>0$, $\operatorname{Re}\nu>-1$	$(2^{-1}\pi\alpha y)^{-1/2}\exp\left(-\frac{y^2}{2\alpha}\right)$
(4)	$x^{\nu/3+1/6}\exp(-2^{-2}\alpha x^2)\times$ $\times I_{\nu/3+1/6}(2^{-2}\alpha x^2)$, $\operatorname{Re}\alpha>0$, $-1<\operatorname{Re}\nu<5/2$	$\pi^{-1}\alpha^{-\nu/3-2/3}y^{\nu/3+1/6}\exp\left(-\frac{y^2}{4\alpha}\right)\times$ $\times K_{\nu/3+1/6}\left(\frac{y^2}{4\alpha}\right)$
(5)	$x^{1/6-\nu/3}\exp(-2^{-2}\alpha x^2)\times$ $\times I_{\nu/3-1/6}(2^{-2}\alpha x^2)$, $\operatorname{Re}\alpha>0$, $\operatorname{Re}\nu>-1$	$\alpha^{\nu/3-2/3}y^{1/6-\nu/3}\exp\left(-\frac{y^2}{4\alpha}\right)\times$ $\times I_{\nu/3-1/6}\left(\frac{y^2}{4\alpha}\right)$
(6)	$x^{1/2+2\mu-\nu}\exp(-2^{-2}\alpha x^2)\times$ $\times I_\mu(2^{-2}\alpha x^2)$, $\operatorname{Re}\alpha>0$ $\operatorname{Re}\nu>2\operatorname{Re}\mu+1/2>-1/2$	$2^{2\mu-\nu+1/2}(\pi\alpha)^{-1/2}\Gamma(1/2+\mu)\times$ $\times[\Gamma(1/2-\mu+\nu)]^{-1}y^{\nu-2\mu-1/2}\times$ $\times{}_1F_1\left(1/2+\mu;\ 1/2-\mu+\nu;\ -\frac{y^2}{2\alpha}\right)$
(7)	$x^{1/2+\nu-2\mu}\exp(-2^{-2}\alpha^2x^2)\times$ $\times I_\mu(2^{-2}\alpha^2x^2)$, $\lvert\arg\alpha\rvert<\pi/4$ $-1<\operatorname{Re}\nu<2\operatorname{Re}\mu+1/2$	$\pi^{-1/2}2^{(3+2\nu-6\mu)/4}\alpha^{-1/2-\nu+\mu}y^{\mu-1}\times$ $\times\exp\left(-\frac{y^2}{4\alpha^2}\right)W_{k,m}\left(\frac{y^2}{2\alpha^2}\right)$, $2k=1/2+\nu-3\mu$, $2m=-1/2+\mu-\nu$
(8)	$x^\lambda\exp(-2^{-2}\alpha^2x^2)\,I_\mu(2^{-2}\alpha^2x^2)$, $\lvert\arg\alpha\rvert<\pi/4$ $-3/2-\operatorname{Re}(2\mu+\nu)<\operatorname{Re}\lambda<0$	$(2\pi)^{-1/2}\left(\frac{2}{y}\right)^{\lambda+1}\times$ $\times G_{23}^{21}\left(\frac{y^2}{2\alpha^2}\,\middle\vert\,\begin{matrix}1-\mu,\ 1+\mu\\ h,\ 1/2,\ k\end{matrix}\right)$, $h=3/4+\lambda/2+\nu/2$, $k=3/4+\lambda/2-\nu/2$
(9)	$x^{1/2}K_{\nu/2}(2^{-2}\alpha x^2)$, $\operatorname{Re}\alpha>0$, $\operatorname{Re}\nu>-1$	$\pi\alpha^{-1}y^{1/2}[I_{\nu/2}(y^2/\alpha)-\mathbf{L}_{\nu/2}(y^2/\alpha)]$
(10)	$x^{3/2}K_{\nu/2+1/2}(2^{-2}\alpha x^2)$, $\operatorname{Re}\alpha>0$, $\operatorname{Re}\nu>-1$	$2\pi\alpha^{-2}y^{3/2}[I_{\nu/2-1/2}(y^2/\alpha)-$ $-\mathbf{L}_{\nu/2-1/2}(y^2/\alpha)]$

	$f(x)$	$\int_0^\infty f(x) J_\nu(xy)(xy)^{1/2}\,dx, \quad y>0$
(11)	$x^{\nu/3+1/6}\exp(-2^{-2}\alpha x^2)\times$ $\times K_{\nu/3+1/6}(2^{-2}\alpha x^2)$, $\operatorname{Re}\alpha>0,\ \operatorname{Re}\nu>-1$	$\pi\alpha^{-\nu/3-2/3}y^{\nu/3+1/6}\exp\left(-\frac{y^2}{4\alpha}\right)\times$ $\times I_{\nu/3+1/6}\left(\frac{y^2}{4\alpha}\right)$
(12)	$x^{\nu/3+1/6}\exp(2^{-2}\alpha x^2)\times$ $\times K_{\nu/3+1/6}(2^{-2}\alpha x^2)$, $-1<\operatorname{Re}\nu<5/2$	$\alpha^{-\nu/3-2/3}y^{\nu/3+1/6}\exp\left(\frac{y^2}{4\alpha}\right)\times$ $\times K_{\nu/3+1/6}\left(\frac{y^2}{4\alpha}\right)$
(13)	$x^{2\mu+\nu+1/2}\exp(-2^{-2}\alpha^2x^2)\times$ $\times K_\mu(2^{-2}\alpha^2x^2)$, $\lvert\arg\alpha\rvert<\pi/4$ $\operatorname{Re}\nu>-1,\ \operatorname{Re}(2\mu+\nu)>-1$	$\pi^{1/2}2^\mu\alpha^{-2\mu-2\nu-2}y^{\nu+1/2}\times$ $\times\Gamma(1+2\mu+\nu)\times$ $\times[\Gamma(\mu+\nu+3/2)]^{-1}\times$ $\times\,{}_1F_1\left(1+2\mu+\nu;\ \mu+\nu+\frac{3}{2};\ -\frac{y^2}{2\alpha^2}\right)$
(14)	$x^{2\mu+\nu+1/2}\exp(2^{-2}\alpha^2x^2)\times$ $\times K_\mu(2^{-2}\alpha^2x^2)$, $\operatorname{Re}\alpha>0,\ \operatorname{Re}\nu>-1$ $-1<\operatorname{Re}(2\mu+\nu)<-1/2$	$\frac{\pi^{1/2}\,\Gamma(1+2\mu+\nu)\,2^{3/2-k}}{\Gamma(1/2-\mu)}\times$ $\times\alpha^{-2m}y^{-\mu-1}\times$ $\times\exp\left(\frac{y^2}{4\alpha^2}\right)W_{k,m}\left(\frac{y^2}{2\alpha^2}\right)$, $2k=-1/2-3\mu-\nu,\ 2m=1/2+\mu+\nu$
(15)	$x^\lambda\exp(-2^{-2}\alpha^2x^2)K_\mu(2^{-2}\alpha^2x^2)$, $\lvert\arg\alpha\rvert<\pi/4$ $\operatorname{Re}(\lambda+\nu\pm2\mu)>-3/2$	$(\pi/2)^{1/2}\left(\frac{2}{y}\right)^{\lambda+1}\times$ $\times G_{23}^{12}\left(\frac{y^2}{2\alpha^2}\middle\vert\begin{matrix}1-\mu,\ 1+\mu\\ h,\ 1/2,\ k\end{matrix}\right)$, $h=3/4+\lambda/2+\nu/2,\ k=3/4+\lambda/2-\nu/2$
(16)	$x^\lambda\exp(2^{-2}\alpha^2x^2)K_\mu(2^{-2}\alpha^2x^2)$, $\operatorname{Re}\alpha>0$ $-3/2-\operatorname{Re}(\nu\pm2\mu)<\operatorname{Re}\lambda<0$	$(2\pi)^{-1/2}\cos(\mu\pi)(2/y)^{\lambda+1}\times$ $\times G_{23}^{22}\left(\frac{y^2}{2\alpha^2}\middle\vert\begin{matrix}1-\mu,\ 1+\mu\\ h,\ 1/2,\ k\end{matrix}\right)$, $h=3/4+\lambda/2+\nu/2$ $k=3/4+\lambda/2-\nu/2$
(17)	$x^{1/2}I_{\nu/4}(2^{-2}\alpha x^2)K_{\nu/4}(2^{-2}\alpha x^2)$, $\operatorname{Re}\alpha>0,\ \operatorname{Re}\nu>-1$	$\frac{y^{1/2}}{\alpha}I_{\nu/4}\left(\frac{y^2}{4\alpha}\right)K_{\nu/4}\left(\frac{y^2}{4\alpha}\right)$
(18)	$x^{1/2}I_{(\nu-\mu)/4}(2^{-1}\alpha x^2)\times$ $\times K_{(\nu+\mu)/4}(2^{-1}\alpha x^2)$, $\operatorname{Re}\alpha>0$ $\operatorname{Re}\nu>-1,\ \operatorname{Re}(\nu-\mu)>-2$	$\frac{2\,\Gamma(1/2+\nu/4-\mu/4)}{\Gamma(1+\nu/2)\,y^{3/2}}W_{\mu/4,\,\nu/4}\left(\frac{y^2}{4\alpha}\right)\times$ $\times M_{-\mu/4,\,\nu/4}\left(\frac{y^2}{4\alpha}\right)$

	$f(x)$	$\int_0^\infty f(x)\, J_\nu(xy)\,(xy)^{1/2}\,dx, \quad y>0$
(19)	$x^{-5/2} K_\nu(\alpha x^{-1})$, $\operatorname{Re}\alpha>0$, $\lvert\operatorname{Re}\nu\rvert<5/2$	$i\alpha^{-1}y^{1/2}\left[e^{\nu\pi i/2}K_{2\nu}(2\alpha^{1/2}e^{\pi i/4}y^{1/2})-\right.$ $\left.-e^{-\nu\pi i/2}K_{2\nu}(2\alpha^{1/2}e^{-\pi i/4}y^{1/2})\right]$
(20)	$x^{-2\nu-2}K_{1/2-\nu}(\alpha x^{-1})$, $\operatorname{Re}\alpha>0$, $-1/2<\operatorname{Re}\nu<2$	$(2\pi)^{1/2}\alpha^{-\nu-1/2}y^{\nu+1/2}\times$ $\times K_{2\nu}(2^{1/2}\alpha^{1/2}y^{1/2})\,J_{2\nu}(2^{1/2}\alpha^{1/2}y^{1/2})$
(21)	$K_{-2\nu-1}(2\alpha x^{1/2})$, $\operatorname{Re}\alpha>0$, $\operatorname{Re}\nu>-1$	$-\dfrac{\pi\alpha\left[\mathbf{H}_{-\nu-1}(\alpha^2y^{-1})-Y_{-\nu-1}(\alpha^2y^{-1})\right]}{4\cos(\nu\pi)\,y^{3/2}}$
(22)	$x^{-1/2}K_{2\nu}(2\alpha x^{1/2})$, $\operatorname{Re}\alpha>0$, $\operatorname{Re}\nu>-1/2$	$\dfrac{\pi\left[\mathbf{H}_{-\nu}(\alpha^2y^{-1}-Y_{-\nu}(\alpha^2y^{-1})\right]}{4\cos(\nu\pi)\,y^{1/2}}$
(23)	$x^{1/2}J_\nu(2\alpha^{1/2}x^{1/2})\,K_\nu(2\alpha^{1/2}x^{1/2})$, $\operatorname{Re}\alpha>0$, $\operatorname{Re}\nu>-1$	$2^{-1}y^{-3/2}e^{-2\alpha/y}$
(24)	$x^{\nu+1/2}J_{2\nu}(2\alpha^{1/2}x^{1/2})\,K_{2\nu}(2\alpha^{1/2}x^{1/2})$, $\operatorname{Re}\alpha>0$, $\operatorname{Re}\nu>-1/2$	$\pi^{-1/2}2^\nu\alpha^{\nu+1/2}y^{-2\nu-2}K_{1/2-\nu}(2\alpha/y)$
(25)	$x^{-\nu-1/2}J_{2\nu+1}(2\alpha^{1/2}x^{1/2})\times$ $\times K_{2\nu+1}(2\alpha^{1/2}x^{1/2})$, $\operatorname{Re}\alpha>0$, $\operatorname{Re}\nu>-1$	$\pi^{1/2}2^{-\nu-2}\alpha^{-\nu-1/2}y^{2\nu}\left[I_{\nu+1/2}(2\alpha/y)-\right.$ $\left.-\mathbf{L}_{\nu+1/2}(2\alpha/y)\right]$
(26)	$x^{-1/2}J_\mu(2\alpha^{1/2}x^{1/2})\,K_\mu(2\alpha^{1/2}x^{1/2})$, $\operatorname{Re}\alpha>0$, $\operatorname{Re}\nu>-1$, $\operatorname{Re}(\nu+\mu)>-1$	$\dfrac{\Gamma(1/2+\mu/2+\nu/2)y^{1/2}}{4\alpha\,\Gamma(1+\mu)}W_{-\nu/2,\,\mu/2}\left(\dfrac{2\alpha}{y}\right)\times$ $\times M_{\nu/2,\,\mu/2}\left(\dfrac{2\alpha}{y}\right)$
(27)	$x^{-1/2}\left[K_{2\nu}(2a^{1/2}x^{1/2})-\right.$ $\left.-2^{-1}\pi Y_{2\nu}(2a^{1/2}x^{1/2})\right]$, $a>0$, $\operatorname{Re}\nu>-1/2$	$-2^{-1}\pi y^{-1/2}Y_\nu(a/y)$
(28)	$x^{-1/2}K_\nu(2\alpha^{1/2}x^{1/2})\,Y_\nu(2\alpha^{1/2}x^{1/2})$, $\operatorname{Re}\alpha>0$, $\operatorname{Re}\nu>-1/2$	$-2^{-2}\alpha^{-1}y^{1/2}W_{\nu/2,\,\nu/2}(2\alpha/y)\times$ $\times W_{-\nu/2,\,\nu/2}(2\alpha/y)$
(29)	$x^{-1/2}K_\mu(2\alpha^{1/2}x^{1/2})\times$ $\times\{\sin[2^{-1}(\mu-\nu)\pi]\,J_\mu(2\alpha^{1/2}x^{1/2})+$ $+\cos[2^{-1}(\mu-\nu)\pi]\,Y_\mu(2\alpha^{1/2}x^{1/2})\}$, $\operatorname{Re}\alpha>0$, $\operatorname{Re}(\nu\pm\mu)>-1$	$-2^{-2}\alpha^{-1}y^{1/2}W_{\nu/2,\,\mu/2}(2\alpha/y)\times$ $\times W_{-\nu/2,\,\mu/2}(2\alpha/y)$

	$f(x)$	$\int_0^\infty f(x)\,J_\nu(xy)\,(xy)^{1/2}\,dx, \quad y>0$
(30)	$x^{-1/2}K_\mu[(2\alpha x)^{1/2}\exp(2^{-2}\pi i)]\times$ $\times K_\mu[(2\alpha x)^{1/2}\exp(-2^{-2}\pi i)]$, $\operatorname{Re}\alpha>0,\ \operatorname{Re}(\nu\pm\mu)>-1$	$2^{-2}\alpha^{-1}y^{1/2}\,\Gamma\left(\frac{1+\mu+\nu}{2}\right)\Gamma\left(\frac{1-\mu+\nu}{2}\right)\times$ $\times W_{-\nu/2,\,\mu/2}(\alpha y^{-1}\exp(2^{-1}\pi i))\times$ $\times W_{-\nu/2,\,\mu/2}(\alpha y^{-1}\exp(-2^{-1}\pi i))$
(31)	$x^{-\nu-1/2}K_{2\nu+1}[(2\alpha x)^{1/2}\exp(2^{-2}\pi i)]\times$ $\times K_{2\nu+1}[(2\alpha x)^{1/2}\exp(-2^{-2}\pi i)]$, $\operatorname{Re}\alpha>0,\ -1<\operatorname{Re}\nu<0$	$-\dfrac{\pi^{3/2}y^{2\nu}}{2^{5/2}\sin(\nu\pi)\,\alpha^{\nu+1/2}}\times$ $\times[\mathbf{H}_{\nu+1/2}(\alpha/y)-Y_{\nu+1/2}(\alpha/y)]$
(32)	$x^{\nu+1/2}(x^2+\beta^2)^{-\nu/2-1/4}\times$ $\times K_{\nu+1/2}[\alpha(x^2+\beta^2)^{1/2}]$, $\operatorname{Re}\alpha>0,\ \operatorname{Re}\beta>0$ $\operatorname{Re}\nu>-1$	$\pi^{1/2}2^{-1/2}\alpha^{-\nu-1/2}y^{\nu+1/2}(\alpha^2+y^2)^{-1/2}\times$ $\times\exp[-\beta(\alpha^2+y^2)^{1/2}]$
(33)	$x^{\nu+1/2}(x^2+\beta^2)^{-\nu/2-3/4}\times$ $\times K_{\nu+3/2}[\alpha(x^2+\beta^2)^{1/2}]$, $\operatorname{Re}\alpha>0,\ \operatorname{Re}\beta>0$ $\operatorname{Re}\nu>-1$	$\pi^{1/2}2^{-1/2}\alpha^{-\nu-3/2}\beta^{-1}y^{\nu+1/2}\times$ $\times\exp[-\beta(\alpha^2+y^2)^{1/2}]$
(34)	$x^{\nu+1/2}(x^2+\alpha^2)^{-(\nu+1)/4}\times$ $\times K_{(\nu+1)/2}[\alpha(x^2+\alpha^2)^{1/2}]$, $\operatorname{Re}\alpha>0,\ \operatorname{Re}\nu>-1$	$y^{\nu+1/2}(y^2+\alpha^2)^{-(\nu+1)/4}\times$ $\times K_{(\nu+1)/2}[\alpha(y^2+\alpha^2)^{1/2}]$
(35)	$x^{\nu+1/2}(x^2+\beta^2)^{-\mu/2}\times$ $\times K_\mu[\alpha(x^2+\beta^2)^{1/2}]$, $\operatorname{Re}\alpha>0$ $\operatorname{Re}\beta>0,\ \operatorname{Re}\nu>-1$	$\alpha^{-\mu}\beta^{\nu+1-\mu}\times$ $\times y^{\nu+1/2}(\alpha^2+y^2)^{\mu/2-\nu/2-1/2}\times$ $\times K_{\mu-\nu-1}[\beta(\alpha^2+y^2)^{1/2}]$
(36)	$x^{\nu+1/2}(b^2-x^2)^{\mu/2}\times$ $\times Y_\mu[\alpha(b^2-x^2)^{1/2}]$, $0<x<b$ $-2\pi^{-1}x^{\nu+1/2}(x^2-b^2)^{\mu/2}\times$ $\times K_\mu[\alpha(x^2-b^2)^{1/2}]$, $b<x<\infty$ $\operatorname{Re}\alpha>0,\ \operatorname{Re}\nu>-1,\ \operatorname{Re}\mu>-1$	$\alpha^\mu b^{\mu+\nu+1}\times$ $\times y^{\nu+1/2}(\alpha^2+y^2)^{-(\mu+\nu+1)/2}\times$ $\times Y_{\mu+\nu+1}[b(\alpha^2+y^2)^{1/2}]$
(37)	$x^{1/2}I_{\nu/2}\{2^{-1}\beta[(\alpha^2+x^2)^{1/2}-\alpha]\}\times$ $\times K_{\nu/2}\{2^{-1}\beta[(\alpha^2+x^2)^{1/2}+\alpha]\}$, $\operatorname{Re}\alpha>0,\ \operatorname{Re}\beta>0$ $\operatorname{Re}\nu>-1$	$y^{-1/2}(\beta^2+y^2)^{-1/2}\times$ $\times\exp[-\alpha(\beta^2+y^2)^{1/2}]$

8.15. Функции, родственные функциям Бесселя

	$f(x)$	$\int\limits_0^\infty f(x)\, J_\nu(xy)\,(xy)^{1/2}\,dx, \quad y>0$
(1)	$\mathbf{H}_{\nu-1/2}(ax)$, $a>0,\ -1<\operatorname{Re}\nu<3/2$	$2^{1/2}\pi^{-1/2}a^{\nu-1/2}y^{1/2-\nu}(a^2-y^2)^{-1/2}$, $0<y<a$; 0, $a<y<\infty$
(2)	$x^\lambda\,\mathbf{H}_\mu(ax)$, $a>0$, $-5/2-\operatorname{Re}\nu<\operatorname{Re}(\lambda+\mu)<0$	$2^{\lambda+1/2}y^{-\lambda-1}\times$ $\times G_{33}^{21}\left(\frac{y^2}{a^2}\middle\vert\begin{matrix}\frac{1-\mu}{2}, & 1-\frac{\mu}{2}, & 1+\frac{\mu}{2}\\ \frac{3}{4}+\frac{\lambda+\nu}{2}, & \frac{1-\mu}{2}, & \frac{3}{4}+\frac{\lambda-\nu}{2}\end{matrix}\right)$
(3)	$x^{1/2}\,\mathbf{H}_{\nu/2}(2^{-2}ax^2)$, $a>0,\ -2<\operatorname{Re}\nu<3/2$	$-2a^{-1}y^{1/2}\,Y_{\nu/2}(y^2/a)$
(4)	$x^\lambda\,\mathbf{H}_\mu(a/x)$, $a>0,\ \operatorname{Re}(\lambda+\nu)>-2$, $-\operatorname{Re}\nu-5/2<\operatorname{Re}(\lambda-\mu)<1$	$2^{\lambda+1/2}y^{-\lambda-1}\times$ $\times G_{15}^{21}\left(\frac{a^2y^2}{16}\middle\vert\begin{matrix}\frac{1+\mu}{2}\\ h,\ \frac{1+\mu}{2},\ \frac{\mu}{2},\ -\frac{\mu}{2},\ k\end{matrix}\right)$, $h=\frac{3}{4}+\frac{\lambda+\nu}{2},\quad k=\frac{3}{4}+\frac{\lambda-\nu}{2}$
(5)	$x^{1/2}\,[\mathbf{H}_{-\nu}(\alpha x)-Y_{-\nu}(\alpha x)]$, $\lvert\arg\alpha\rvert<\pi,\ -1/2<\operatorname{Re}\nu$	$2\alpha^{-\nu}\pi^{-1}\cos(\nu\pi)\,y^{\nu-1/2}(y+\alpha)^{-1}$
(6)	$x^\lambda\,[\mathbf{H}_\mu(\alpha x)-Y_\mu(\alpha x)]$, $\lvert\arg\alpha\rvert<\pi,\ \operatorname{Re}(\lambda+\mu)<1$, $\operatorname{Re}(\lambda+\nu)+3/2>\lvert\operatorname{Re}\mu\rvert$	$2^{\lambda+1/2}\pi^{-2}\cos(\mu\pi)\,y^{-\lambda-1}\times$ $\times G_{33}^{23}\left(\frac{y^2}{\alpha^2}\middle\vert\begin{matrix}\frac{1-\mu}{2}, & 1-\frac{\mu}{2}, & 1+\frac{\mu}{2}\\ \frac{3}{4}+\frac{\lambda+\nu}{2}, & \frac{1-\mu}{2}, & \frac{3}{4}+\frac{\lambda-\nu}{2}\end{matrix}\right)$
(7)	$x^{-1/2}\,[\mathbf{H}_{-\nu}(\alpha x^{-1})-Y_{-\nu}(\alpha x^{-1})]$, $\lvert\arg\alpha\rvert<\pi,\ \lvert\operatorname{Re}\nu\rvert<1/2$	$4\pi^{-1}\cos(\nu\pi)\,y^{-1/2}K_{2\nu}(2\alpha^{1/2}y^{1/2})$
(8)	$x^{-3/2}\,[\mathbf{H}_{-\nu-1}(\alpha x^{-1})-$ $-Y_{-\nu-1}(\alpha x^{-1})]$, $\lvert\arg\alpha\rvert<\pi,\ \lvert\operatorname{Re}\nu\rvert<1/2$	$-4\pi^{-1}\alpha^{-1/2}\cos(\nu\pi)\times$ $\times K_{-2\nu-1}(2\alpha^{1/2}y^{1/2})$
(9)	$x^{2\nu}\,[\mathbf{H}_{\nu+1/2}(\alpha x^{-1})-$ $-Y_{\nu+1/2}(\alpha x^{-1})]$, $\lvert\arg\alpha\rvert<\pi,\ -1<\operatorname{Re}\nu<-1/6$	$-2^{5/2}\pi^{-3/2}\alpha^{\nu+1/2}y^{-\nu-1/2}\sin(\nu\pi)\times$ $\times K_{2\nu+1}(2^{1/2}\alpha^{1/2}e^{\pi i/4}y^{1/2})\times$ $\times K_{2\nu+1}(2^{1/2}\alpha^{1/2}e^{-\pi i/4}y^{1/2})$

	$f(x)$	$\int_0^\infty f(x) J_\nu(xy)(xy)^{1/2}\,dx,\quad y>0$
(10)	$x^\lambda[\mathbf{H}_\mu(\alpha x^{-1}) - Y_\mu(\alpha x^{-1})]$, $\lvert\arg\alpha\rvert<\pi$, $\operatorname{Re}\lambda<-\lvert\operatorname{Re}\mu\rvert$ $\operatorname{Re}(\nu-\mu+\lambda)>-5/2$	$2^{\lambda+1/2}\pi^{-2}\cos(\mu\pi)\,y^{-\lambda-1}\times$ $\times G_{15}^{41}\left(\frac{\alpha^2y^2}{16}\middle\vert\begin{matrix}\frac{1+\mu}{2}\\ h,\ \frac{1+\mu}{2},\ -\frac{\mu}{2},\ \frac{\mu}{2},\ k\end{matrix}\right)$, $h=\frac{3}{4}+\frac{\lambda+\nu}{2},\quad k=\frac{3}{4}+\frac{\lambda-\nu}{2}$
(11)	$I_{\nu-1/2}(\alpha x)-\mathbf{L}_{\nu-1/2}(\alpha x)$, $\operatorname{Re}\alpha>0$, $\lvert\operatorname{Re}\nu\rvert<1/2$	$2^{1/2}\pi^{-1/2}\alpha^{\nu-1/2}y^{1/2-\nu}(y^2+\alpha^2)^{-1/2}$
(12)	$x^{1/2}[I_\nu(\alpha x)-\mathbf{L}_\nu(\alpha x)]$, $\operatorname{Re}\alpha>0$, $-1<\operatorname{Re}\nu<-1/2$	$2\pi^{-1}\alpha^{\nu+1}y^{-\nu-1/2}(y^2+\alpha^2)^{-1}$
(13)	$x^{\mu-\nu+1/2}[I_\mu(\alpha x)-\mathbf{L}_\mu(\alpha x)]$, $\operatorname{Re}\alpha>0$ $-1<2\operatorname{Re}\mu+1<\operatorname{Re}\nu+1/2$	$\frac{2^{\mu-\nu+1}\alpha^{\mu-1}y^{\nu-2\mu-1/2}}{\pi^{1/2}\Gamma(\nu-\mu+1/2)}\times$ $\times{}_2F_1(1,\ 1/2;\ \nu-\mu+1/2;\ -y^2/\alpha^2)$
(14)	$x^{\nu-\mu+1/2}[I_\mu(\alpha x)-\mathbf{L}_\mu(\alpha x)]$, $\operatorname{Re}\alpha>0$, $-1<\operatorname{Re}\nu<-1/2$	$\frac{2^{\nu-\mu+1}\Gamma(3/2+\nu)\alpha^{\mu+1}}{\pi\Gamma(3/2+\mu)y^{\nu+5/2}}\times$ $\times{}_2F_1(1,\ 3/2+\nu;\ 3/2+\mu;\ -\alpha^2/y^2)$
(15)	$x^{\nu-\mu-1/2}[I_\mu(\alpha x)-\mathbf{L}_\mu(\alpha x)]$, $\operatorname{Re}\alpha>0$, $\lvert\operatorname{Re}\nu\rvert<1/2$	$\frac{2^{\nu-\mu}\Gamma(1/2+\nu)\alpha^{\mu}}{\pi^{1/2}\Gamma(1+\mu)y^{\nu+1/2}}\times$ $\times{}_2F_1(1/2+\nu,\ 1/2;\ 1+\mu;\ -\alpha^2/y^2)$
(16)	$x^\lambda[I_\mu(\alpha x)-\mathbf{L}_\mu(\alpha x)]$, $\operatorname{Re}\alpha>0$ $-\operatorname{Re}\nu-3/2<\operatorname{Re}(\lambda+\mu)<0$	$2^{\lambda+1/2}\pi^{-1}y^{-\lambda-1}\times$ $\times G_{33}^{22}\left(\frac{y^2}{\alpha^2}\middle\vert\begin{matrix}1-\frac{\mu}{2},\ \frac{1-\mu}{2},\ 1+\frac{\mu}{2}\\ \frac{3}{4}+\frac{\lambda+\nu}{2},\ \frac{1-\mu}{2},\ \frac{3}{4}+\frac{\lambda-\nu}{2}\end{matrix}\right)$
(17)	$x^{1/2}[I_\nu(\alpha x)-\mathbf{L}_{-\nu}(\alpha x)]$, $\operatorname{Re}\alpha>0$, $\operatorname{Re}\nu>1/2$	$2\pi^{-1}\alpha^{1-\nu}y^{\nu-1/2}\cos(\nu\pi)(y^2+\alpha^2)^{-1}$
(18)	$x^{\mu-\nu+1/2}[I_\mu(\alpha x)-\mathbf{L}_{-\mu}(\alpha x)]$, $\operatorname{Re}\alpha>0$, $\operatorname{Re}\nu>-1/2$, $\operatorname{Re}\mu>-1$	$\frac{2^{\mu-\nu+1}\alpha^{-\mu-1}y^{\nu-1/2}}{\Gamma(1/2-\mu)\Gamma(1/2+\nu)}\times$ $\times{}_2F_1(1,\ 1/2+\mu;\ 1/2+\nu;\ -y^2/\alpha^2)$
(19)	$x^{\nu-\mu+1/2}[I_\mu(\alpha x)-\mathbf{L}_{-\mu}(\alpha x)]$, $\operatorname{Re}\alpha>0$, $\operatorname{Re}\nu>-1/2$ $\operatorname{Re}(\nu-\mu)>-1$, $\operatorname{Re}(\nu-2\mu)<1/2$	$2^{2+\nu-\mu}\pi^{-3/2}\cos(\mu\pi)\times$ $\times\Gamma(3/2+\nu-\mu)\,\alpha^{1-\mu}y^{-5/2+2\mu-\nu}\times$ $\times{}_2F_1(3/2+\nu-\mu,\ 1;\ 3/2;\ -\alpha^2/y^2)$

	$f(x)$	$\int\limits_0^\infty f(x)\, J_\nu(xy)\,(xy)^{1/2}\,dx, \quad y>0$
(20)	$x^{\mu+\nu-1/2}\,[I_\mu(\alpha x)-\mathbf{L}_{-\mu}(\alpha x)]$, $\operatorname{Re}\alpha>0,\ -1<\operatorname{Re}\nu<3/2$, $\operatorname{Re}(\mu+\nu)>-1/2$	$\dfrac{2^{\mu+\nu}\,\Gamma(1/2+\mu+\nu)}{\Gamma(1+\mu)\,\Gamma(1/2-\mu)}\,\alpha^\mu y^{-1/2-2\mu-\nu}\times$ $\times\,{}_2F_1(1/2+\mu+\nu,\ 1/2+\mu;\ 1+\mu;\ -\alpha^2/y^2)$
(21)	$x^\lambda\,[I_\mu(\alpha x)-\mathbf{L}_{-\mu}(\alpha x)]$, $\operatorname{Re}\alpha>0,\ \operatorname{Re}(\mu+\nu+\lambda)>-3/2$, $-\operatorname{Re}\nu-5/2<\operatorname{Re}(\lambda-\mu)<1$	$2^{\lambda+1/2}\pi^{-1}\cos(\mu\pi)\,y^{-\lambda-1}\times$ $\times G_{33}^{22}\left(\dfrac{y^2}{\alpha^2}\,\middle\vert\,\begin{matrix}\frac{1+\mu}{2},\ 1-\frac{\mu}{2},\ 1+\frac{\mu}{2}\\ \frac{3}{4}+\frac{\lambda+\nu}{2},\ \frac{1+\mu}{2},\ \frac{3}{4}+\frac{\lambda-\nu}{2}\end{matrix}\right)$
(22)	$x^{2\nu}\left[I_{\nu+1/2}(\alpha x^{-1})-\mathbf{L}_{\nu+1/2}(\alpha x^{-1})\right]$, $\operatorname{Re}\alpha>0,\ -1<\operatorname{Re}\nu<1/2$	$2^{3/2}\pi^{-1/2}y^{-\nu-1/2}\alpha^{\nu+1/2}\times$ $\times\,J_{2\nu+1}\left[(2\alpha y)^{1/2}\right]K_{2\nu+1}\left[(2\alpha y)^{1/2}\right]$
(23)	$x^{\nu+1/2}\,U_{\nu+1}(2a^2\beta,\ ax)$, $a>0,\ \operatorname{Re}\nu>-1$	$(2\beta)^{\nu+1}y^{\nu+1/2}\cos[\beta(a^2-y^2)]$, $0<y<a$; 0, $a<y<\infty$
(24)	$x^{\nu+1/2}\,U_{\nu+2}(2a^2\beta,\ ax)$, $a>0,\ \operatorname{Re}\nu>-1$	$(2\beta)^{\nu+1}y^{\nu+1/2}\sin[\beta(a^2-y^2)]$, $0<y<a$; 0, $a<y<\infty$

8.16. Функции параболического цилиндра

(1)	$x^{\nu-1/2}\exp\left(-2^{-2}x^2\right)D_{2\nu-1}(x)$, $\operatorname{Re}\nu>-1/2$	$-\dfrac{y^{\nu-1/2}\exp\left(-2^{-2}y^2\right)}{2\cos(\nu\pi)}\times$ $\times\,[D_{2\nu-1}(y)-D_{2\nu-1}(-y)]$
(2)	$x^{\nu-1/2}\exp\left(-2^{-2}x^2\right)\times$ $\times\,\{[1-2\cos(\nu\pi)]\,D_{2\nu-1}(x)-$ $-D_{2\nu-1}(-x)\}$, $\operatorname{Re}\nu>-1/2$	$y^{\nu-1/2}\exp\left(-2^{-2}y^2\right)\times$ $\times\,\{[1-2\cos(\nu\pi)]\,D_{2\nu-1}(y)-$ $-D_{2\nu-1}(-y)\}$
(3)	$x^{\nu-1/2}\exp\left(-2^{-2}x^2\right)\times$ $\times\,\{[1+2\cos(\nu\pi)]\,D_{2\nu-1}(x)-$ $-D_{2\nu-1}(-x)\}$, $\operatorname{Re}\nu>-1/2$	$-y^{\nu-1/2}\exp\left(-2^{-2}y^2\right)\times$ $\times\,\{[1+2\cos(\nu\pi)]\,D_{2\nu-1}(y)-$ $-D_{2\nu-1}(-y)\}$
(4)	$x^{\nu-1/2}\exp\left(2^{-2}x^2\right)D_{2\nu-1}(x)$, $-1/2<\operatorname{Re}\nu<1/2$	$2^{1/2-\nu}\pi\sin(\nu\pi)\,\Gamma(2\nu)\,y^{1/2-\nu}\times$ $\times\exp\left(2^{-2}y^2\right)K_\nu\left(2^{-2}y^2\right)$

	$f(x)$	$\int_0^\infty f(x) J_\nu(xy)(xy)^{1/2}\,dx, \quad y>0$
(5)	$x^{\nu-1/2}\exp(-2^{-2}x^2) D_{2\nu+1}(x)$, $\operatorname{Re}\nu>-1/2$	$\frac{\exp(-2^{-2}y^2)\, y^{\nu+1/2}}{2\cos(\nu\pi)} \times [D_{2\nu}(y)+D_{2\nu}(-y)]$
(6)	$x^{\nu-1/2}\exp(-2^{-2}x^2) \times \{[1+2\cos(\nu\pi)] D_{2\nu+1}(x) - D_{2\nu+1}(-x)\}$, $\operatorname{Re}\nu>-1/2$	$y^{\nu+1/2}\exp(-2^{-2}y^2) \times \{[1+2\cos(\nu\pi)] D_{2\nu}(y) + D_{2\nu}(-y)\}$
(7)	$x^{\nu-1/2}\exp(-2^{-2}x^2) \times \{[1-2\cos(\nu\pi)] D_{2\nu+1}(x) - D_{2\nu+1}(-x)\}$, $\operatorname{Re}\nu>-1/2$	$-y^{\nu+1/2}\exp(-2^{-2}y^2) \times \{[1-2\cos(\nu\pi)] D_{2\nu}(y) + D_{2\nu}(-y)\}$
(8)	$x^{\nu-1/2}\exp(-2^{-2}x^2) D_{-2\nu}(x)$, $\operatorname{Re}\nu>-1/2$	$2^{-1/2}\pi^{1/2}y^{1/2-\nu} \times \exp(-2^{-2}y^2) I_\nu(2^{-2}y^2)$
(9)	$x^{\nu-1/2}\exp(2^{-2}x^2) D_{-2\nu}(x)$, $\operatorname{Re}\nu>-1/2$	$y^{\nu-1/2}\exp(2^{-2}y^2) D_{-2\nu}(y)$
(10)	$x^{\nu-1/2}\exp(2^{-2}x^2) D_{-2\nu-2}(x)$, $\operatorname{Re}\nu>-1/2$	$(2\nu+1)^{-1} y^{\nu+1/2} \times \exp(2^{-2}y^2) D_{-2\nu-1}(y)$
(11)	$x^{\nu-1/2}\exp(-2^{-2}\alpha^2x^2) D_{2\mu}(\alpha x)$, $\lvert\arg\alpha\rvert<\pi/4$, $\operatorname{Re}\nu>-1/2$	$\frac{2^{\mu-1/2}\Gamma(\nu+1/2)\, y^{\nu+1/2}}{\Gamma(\nu-\mu+1)\,\alpha^{1+2\nu}} \times {}_1F_1\left(\nu+\frac{1}{2};\ \nu-\mu+1;\ -\frac{y^2}{2\alpha^2}\right)$
(12)	$x^{\nu-1/2}\exp(2^{-2}\alpha^2x^2) D_{2\mu}(\alpha x)$, $\lvert\arg\alpha\rvert<\pi/4$, $-1/2<\operatorname{Re}\nu<\operatorname{Re}(1/2-2\mu)$	$\frac{\Gamma(1/2+\nu)\,\alpha^{2k}2^{m+\mu}}{\Gamma(1/2-\mu)\, y^{\mu+1}} \exp\left(\frac{y^2}{4\alpha^2}\right) \times W_{k,m}\left(\frac{y^2}{2\alpha^2}\right)$, $2k=1/2+\mu-\nu$, $2m=1/2+\mu+\nu$
(13)	$x^{\nu+1/2}\exp(-2^{-2}x^2) D_{2\nu}(x)$, $\operatorname{Re}\nu>-1$	$\frac{y^{\nu-1/2}\exp(-2^{-2}y^2)}{2\cos(\nu\pi)} \times [D_{2\nu+1}(y)-D_{2\nu+1}(-y)]$
(14)	$x^{\nu+1/2}\exp(-2^{-2}x^2) \times \{[1+2\cos(\nu\pi)] D_{2\nu}(x) + D_{2\nu}(-x)\}$, $\operatorname{Re}\nu>-1$	$y^{\nu-1/2}\exp(-2^{-2}y^2) \times \{[1+2\cos(\nu\pi)] D_{2\nu+1}(y) - D_{2\nu+1}(-y)\}$

	$f(x)$	$\int\limits_0^\infty f(x)\, J_\nu(xy)\,(xy)^{1/2}\,dx, \quad y>0$
(15)	$x^{\nu+1/2}\exp\left(-2^{-2}x^2\right)\times$ $\times\{[1-2\cos(\nu\pi)]\, D_{2\nu}(x)+$ $+D_{2\nu}(-x)\}, \quad \operatorname{Re}\nu>-1$	$-y^{\nu-1/2}\exp\left(-2^{-2}y^2\right)\times$ $\times\{[1-2\cos(\nu\pi)]\, D_{2\nu+1}(y)-$ $-D_{2\nu+1}(-y)\}$
(16)	$x^{\nu+1/2}\exp\left(-2^{-2}x^2\right) D_{2\nu+2}(x),$ $\operatorname{Re}\nu>-1$	$-\dfrac{y^{\nu+1/2}\exp\left(-2^{-2}y^2\right)}{2\cos(\nu\pi)}\times$ $\times\left[D_{2\nu+2}(y)+D_{2\nu+2}(-y)\right]$
(17)	$x^{\nu+1/2}\exp\left(-2^{-2}x^2\right)\times$ $\times\{[1-2\cos(\nu\pi)]\, D_{2\nu+2}(x)+$ $+D_{2\nu+2}(-x)\}, \quad \operatorname{Re}\nu>-1$	$y^{\nu+1/2}\exp\left(-2^{-2}y^2\right)\times$ $\times\{[1-2\cos(\nu\pi)]\, D_{2\nu+2}(y)+$ $+D_{2\nu+2}(-y)\}$
(18)	$x^{\nu+1/2}\exp\left(-2^{-2}x^2\right)\times$ $\times\{[1+2\cos(\nu\pi)]\, D_{2\nu+2}(x)+$ $+D_{2\nu+2}(-x)\}, \quad \operatorname{Re}\nu>-1$	$-y^{\nu+1/2}\exp\left(-2^{-2}y^2\right)\times$ $\times\{[1+2\cos(\nu\pi)]\, D_{2\nu+2}(y)+$ $+D_{2\nu+2}(-y)\}$
(19)	$x^{\nu+1/2}\exp\left(2^{-2}x^2\right) D_{2\nu+2}(x),$ $-1<\operatorname{Re}\nu<-5/6$	$\pi^{-1}\sin(\nu\pi)\,\Gamma(2\nu+3)\, y^{-\nu-3/2}\times$ $\times\exp\left(2^{-2}y^2\right) K_{\nu+1}\left(2^{-2}y^2\right)$
(20)	$x^{\nu+1/2}\exp\left(2^{-2}x^2\right) D_{-2\nu-1}(x),$ $\operatorname{Re}\nu>-1/2$	$(2\nu+1)\, y^{\nu-1/2}\times$ $\times\exp\left(2^{-2}y^2\right) D_{-2\nu-2}(y)$
(21)	$x^{\nu+1/2}\exp\left(-2^{-2}x^2\right) D_{-2\nu-3}(x),$ $\operatorname{Re}\nu>-1$	$2^{-1/2}\pi^{1/2}y^{-\nu-3/2}\exp\left(-2^{-2}y^2\right)\times$ $\times I_{\nu+1}\left(2^{-2}y^2\right)$
(22)	$x^{\nu+1/2}\exp\left(2^{-2}x^2\right) D_{-2\nu-3}(x),$ $\operatorname{Re}\nu>-1$	$y^{\nu+1/2}\exp\left(2^{-2}y^2\right) D_{-2\nu-3}(y)$
(23)	$x^{\nu+1/2}\exp\left(-2^{-2}\alpha^2x^2\right) D_{2\mu}(\alpha x),$ $\lvert\arg\alpha\rvert<\pi/4, \quad \operatorname{Re}\nu>-1$	$\dfrac{2^{\mu}\,\Gamma(\nu+3/2)\, y^{\nu+1/2}}{\Gamma(\nu-\mu+3/2)\,\alpha^{2\nu+2}}\times$ $\times\, {}_1F_1\left(\nu+\dfrac{3}{2};\ \nu-\mu+\dfrac{3}{2};\ -\dfrac{y^2}{2\alpha^2}\right)$
(24)	$x^{\nu+1/2}\exp\left(2^{-2}\alpha^2x^2\right) D_{2\mu}(\alpha x),$ $\lvert\arg\alpha\rvert<3\pi/4$ $-1<\operatorname{Re}\nu<-1/2-2\operatorname{Re}\mu$	$\dfrac{\Gamma(3/2+\nu)\, 2^{1/2+m+\mu}\alpha^{2k+1}}{\Gamma(-\mu)\, y^{\mu+3/2}}\times$ $\times\exp\left(\dfrac{y^2}{4\alpha^2}\right) W_{k,\,m}\left(\dfrac{y^2}{2\alpha^2}\right),$ $2k=\mu-\nu-1,\ 2m=\mu+\nu+1$

	$f(x)$	$\int_0^\infty f(x)\, J_\nu(xy)\,(xy)^{1/2}\,dx, \quad y>0$
(25)	$x^\lambda \exp(-2^{-2}\alpha^2x^2)\, D_\mu(\alpha x)$, $\lvert\arg\alpha\rvert<\pi/4$, $\mathrm{Re}(\lambda+\nu)>-3/2$	$\dfrac{2^{-c-3\nu/2}\pi^{1/2}\,\Gamma(2b)\,y^{\nu+1/2}}{\Gamma(\nu+1)\,\Gamma(c+1/2)\,\alpha^{2b}}\times$ $\times\,{}_2F_2\left(b,\ b+\frac{1}{2};\ \nu+1,\ c+\frac{1}{2};\ -\frac{y^2}{2\alpha^2}\right)$, $2b=\lambda+\nu+3/2,\ 2c=\lambda-\mu+3/2$
(26)	$x^\lambda \exp(2^{-2}\alpha^2x^2)\, D_\mu(\alpha x)$, $\lvert\arg\alpha\rvert<3\pi/4$, $\mathrm{Re}\,\mu<-\mathrm{Re}\,\lambda<\mathrm{Re}\,\nu+3/2$	$\dfrac{2^{\lambda-\mu/2-1/2}\pi^{-1/2}}{\Gamma(-\mu)\,y^{\lambda+1}}\times$ $\times\, G_{23}^{22}\left(\frac{y^2}{2\alpha^2}\,\middle\vert\, \begin{matrix}1/2,\ 1\\ \tau+\nu,\ -\mu/2,\ \tau\end{matrix}\right)$, $\tau=\frac{3}{4}+\frac{\lambda-\nu}{2}$
(27)	$D_{-1/2-\nu}(\alpha\exp(2^{-2}\pi i)\,x^{1/2})\times$ $\times D_{-1/2-\nu}(\alpha\exp(-2^{-2}\pi i)\,x^{1/2})$, $\mathrm{Re}\,\alpha>0,\ \mathrm{Re}\,\nu>-1/2$	$2^{-\nu}\pi^{1/2}[\Gamma(\nu+1/2)]^{-1}y^{-\nu-1/2}\times$ $\times(\alpha^2+2y)^{-1/2}[(\alpha^2+2y)^{1/2}-\alpha]^{2\nu}$
(28)	$x^{-1/2}D_{-1/2-\nu}(\alpha\exp(2^{-2}\pi i)\,x^{-1/2})\times$ $\times D_{-1/2-\nu}(\alpha\exp(-2^{-2}\pi i)\,x^{-1/2})$, $\mathrm{Re}\,\alpha>0,\ \mathrm{Re}\,\nu>-1/2$	$2^{1/2}\pi^{1/2}[\Gamma(\nu+1/2)]^{-1}y^{-1}\times$ $\times\exp[-\alpha(2y)^{1/2}]$

8.17. Гипергеометрическая функция Гаусса

(1)	$x^{2\alpha+\nu-1/2}\times$ $\times\,{}_2F_1(\alpha-\nu-1/2,\ \alpha;\ 2\alpha;\ -\lambda^2x^2)$, $\mathrm{Re}\,\nu<-1/2,\ \mathrm{Re}\,\lambda>0$, $\mathrm{Re}(\alpha+\nu)>-1/2$	$\dfrac{i\,\Gamma(1/2+\alpha)\,\Gamma(1/2+\alpha+\nu)}{\pi 2^{1-\nu-2\alpha}\lambda^{2\alpha-1}}\,y^{-\nu-3/2}\times$ $\times W_{1/2-\alpha,\ -1/2-\nu}(y/\lambda)\times$ $\times[W_{1/2-\alpha,\ -1/2-\nu}(e^{-i\pi}y/\lambda)-$ $-W_{1/2-\alpha,\ -1/2-\nu}(e^{i\pi}y/\lambda)]$
(2)	$x^{2\alpha-\nu-1/2}\times$ $\times\,{}_2F_1(\nu+\alpha-1/2,\ \alpha;\ 2\alpha;\ -\lambda^2x^2)$, $\mathrm{Re}\,\alpha>-1/2,\ \mathrm{Re}\,\nu>1/2$, $\mathrm{Re}\,\lambda>0$	$\dfrac{2^{2\alpha-\nu}\,\Gamma(1/2+\alpha)}{\lambda^{2\alpha-1}\,\Gamma(2\nu)}\,y^{\nu-3/2}\times$ $\times M_{\alpha-1/2,\ \nu-1/2}(y/\lambda)\times$ $\times W_{1/2-\alpha,\ \nu-1/2}(y/\lambda)$

	$f(x)$	$\int_0^\infty f(x)\,J_\nu(xy)\,(xy)^{1/2}\,dx,\quad y>0$
(3)	$x^{\nu+1/2}\,{}_2F_1(\alpha,\ \beta;\ \nu+1;\ -\lambda^2x^2)$, $-1<\operatorname{Re}\nu<2\max(\operatorname{Re}\alpha,\ \operatorname{Re}\beta)-3/2$ $\operatorname{Re}\lambda>0$	$\dfrac{2^{\nu-\alpha-\beta+2}\,\Gamma(\nu+1)}{\lambda^{\alpha+\beta}\Gamma(\alpha)\,\Gamma(\beta)}\,y^{\alpha+\beta-\nu-3/2}\times$ $\times K_{\alpha-\beta}(y/\lambda)$
(4)	$x^{\nu+1/2}\times$ $\times\,{}_2F_1\left[\alpha,\ \beta;\ 2^{-1}(\beta+\nu)+1;\ -\lambda^2x^2\right]$, $-1<\operatorname{Re}\nu<2\max(\operatorname{Re}\alpha,\ \operatorname{Re}\beta)-3/2$	$\dfrac{\Gamma(\beta/2+\nu/2+1)}{\pi^{1/2}\,\Gamma(\alpha)\,\Gamma(\beta)}\,\dfrac{y^{\beta-1/2}}{2^{\beta-1}\lambda^{\nu+\beta+1}}\times$ $\times\left[K_{(\nu-\beta+1)/2}\left(\dfrac{y}{2\lambda}\right)\right]^2$
(5)	$x^{\nu+1/2}\,{}_2F_1(\alpha,\ \beta;\ \gamma;\ -\lambda^2x^2)$, $-1<\operatorname{Re}\nu<2\max(\operatorname{Re}\alpha,\ \operatorname{Re}\beta)-3/2$ $\operatorname{Re}\lambda>0$	$\dfrac{2^{\nu+1}\,\Gamma(\gamma)}{\Gamma(\alpha)\,\Gamma(\beta)}\,y^{-\nu-3/2}\times$ $\times\,G_{13}^{30}\left(\dfrac{y^2}{4\lambda^2}\,\middle\vert\,\begin{matrix}\gamma\\ \nu+1,\ \alpha,\ \beta\end{matrix}\right)$
(6)	$x^{\delta-1/2}\,{}_2F_1(\alpha,\ \beta;\ \gamma;\ -\lambda^2x^2)$, $-\operatorname{Re}\nu-1<\operatorname{Re}\delta<$ $<2\max(\operatorname{Re}\alpha,\ \operatorname{Re}\beta)-1/2$ $\operatorname{Re}\lambda>0$	$\dfrac{2^{\delta}\,\Gamma(\gamma)}{\Gamma(\alpha)\,\Gamma(\beta)}\,y^{-\delta-1/2}\times$ $\times\,G_{24}^{31}\left(\dfrac{y^2}{4\lambda^2}\,\middle\vert\,\begin{matrix}1,\ \gamma\\ \tau+\nu,\ \alpha,\ \beta,\ \tau\end{matrix}\right)$, $\tau=\dfrac{1+\delta-\nu}{2}$
(7)	$x^{-2\alpha-3/2}\times$ $\times\,{}_2F_1\left(\dfrac{1}{2}+\alpha,\ 1+\alpha;\ 1+2\alpha;\ -\dfrac{4\lambda^2}{x^2}\right)$, $\operatorname{Re}\nu>-1,\ \operatorname{Re}\lambda>0$ $\operatorname{Re}\alpha>-1/2$	$\lambda^{-2\alpha}y^{1/2}\,I_{\nu/2+\alpha}(\lambda y)\,K_{\nu/2-\alpha}(\lambda y)$
(8)	$x^{\nu-4\alpha+1/2}\times$ $\times\,{}_2F_1(\alpha,\ \alpha+1/2;\ \nu+1;\ -\lambda^2x^{-2})$, $\operatorname{Re}\alpha-1<\operatorname{Re}\nu<4\operatorname{Re}\alpha-3/2$ $\operatorname{Re}\lambda>0$	$\dfrac{\Gamma(\nu)}{\Gamma(2\alpha)}\,2^{\nu}\lambda^{1-2\alpha}y^{2\alpha-\nu-1/2}\times$ $\times\,I_\nu(2^{-1}\lambda y)\,K_{2\alpha-\nu-1}(2^{-1}\lambda y)$
(9)	$x^{\delta-1/2}\,{}_2F_1(\alpha,\ \beta;\ \gamma;\ -\lambda^2x^{-2})$, $-1-\operatorname{Re}\nu-2\min(\operatorname{Re}\alpha,\ \operatorname{Re}\beta)<$ $<\operatorname{Re}\delta<-1/2$ $\operatorname{Re}\lambda>0$	$\dfrac{2^{\delta}\,\Gamma(\gamma)}{\Gamma(\alpha)\,\Gamma(\beta)}\,y^{-\delta-1/2}\times$ $\times\,G_{24}^{22}\left(\dfrac{\lambda^2y^2}{4}\,\middle\vert\,\begin{matrix}1-\alpha,\ 1-\beta\\ \tau+\nu,\ 0,\ 1-\gamma,\ \tau\end{matrix}\right)$, $\tau=\dfrac{1+\delta-\nu}{2}$
(10)	$x^{\nu+1/2}(1+x)^{-2\alpha}\times$ $\times\,{}_2F_1\left[\alpha,\ \nu+\dfrac{1}{2};\ 2\nu+1;\ \dfrac{4x}{(1+x)^2}\right]$, $-1<\operatorname{Re}\nu<2\operatorname{Re}\alpha-3/2$	$\dfrac{\Gamma(\nu+1)\,\Gamma(\nu-\alpha+1)}{\Gamma(\alpha)}\,2^{2\nu-2\alpha+1}\times$ $\times\,y^{2\alpha-2\nu-3/2}\,J_\nu(y)$

8.18. Вырожденные гипергеометрические функции

	$f(x)$	$\int_0^\infty f(x)\, J_\nu(xy)\,(xy)^{1/2}\,dx,\quad y>0$
(1)	$x^{-1}\exp(-2^{-1}x^2)\times$ $\times M_{\nu/2-1/4,\ \nu/2+1/4}(x^2)$, $\operatorname{Re}\nu>-1/2$	$(2\nu+1)\,2^{-\nu}y^{\nu-1/2}\operatorname{Erfc}(y/2)$
(2)	$x^{-3/2}\exp(-2^{-1}x^2)\times$ $\times M_{\nu/2+1/2,\ \nu/2+1/2}(x^2)$, $\operatorname{Re}\nu>-1$	$\dfrac{\Gamma(\nu+2)\,y^{\nu+1/2}}{\Gamma(\nu+3/2)\,2^{\nu}}\operatorname{Erfc}(y/2)$
(3)	$x^{2\mu-\nu-1/2}\exp(-2^{-2}x^2)\times$ $\times M_{3\mu-\nu+1/2,\ \mu}(2^{-1}x^2)$, $\operatorname{Re}\mu>-1/2,\ \operatorname{Re}(4\mu-\nu)>-1/2$	$y^{2\mu-\nu-1/2}\exp(-2^{-2}y^2)\times$ $\times M_{3\mu-\nu+1/2,\ \mu}(2^{-1}y^2)$
(4)	$x^{\nu-2\mu-1/2}\exp(-2^{-2}x^2)\times$ $\times M_{\nu-\mu,\ \mu}(2^{-1}x^2)$, $\operatorname{Re}\nu>-1/2$	$\dfrac{\Gamma(2\mu+1)}{2^{\nu-\mu}\,\Gamma(\nu+1/2)}\,y^{\nu-1/2}\exp(-2^{-2}y^2)\times$ $\times D_{2\nu-4\mu}(y)$
(5)	$x^{\nu-2\mu-1/2}\exp(-2^{-2}x^2)\times$ $\times M_{\nu-\mu+1,\ \mu}(2^{-1}x^2)$, $\operatorname{Re}\nu>-1$	$\dfrac{\Gamma(2\mu+1)}{\Gamma(\nu+3/2)\,2^{\nu-\mu+1}}\,y^{\nu+1/2}\times$ $\times\exp(-2^{-2}y^2)\,D_{2\nu-4\mu+1}(y)$
(6)	$x^{-\lambda-1}\exp(-2^{-2}x^2)\times$ $\times M_{\lambda+\mu,\ \mu}(2^{-1}x^2)$, $\lambda=2\mu-\nu-1/2$ $-1<\operatorname{Re}\nu<4\operatorname{Re}\mu$	$\pi^{-1/2}2^{-5(\mu+\nu/3)}\dfrac{\Gamma(2\mu+1)}{\Gamma(4\mu-\nu)}\,y^{\lambda+2\mu}\times$ $\times\exp(-2^{-2}y^2)\,K_\lambda(2^{-2}y^2)$
(7)	$x^{-1/2}\exp(-2^{-2}x^2)\times$ $\times M_{\varkappa,\ \nu/2}(2^{-1}x^2)$, $\operatorname{Re}\nu>-1,\ \operatorname{Re}\varkappa<1/2$	$\dfrac{2^{-\varkappa}\,\Gamma(\nu+1)}{\Gamma(\varkappa+\nu/2+1/2)}\,y^{2\varkappa-1/2}\times$ $\times\exp(-2^{-1}y^2)$
(8)	$x^{2\mu-\nu-1/2}\exp(-2^{-2}x^2)\times$ $\times M_{\varkappa,\ \mu}(2^{-1}x^2)$, $-1/2<\operatorname{Re}\mu<\operatorname{Re}(\varkappa+\nu/2)-1/4$	$\dfrac{\Gamma(2\mu+1)\,2^{\alpha-\varkappa}}{\Gamma(1/2+\varkappa-\mu+\nu)\,y^{1-\varkappa+\mu}}\times$ $\times\exp(-2^{-2}y^2)\,M_{\alpha,\ \beta}(2^{-1}y^2)$, $2\alpha=1/2+\varkappa+3\mu-\nu$ $2\beta=-1/2+\varkappa-\mu+\nu$

	$f(x)$	$\int\limits_0^\infty f(x)\, J_\nu(xy)\,(xy)^{1/2}\,dx, \quad y>0$
(9)	$x^{\nu-2\mu-1/2}\exp(-2^{-2}x^2)\times$ $\times M_{\varkappa,\mu}(2^{-1}x^2)$, $-1<\operatorname{Re}\nu<2\operatorname{Re}(\varkappa+\mu)-1/2$	$2^{\alpha-\varkappa}\frac{\Gamma(2\mu+1)}{\Gamma(\mu+\varkappa+1/2)}y^{\varkappa+\mu-1}\times$ $\times\exp(-2^{-2}y^2)\,W_{\alpha,\beta}(2^{-1}y^2)$, $2\alpha=\varkappa-3\mu+\nu+1/2$ $2\beta=\varkappa+\mu-\nu-1/2$
(10)	$x^{2\rho-1/2}\exp(-2^{-1}\alpha x^2)\times$ $\times M_{\varkappa,\mu}(\alpha x^2)$, $-1-\operatorname{Re}(\nu/2+\mu)<$ $<\operatorname{Re}\rho<\operatorname{Re}\varkappa-1/4$ $\operatorname{Re}\alpha>0$	$\frac{\Gamma(2\mu+1)}{\Gamma(\mu+\varkappa+1/2)}2^{2\rho}y^{-2\rho-1/2}\times$ $\times G_{23}^{21}\left(\frac{y^2}{4\alpha}\,\middle\vert\,\begin{matrix}1/2-\mu,\ 1/2+\mu\\ 1/2+\rho+\nu/2,\ \varkappa,\ 1/2+\rho-\nu/2\end{matrix}\right)$
(11)	$x^{\nu-2\mu-1/2}\exp(-2^{-2}x^2)\times$ $\times W_{3\mu-\nu-1/2,\ \pm\mu}(2^{-1}x^2)$, $\operatorname{Re}\nu>-1,\ \operatorname{Re}(\nu-2\mu)>-1$	$\pi^{1/2}2^{\nu-3\mu}y^{4\mu-\nu-1/2}\exp(-2^{-2}y^2)\times$ $\times I_{\nu-2\mu+1/2}(2^{-2}y^2)$
(12)	$x^{\nu-2\mu-1/2}\exp(2^{-2}x^2)\times$ $\times W_{\nu-3\mu+1/2,\ \pm\mu}(2^{-1}x^2)$, $\operatorname{Re}\nu>-1,\ \operatorname{Re}(\nu-2\mu)>-1$ $\operatorname{Re}(3\nu-8\mu)<-3/2$	$\pi^{-1/2}2^{\nu-3\mu}\frac{\Gamma(1+\nu-2\mu)}{\Gamma(4\mu-\nu)}y^{4\mu-\nu-1/2}\times$ $\times\exp(2^{-2}y^2)\,K_{\nu-2\mu+1/2}(2^{-2}y^2)$
(13)	$x^{\nu-2\mu-1/2}\exp(2^{-2}x^2)\times$ $\times W_{3\mu-\nu-1/2,\ \pm\mu}(2^{-1}x^2)$, $\operatorname{Re}\nu>-1,\ \operatorname{Re}(\nu-2\mu)>-1$ $\operatorname{Re}(\nu-4\mu)>-1/2$	$y^{\nu-2\mu-1/2}\exp(2^{-2}y^2)\times$ $\times W_{3\mu-\nu-1/2,\ \pm\mu}(2^{-1}y^2)$
(14)	$x^{\nu-2\mu-1/2}\exp(-2^{-2}x^2)\times$ $\times W_{\varkappa,\ \pm\mu}(2^{-1}x^2)$, $\operatorname{Re}\nu>-1,\ \operatorname{Re}(\nu-2\mu)>-1$	$\frac{\Gamma(1+\nu-2\mu)}{\Gamma(1+2\beta)}2^{\beta-\mu}y^{\varkappa+\mu-1}\times$ $\times\exp(-2^{-2}y^2)\,M_{\alpha,\beta}(2^{-1}y^2)$, $2\alpha=1/2+\varkappa+\nu-3\mu$ $2\beta=1/2-\varkappa+\nu-\mu$
(15)	$x^{\nu-2\mu-1/2}\exp(2^{-2}x^2)\times$ $\times W_{\varkappa,\ \pm\mu}(2^{-1}x^2)$, $\operatorname{Re}\nu>-1,\ \operatorname{Re}(\nu-2\mu)>-1$ $\operatorname{Re}(\varkappa-\mu+\nu/2)<-1/4$	$\frac{\Gamma(1+\nu-2\mu)}{\Gamma(1/2+\mu-\varkappa)}2^{\varkappa-\alpha}y^{\mu-\varkappa-1}\times$ $\times\exp(2^{-2}y^2)\,W_{\alpha,\beta}(2^{-1}y^2)$, $2\alpha=\varkappa+3\mu-\nu-1/2$ $2\beta=\varkappa-\mu+\nu+1/2$

	$f(x)$	$\int\limits_0^\infty f(x)\, J_\nu(xy)\,(xy)^{1/2}\,dx, \quad y>0$
(16)	$x^{2\rho-1/2}\exp(-2^{-1}\alpha x^2)\times$ $\times W_{\varkappa,\mu}(\alpha x^2),\quad \operatorname{Re}\alpha>0$ $\operatorname{Re}(\rho\pm\mu+\nu/2)>-1$	$\dfrac{\Gamma(1+\mu+\nu/2+\rho)\,\Gamma(1-\mu+\nu/2+\rho)}{\Gamma(\nu+1)\,\Gamma(3/2-\varkappa-\nu/2+\rho)}\times$ $\times 2^{-\nu-1}\alpha^{-\nu/2-\rho-1/2}y^{\nu+1/2}\times$ $\times {}_2F_2\left(\lambda+\mu,\ \lambda-\mu:\ \nu+1,\ \frac{1}{2}-\varkappa+\lambda;\ -\frac{y^2}{4\alpha}\right),$ $\lambda=1+\nu/2+\rho$
(17)	$x^{2\rho-1/2}\exp(2^{-1}\alpha x^2)\times$ $\times W_{\varkappa,\mu}(\alpha x^2),\ \lvert\arg\alpha\rvert<\pi$ $-1-\operatorname{Re}(\nu/2\pm\mu)<$ $<\operatorname{Re}\rho<-1/4-\operatorname{Re}\varkappa$	$\dfrac{2^{2\rho}y^{-2\rho-1/2}}{\Gamma(1/2+\mu-\varkappa)\,\Gamma(1/2-\mu-\varkappa)}\times$ $\times G_{23}^{22}\left(\frac{y^2}{4\alpha}\,\middle\vert\,\begin{matrix}1/2-\mu,\ 1/2+\mu\\ 1/2+\rho+\nu/2,\ -\varkappa,\ 1/2+\rho-\nu/2\end{matrix}\right)$
(18)	$x^{-1/2}M_{\varkappa,\nu/2}(-iax)\times$ $\times M_{-\varkappa,\nu/2}(-iax),$ $a>0,\ \operatorname{Re}\nu>-1,\ \lvert\operatorname{Re}\varkappa\rvert<1/4$	$\dfrac{a\exp[-2^{-1}(\nu+1)\pi i]\,[\Gamma(1+\nu)]^2}{\Gamma(1/2+\varkappa+\nu/2)\,\Gamma(1/2-\varkappa+\nu/2)}\times$ $\times y^{-1/2-2\varkappa}(a^2-y^2)^{-1/2}\times$ $\times\{[a+(a^2-y^2)^{1/2}]^{2\varkappa}+$ $+[a-(a^2-y^2)^{1/2}]^{2\varkappa}\},\quad 0<y<a$ $0,\quad a<y<\infty$
(19)	$x^{-1/2}M_{-\mu/2,\nu/2}(\alpha x)\times$ $\times W_{\mu/2,\nu/2}(\alpha x),\quad \operatorname{Re}\nu>-1$ $\operatorname{Re}\mu<1/2,\ \operatorname{Re}\alpha>0$	$\alpha\dfrac{\Gamma(\nu+1)}{\Gamma(1/2-\mu/2+\nu/2)}y^{-\mu-1/2}\times$ $\times[\alpha+(\alpha^2+y^2)^{1/2}]^{\mu}(\alpha^2+y^2)^{-1/2}$
(20)	$x^{2\mu-\nu-1/2}W_{\varkappa,\mu}(\alpha x)\,M_{-\varkappa,\mu}(\alpha x),$ $\operatorname{Re}\mu>-1/2,\ \operatorname{Re}\alpha>0$ $\operatorname{Re}(2\mu+2\varkappa-\nu)<1/2$	$2^{2\mu-\nu+2\varkappa}\alpha^{2\varkappa}y^{\nu-2\mu-2\varkappa-1/2}\times$ $\times\dfrac{\Gamma(2\mu+1)}{\Gamma(\nu-\varkappa-\mu+1/2)}\times$ $\times {}_3F_2(1/2-\varkappa,\ 1-\varkappa,\ 1/2-\varkappa+\mu;$ $1-2\varkappa,\ 1/2-\varkappa-\mu+\nu;\ -y^2/\alpha^2)$
(21)	$x^{2\rho-\nu-5/2}W_{\varkappa,\mu}(\alpha x)\times$ $\times M_{-\varkappa,\mu}(\alpha x),\quad \operatorname{Re}\alpha>0$ $\operatorname{Re}\rho>0,\ \operatorname{Re}(\rho+\mu)>0$ $\operatorname{Re}(2\rho+2\varkappa-\nu)<5/2$	$\dfrac{2^{2\rho-\nu-2}\,\Gamma(2\mu+1)}{\pi^{1/2}\,\Gamma(1/2-\varkappa+\mu)}y^{\nu-2\rho+3/2}\times$ $\times G_{44}^{23}\left(\frac{y^2}{\alpha^2}\,\middle\vert\,\begin{matrix}1/2,\ 0,\ 1/2-\mu,\ 1/2+\mu\\ \rho-1/2,\ -\varkappa,\ \varkappa,\ \rho-\nu-1/2\end{matrix}\right)$
(22)	$x^{2\rho-\nu-5/2}W_{\varkappa,\mu}(\alpha x)\,W_{-\varkappa,\mu}(\alpha x),$ $\operatorname{Re}\rho>\lvert\operatorname{Re}\mu\rvert,\ \operatorname{Re}\alpha>0$	$\dfrac{\Gamma(\rho+\mu)\,\Gamma(\rho-\mu)\,\Gamma(2\rho)}{\Gamma(1/2+\varkappa+\rho)\,\Gamma(1/2-\varkappa+\rho)\,\Gamma(1+\nu)}\times$ $\times 2^{-\nu-1}\alpha^{1-2\rho}y^{\nu+1/2}\times$ $\times {}_4F_3(\rho,\ \rho+1/2,\ \rho+\mu,\ \rho-\mu;$ $1/2+\varkappa+\rho,\ 1/2-\varkappa+\rho,\ 1+\nu;$ $-y^2/\alpha^2)$

	$f(x)$	$\int\limits_0^\infty f(x)\, J_\nu(xy)\,(xy)^{1/2}\,dx, \quad y>0$
(23)	$x^{2\rho-\nu-5/2}\, W_{\varkappa,\mu}(iax)\times$ $\times W_{\varkappa,\mu}(-iax), \quad \operatorname{Re} a>0$ $\operatorname{Re}(2\rho+2\varkappa-\nu)<5/2$	$\dfrac{2^{2\rho-\nu-2}y^{\nu-2\rho+3/2}}{\pi^{1/2}\,\Gamma(1/2-\varkappa+\mu)\,\Gamma(1/2-\varkappa-\mu)}\times$ $\times G_{41}^{24}\left(\dfrac{y^2}{a^2}\,\middle\vert\, \begin{matrix}1/2,\ 0,\ 1/2-\mu,\ 1/2+\mu\\ \rho-1/2,\ -\varkappa,\ \varkappa,\ \rho-\nu-1/2\end{matrix}\right)$
(24)	$x^{-3/2}\, M_{-\mu,\,\nu/4}(2^{-1}x^2)\times$ $\times W_{\mu,\,\nu/4}(2^{-1}x^2), \quad \operatorname{Re}\nu>-1$	$\dfrac{\Gamma(1+\nu/2)\, y^{1/2}}{\Gamma(1/2+\nu/4-\mu)}\, I_{\nu/4-\mu}(2^{-2}y^2)\times$ $\times K_{\nu/4+\mu}(2^{-2}y^2)$
(25)	$x^{-3/2}\, M_{\alpha-\beta,\,\nu/4-\gamma}(2^{-1}x^2)\times$ $\times W_{\alpha+\beta,\,\nu/4+\gamma}(2^{-1}x^2),$ $\operatorname{Re}\beta<1/8, \ \operatorname{Re}\nu>-1$ $\operatorname{Re}(\nu-4\gamma)>-2$	$\dfrac{\Gamma(1+\nu/2-2\gamma)}{\Gamma(1+\nu/2-2\beta)}\, y^{-3/2}\times$ $\times M_{\alpha-\gamma,\,\nu/4-\beta}(2^{-1}y^2)\times$ $\times W_{\alpha+\gamma,\,\nu/4+\beta}(2^{-1}y^2)$
(26)	$x^{1/2}\, M_{\nu/2,\,\mu}(2/x)\, W_{-\nu/2,\,\mu}(2/x),$ $\operatorname{Re}\nu>-1, \ \operatorname{Re}\mu>-1/4$	$\dfrac{4\,\Gamma(1+2\mu)}{\Gamma(1/2+\nu/2+\mu)}\, y^{-1/2} J_{2\mu}(2y^{1/2})\times$ $\times K_{2\mu}(2y^{1/2})$
(27)	$x^{1/2}\, W_{\nu/2,\,\mu}(2/x)\, W_{-\nu/2,\,\mu}(2/x),$ $\operatorname{Re}(\nu\pm 2\mu)>-1$	$-4y^{-1/2}\times$ $\times\{\sin[(\mu-\nu/2)\pi]\, J_{2\mu}(2y^{1/2})+$ $+\cos[(\mu-\nu/2)\pi]\, Y_{2\mu}(2y^{1/2})\}\times$ $\times K_{2\mu}(2y^{1/2})$
(28)	$x^{1/2}\, W_{-\nu/2,\,\mu}(ia/x)\times$ $\times W_{-\nu/2,\,\mu}(-ia/x), \quad \operatorname{Re} a>0$ $\vert\operatorname{Re}\mu\vert<1/2, \ \operatorname{Re}\nu>-1$	$4ay^{-1/2}\,[\Gamma(1/2+\mu+\nu/2)\times$ $\times\Gamma(1/2-\mu+\nu/2)]^{-1}\times$ $\times K_\mu[(2iay)^{1/2}]\, K_\mu[(-2iay)^{1/2}]$
(29)	$x^{-1/2}\, M_{-\mu,\,\nu/2}\{a[(\beta^2+x^2)^{1/2}-\beta]\}\times$ $\times W_{\mu,\,\nu/2}\{a[(\beta^2+x^2)^{1/2}+\beta]\},$ $\operatorname{Re}\nu>-1, \ \operatorname{Re}\mu<1/4$ $\operatorname{Re} a>0, \ \operatorname{Re}\beta>0$	$\dfrac{a\,\Gamma(1+\nu)\,[(a^2+y^2)^{1/2}+a]^{2\mu}}{\Gamma(1/2+\nu/2-\mu)\, y^{1/2+2\mu}\,(a^2+y^2)^{1/2}}\times$ $\times\exp[-\beta(a^2+y^2)^{1/2}]$

8.19. Обобщенные гипергеометрические ряды и разные функции

(1)	$x^{\nu+1/2}\, {}_1F_1(2\alpha-\nu;\ \alpha+1;\ -2^{-1}x^2),$ $\operatorname{Re}\nu>-1, \ \operatorname{Re}(4\alpha-3\nu)>1/2$	$\dfrac{2^{\nu-\alpha+1/2}\,\Gamma(\alpha+1)}{\pi^{1/2}\,\Gamma(2\alpha-\nu)}\, y^{2\alpha-\nu-1/2}\times$ $\times\exp(-2^{-2}y^2)\, K_{\alpha-\nu-1/2}(2^{-2}y^2)$

	$f(x)$	$\int\limits_0^\infty f(x)\,J_\nu(xy)\,(xy)^{1/2}\,dx,\quad y>0$
(2)	$x^{\alpha-1/2}\,{}_1F_1\left(\alpha;\ \frac{1+\alpha+\nu}{2};\ -\frac{x^2}{2}\right)$, $\operatorname{Re}\alpha>-1/2,\ \operatorname{Re}(\alpha+\nu)>-1$	$y^{\alpha-1/2}\,{}_1F_1\left(\alpha;\ \frac{1+\alpha+\nu}{2};\ -\frac{y^2}{2}\right)$
(3)	$x^{\nu+1/2-2\alpha}\times$ $\times\,{}_1F_1(\alpha;\ 1+\nu-\alpha;\ -2^{-1}x^2)$, $\operatorname{Re}\alpha-1<\operatorname{Re}\nu<4\operatorname{Re}\alpha-1/2$	$\frac{\pi^{1/2}\,\Gamma(1+\nu-\alpha)}{2^{2\alpha-\nu-1/2}\,\Gamma(\alpha)}\,y^{2\alpha-\nu-1/2}\times$ $\times\exp(-2^{-2}y^2)\,I_{\alpha-1/2}(2^{-2}y^2)$
(4)	$x^{\nu+1/2}\,{}_1F_1(\alpha;\ \beta;\ -\lambda x^2)$, $-1<\operatorname{Re}\nu<2\operatorname{Re}\alpha-1/2$ $\operatorname{Re}\lambda>0$	$\frac{2^{1-\alpha}\,\Gamma(\beta)}{\Gamma(\alpha)\,\lambda^{\alpha/2+\nu/2}}\,y^{\alpha-3/2}\exp\left(-\frac{y^2}{8\lambda}\right)\times$ $\times\,W_{\varkappa,\mu}\left(\frac{y^2}{4\lambda}\right)$, $2\varkappa=\alpha-2\beta+\nu+2$ $2\mu=\alpha-\nu-1$
(5)	$x^{2\beta-\nu-5/2}\,{}_1F_1(\alpha;\ \beta;\ -\lambda x^2)$, $0<\operatorname{Re}\beta<3/4+\operatorname{Re}(\alpha+\nu/2)$ $\operatorname{Re}\lambda>0$	$\frac{2^{2\beta-2\alpha-\nu-1}\,\Gamma(\beta)}{\Gamma(\alpha-\beta+\nu+1)\,\lambda^\alpha}\,y^{2\alpha-2\beta+\nu+1/2}\times$ $\times\,{}_1F_1\left(\alpha;\ 1+\alpha-\beta+\nu;\ -\frac{y^2}{4\lambda}\right)$
(6)	$x^{2\rho-1/2}\,{}_1F_1(\alpha;\ \beta;\ -\lambda x^2)$, $-1-\operatorname{Re}\nu<2\operatorname{Re}\rho<1/2+2\operatorname{Re}\alpha$ $\operatorname{Re}\lambda>0$	$\frac{2^{2\rho}\,\Gamma(\beta)}{\Gamma(\alpha)\,y^{2\rho+1/2}}\times$ $\times\,G_{23}^{21}\left(\frac{y^2}{4\lambda}\,\middle\vert\,\begin{matrix}1,\ \beta\\ 1/2+\rho+\nu/2,\ \alpha,\ 1/2+\rho-\nu/2\end{matrix}\right)$
(7)	$x^{\nu+1/2}\,{}_1F_2(\alpha;\ \beta,\ \nu+1;\ -2^{-2}x^2)$, $\operatorname{Re}>-1,\ \operatorname{Re}\beta>\operatorname{Re}\alpha>0$	$\frac{\Gamma(\nu+1)\,\Gamma(\beta)}{\Gamma(\alpha)\,\Gamma(\beta-\alpha)}\,2^{\nu+1}\,y^{2\alpha-\nu-3/2}\times$ $\times(1-y^2)^{\beta-\alpha-1},\quad 0<y<1$ $0,\quad 1<y<\infty$
(8)	$x^{\nu+1/2}(1-x^2)^{\mu-\nu-1}\times$ $\times\,{}_1F_2[\mu+1/2;\ 2\mu+1,\ \mu-\nu;$ $-a(1-x^2)]$, $0<x<1$ $0,\quad 1<x<\infty$ $\operatorname{Re}\mu>\operatorname{Re}\nu>-1$	$2^{2\mu-\nu-1}a^{-\mu}\,\Gamma(1+\mu)\times$ $\times\,\Gamma(\mu-\nu)\,y^{\nu+1/2}\times$ $\times\,J_\mu[2^{-1}(y^2+4a^2)^{1/2}+y/2]\times$ $\times\,J_\mu[2^{-1}(y^2+4a^2)^{1/2}-y/2]$
(9)	$x^{\nu+1/2}(1-x^2)^{\mu-3/2}\times$ $\times\,{}_1F_2\left[1;\ \frac{\mu}{2},\ \frac{\mu+1}{2};\ -a^2(1-x^2)^2\right]$, $0<x<1$ $0,\quad 1<x<\infty$ $\operatorname{Re}\nu>-1,\ \operatorname{Re}\mu>0$	$2^{-\mu-2\nu-1}\,\Gamma(\mu)\,a^{-\mu-\nu}y^{\nu+1/2}\times$ $\times\,U_{\nu+\mu}(4a,\ y)$

	$f(x)$	$\int\limits_0^\infty f(x)\, J_\nu(xy)\,(xy)^{1/2}\,dx, \quad y>0$
(10)	$x^{\rho-1/2}\,{}_2F_2\left(\rho, \frac{\rho+m+1}{2}; \frac{\rho-m+1}{2}, \frac{\rho+\nu+1}{2}; \frac{x^2}{2}\right)$, $\mathrm{Re}(\rho+\nu+1)>0$, $\mathrm{Re}\,\rho>1/2$, $\mathrm{Re}(\rho-m+1)>0$	$(-1)^m y^{\rho-1/2}\,{}_2F_2\left(\rho, \frac{\rho+m+1}{2}; \frac{\rho-m+1}{2}, \frac{\rho+\nu+1}{2}; -\frac{y^2}{2}\right)$
(11)	$x^{\nu+1/2}{}_3F_2(\alpha, \alpha+\beta, \alpha-\beta; \alpha+1/2, \nu+1; -x^2)$, $\mathrm{Re}\,\alpha>2^{-1}\,\mathrm{Re}\,\nu+1/4>-1/4$, $\lvert\mathrm{Re}\,\beta\rvert<\mathrm{Re}\,\alpha-2^{-1}\,\mathrm{Re}\,\nu-1/4$	$\frac{2^{\nu-2\alpha+2}\,\Gamma(\alpha+1/2)\,\Gamma(\nu+1)}{\pi^{1/2}\,\Gamma(\alpha)\,\Gamma(\alpha+\beta)\,\Gamma(\alpha-\beta)}\times$ $\times\, y^{-2\alpha-\nu-3/2}\,[K_\beta(2^{-1}y)]^2$
(12)	$x^{\nu+1/2}{}_3F_2(\beta+\rho, \beta-\rho, 2\beta-\nu-1; \beta, \beta+1/2; -2^{-2}x^2)$, $\lvert\mathrm{Re}\,\rho\rvert<\mathrm{Re}\,\beta-2^{-1}\,\mathrm{Re}\,\nu-1/4$, $\mathrm{Re}\,\nu>-1$, $\mathrm{Re}(4\beta-3\nu)>5/2$	$\frac{2^{\nu+3-2\beta}\,\Gamma(2\beta)\,y^{2\beta-\nu-3/2}}{\Gamma(\beta+\rho)\,\Gamma(\beta-\rho)\,\Gamma(2\beta-\nu-1)}\times$ $\times\, K_{\nu+1-\beta+\rho}(y)\,K_{\nu+1-\beta-\rho}(y)$
(13)	$x^{\nu+1/2}{}_3F_2(\alpha, \alpha+1/2, 2\alpha-\nu-1; \alpha+1/2+\varkappa, \alpha+1/2-\varkappa; -x^2)$, $-1<\mathrm{Re}\,\nu<2\,\mathrm{Re}\,\alpha-1/2$, $\mathrm{Re}(4\alpha-3\nu)>5/2$	$\frac{\Gamma(\alpha+1/2+\varkappa)\,\Gamma(\alpha+1/2-\varkappa)}{\Gamma(2\alpha)\,\Gamma(2\alpha-\nu-1)}\,2^{\nu+1}\times$ $\times\, y^{2\alpha-\nu-5/2}\,W_{\varkappa,\,\alpha-\nu-1}(y)\times$ $\times\, W_{-\varkappa,\,\alpha-\nu-1}(y)$
(14)	$x^{\nu+1/2}{}_3F_2(\nu+1/2, \nu+1/2+\mu, \nu+1/2-\mu; \nu+1+\varkappa, \nu+1-\varkappa; -x^2)$, $\mathrm{Re}\,\nu>-1/2$, $\lvert\mathrm{Re}\,\mu\rvert<2^{-1}\,\mathrm{Re}\,\nu+1/4$	$\frac{\Gamma(\nu+\varkappa+1)\,\Gamma(\nu-\varkappa+1)\,\pi^{1/2}2^{1-\nu}}{\Gamma(\nu+1/2)\,\Gamma(\nu+1/2+\mu)\,\Gamma(\nu+1/2-\mu)}\times$ $\times\, y^{\nu-3/2}\,W_{\varkappa,\,\mu}(y)\,W_{-\varkappa,\,\mu}(y)$
(15)	$x^{\nu+1/2}\,{}_3F_2\left(\nu+\frac{3}{2}, \nu+1+\mu, \nu+1-\mu; \nu+\frac{3}{2}+\varkappa, \nu+\frac{3}{2}-\varkappa; -x^2\right)$, $\mathrm{Re}\,\nu>-1$, $\lvert\mathrm{Re}\,\mu\rvert<2^{-1}\,\mathrm{Re}\,\nu+3/4$	$\frac{\Gamma(\nu+\varkappa+3/2)\,\Gamma(\nu-\varkappa+3/2)\,\pi^{1/2}2^{-\nu}}{\Gamma(\nu+3/2)\,\Gamma(\nu+1+\mu)\,\Gamma(\nu+1-\mu)}\times$ $\times\, y^{\nu-1/2}\,W_{\varkappa,\,\mu}(y)\,W_{-\varkappa,\,\mu}(y)$
(16)	$x^{\nu+1/2}\times$ $\times\,{}_4F_3(\alpha+\beta, \alpha-\beta, \alpha+\gamma, \alpha-\gamma; \alpha, \alpha+1/2, \nu+1; -x^2)$, $-1<\mathrm{Re}\,\nu<2\,\mathrm{Re}\,\alpha-1/2$, $\lvert\mathrm{Re}\,\beta\rvert<\mathrm{Re}\,\alpha-2^{-1}\,\mathrm{Re}\,\nu-1/4$, $\lvert\mathrm{Re}\,\gamma\rvert<\mathrm{Re}\,\alpha-2^{-1}\,\mathrm{Re}\,\nu-1/4$	$\frac{2^{\nu-4\alpha+3}\,\Gamma(2\alpha)\,\Gamma(\nu+1)\,y^{2\alpha-\nu-3/2}}{\Gamma(\alpha+\beta)\,\Gamma(\alpha-\beta)\,\Gamma(\alpha-\gamma)\,\Gamma(\alpha+\gamma)}\times$ $\times\, K_{\beta+\gamma}(y/2)\,K_{\beta-\gamma}(y/2)$

	$f(x)$	$\int_0^\infty f(x) J_\nu(xy)(xy)^{1/2}\,dx, \quad y>0$
(17)	$x^{\nu+1/2} \times$ $\times\, {}_4F_3(\alpha, \alpha+1/2, \alpha+\mu, \alpha-\mu;$ $1/2+\alpha+\varkappa, 1/2+\alpha-\varkappa; -x^2),$ $-1<\operatorname{Re}\nu<2\operatorname{Re}\alpha-1/2$ $\lvert\operatorname{Re}\mu\rvert<\operatorname{Re}\alpha-2^{-1}\operatorname{Re}\nu-1/4$	$\dfrac{\Gamma(1/2+\alpha+\varkappa)\,\Gamma(1/2+\alpha-\varkappa)\,\Gamma(\nu+1)}{\Gamma(2\alpha)\,\Gamma(\alpha+\mu)\,\Gamma(\alpha-\mu)} \times$ $\times\, 2^{\nu+1} y^{2\alpha-\nu-3/2} W_{\varkappa,\mu}(y)\, W_{-\varkappa,\mu}(y)$
(18)	$x^{2\rho-1/2}\, {}_pF_p(\alpha_1, \ldots, \alpha_p;$ $\beta_1, \ldots, \beta_p; -\lambda x^2),$ $-1-\operatorname{Re}\nu<2\operatorname{Re}\rho<1/2+2\operatorname{Re}\alpha_r$ $\operatorname{Re}\lambda>0,\ r=1, \ldots, p$	$\dfrac{\Gamma(\beta_1)\cdots\Gamma(\beta_p)}{\Gamma(\alpha_1)\cdots\Gamma(\alpha_p)}\, 2^{2\rho} y^{-2\rho-1/2} \times$ $\times\, G^{p+1,\,1}_{p+1,\,p+2}\left(\dfrac{y^2}{4\lambda}\,\middle\vert\, \begin{matrix}1, \beta_1, \ldots, \beta_p\\ h, \alpha_1, \ldots, \alpha_p, k\end{matrix}\right),$ $h=1/2+\rho+\nu/2,\ k=1/2+\rho-\nu/2$
(19)	$x^{2\rho-1/2}\, {}_{m+1}F_m(\alpha_1, \ldots, \alpha_{m+1};$ $\beta_1, \ldots, \beta_m; -\lambda^2x^2),$ $\operatorname{Re}(2\rho+\nu)>-1,\ \operatorname{Re}\lambda>0$ $\operatorname{Re}(\rho-\alpha_r)<1/4,\ r=1, \ldots, m+1$	$\dfrac{2^{2\rho}\,\Gamma(\beta_1)\cdots\Gamma(\beta_m)}{\Gamma(\alpha_1)\cdots\Gamma(\alpha_{m+1})\, y^{2\rho+1/2}} \times$ $\times\, G^{m+2,\,1}_{m+1,\,m+3}\left(z\,\middle\vert\, \begin{matrix}1, \beta_1, \ldots, \beta_m\\ h, \alpha_1, \ldots, \alpha_{m+1}, k\end{matrix}\right),$ $h=1/2+\rho+\nu/2,\ k=1/2+\rho-\nu/2$ $z=\dfrac{y^2}{4\lambda^2}$
(20)	$x^{2\rho-1/2}\, G^{mn}_{pq}\left(\lambda x^2\,\middle\vert\, \begin{matrix}\alpha_1, \ldots, \alpha_p\\ \beta_1, \ldots, \beta_q\end{matrix}\right),$ $p+q<2(m+n)$ $\lvert\arg\lambda\rvert<(m+n-p/2-q/2)\pi$ $\operatorname{Re}(\beta_j+\rho+\nu/2)>-1/2$ $j=1, \ldots, m$ $\operatorname{Re}(\alpha_j+\rho)<3/4,\ j=1, \ldots, n$	$\dfrac{2^{2\rho}}{y^{2\rho+1/2}} \times$ $\times\, G^{m,\,n+1}_{p+2,\,q}\left(\dfrac{4\lambda}{y^2}\,\middle\vert\, \begin{matrix}h, \alpha_1, \ldots, \alpha_p, k\\ \beta_1, \ldots, \beta_q\end{matrix}\right)$ $h=1/2-\rho-\nu/2,\ k=1/2-\rho+\nu/2$
(21)	$x^{\nu+2n-1}\,[\mathrm{B}(a+x, a-x)]^{-1},$ $-1<\operatorname{Re}\nu<2a-2n-7/2$	$0, \qquad \pi\leqslant y<\infty$
(22)	$x^{\nu+1/2}\operatorname{Erfc}(\alpha x),$ $\lvert\arg\alpha\rvert<\pi/4,\ \operatorname{Re}\nu>-1$	$\alpha^{-\nu}\dfrac{\Gamma(\nu+3/2)}{\Gamma(\nu+2)}\, y^{-3/2}\exp\left(-\dfrac{y^2}{8\alpha^2}\right) \times$ $\times\, M_{\nu/2+1/2,\ \nu/2+1/2}\left(\dfrac{y^2}{4\alpha^2}\right)$
(23)	$x^{\nu-1/2}\operatorname{Erfc}(\alpha x),$ $\lvert\arg\alpha\rvert<\pi/4,\ \operatorname{Re}\nu>-1/2$	$\dfrac{\alpha^{1/2-\nu}\,\Gamma(\nu+1/2)}{2^{1/2}\,\Gamma(\nu+3/2)\,y}\exp\left(-\dfrac{y^2}{8\alpha^2}\right)\times$ $\times\, M_{\nu/2-1/4,\ \nu/2+1/4}\left(\dfrac{y^2}{4\alpha^2}\right)$
(24)	$x^{-\mu-1/2}\, s_{\nu+\mu,\,-\nu+\mu+1}(x),$ $\operatorname{Re}\mu>-1,\ -1<\operatorname{Re}\nu<3/2$	$2^{\nu-1}\,\Gamma(\nu)\, y^{1/2-\nu}(1-y^2)^{\mu},$ $0<y<1$ $0, \qquad 1<y<\infty$

ГЛАВА IX

Y-ПРЕОБРАЗОВАНИЯ

Мы называем *Y-преобразованием порядка* ν функции $f(x)$ функцию

$$\mathfrak{Y}_\nu \{f(x);\ y\} = \int_0^\infty f(x)\, Y_\nu(xy)\,(xy)^{1/2}\,dx$$

положительного аргумента y. Формула обращения 9.1 (1) дана Титчмаршем (1948, стр. 280). Обратным преобразованием является $\mathbf{H}$-преобразование (см. главу XI).

Из пар преобразований, данных в этой главе, можно вывести дальнейшие пары преобразований с помощью методов, указанных во введении к первому тому, а также с помощью общих формул из п. 9.1. Кроме того, Y-преобразования связаны с преобразованиями Ганкеля соотношением

$$\mathfrak{Y}_\nu \{f(x);\ y\} = \operatorname{ctg}(\nu\pi)\, \mathfrak{H}_\nu \{f(x);\ y\} - \frac{\mathfrak{H}_{-\nu}\{f(x);\ y\}}{\sin(\nu\pi)},$$

которое является непосредственным следствием соотношения между функциями Бесселя первого и второго рода. Это соотношение можно использовать для вычисления Y-преобразований с помощью таблицы преобразований Ганкеля, приведенной в главе VIII.

9.1. Общие формулы

	$f(x)$	$\int_0^\infty f(x)\, Y_\nu(xy)\,(xy)^{1/2}\,dx = g(y;\ \nu)$, $y > 0$
(1)	$\int_0^\infty g(y)\, \mathbf{H}_\nu(xy)\,(xy)^{1/2}\,dy$, $-1/2 < \operatorname{Re}\nu < 1/2$	$g(y)$
(2)	$f(ax)$, $a > 0$	$a^{-1} g(a^{-1}y;\ \nu)$
(3)	$x^m f(x)$, $m = 0, 1, 2, \ldots$	$y^{1/2-\nu} \times \left(\frac{d}{y\,dy}\right)^m [y^{\nu-1/2+m} g(y;\ \nu+m)]$

	$f(x)$	$\int\limits_0^\infty f(x)\, Y_\nu(xy)\,(xy)^{1/2}\,dx = g(y;\nu)$, $y > 0$
(4)	$x^m f(x)$, $m = 0, 1, 2, \ldots$	$(-1)^m y^{1/2+\nu} \times$ $\times \left(\frac{d}{y\,dy}\right)^m \left[y^{-1/2+m-\nu} g(y;\nu-m)\right]$
(5)	$2\nu x^{-1} f(x)$	$y\,g(y;\nu-1) + y\,g(y;\nu+1)$
(6)	$2\nu f'(x)$	$(\nu - 1/2)\,y\,g(y;\nu+1) -$ $-(\nu+1/2)\,y\,g(y;\nu-1)$
(7)	$x^{1/2-\nu}\left(\frac{d}{x\,dx}\right)^m \left[x^{\nu+m-1/2} f(x)\right]$, $m = 0, 1, 2, \ldots$	$y^m g(y;\nu+m)$
(8)	$x^{1/2+\nu}\left(\frac{d}{x\,dx}\right)^m \left[x^{m-\nu-1/2} f(x)\right]$, $m = 0, 1, 2, \ldots$	$(-y)^m g(y;\nu-m)$
(9)	$x^{1/2-\nu}\int\limits_0^x \xi^{\nu-\mu+1/2}(x^2-\xi^2)^{\mu-1}\times$ $\times f(\xi)\,d\xi$, $\operatorname{Re}\nu + 3/2 > \operatorname{Re}\mu > 0$	$2^{\mu-1}\Gamma(\mu)\,y^{-\mu} g(y;\nu-\mu)$
(10)	$x^{-\mu} f(x)$, $\operatorname{Re}\mu > 0$, $\operatorname{Re}\nu > -3/2$	$2^{1-\mu}[\Gamma(\mu)]^{-1} y^{\nu+1/2}\times$ $\times \int\limits_y^\infty \eta^{1/2-\mu-\nu}(\eta^2-y^2)^{\mu-1}\times$ $\times g(\eta;\nu+\mu)\,d\eta$

9.2. Алгебраические функции и степени с произвольными показателями

(1)	$x^{-1/2}$, $-1 < \operatorname{Re}\nu < 1$	$-\operatorname{tg}(\nu\pi/2)\,y^{-1/2}$
(2)	0, $0 < x < a$; $x^{1/2+\nu}$, $a < x < \infty$; $\operatorname{Re}\nu < -1/2$	$-a^{\nu+1} y^{-1/2} Y_{\nu+1}(ay)$
(3)	x^μ, $\lvert\operatorname{Re}\nu\rvert - 3/2 < \mu < 0$	$2^{\mu+1/2}\times$ $\times \operatorname{ctg}\left[2^{-1}(\nu+1/2-\mu)\pi\right] y^{-\mu-1}\times$ $\times \frac{\Gamma(3/4+\nu/2+\mu/2)}{\Gamma(1/4+\nu/2-\mu/2)}$

	$f(x)$	$\int_0^\infty f(x)\,Y_\nu(xy)\,(xy)^{1/2}\,dx, \quad y>0$
(4)	$0, \quad 0<x<a$ $x^\mu, \quad a<x<\infty$ $\operatorname{Re}\mu<0$	$ay^{-\mu}\,[Y_{\nu-1}(ay)\,S_{\mu+1/2,\,\nu}(ay)-$ $-(\mu+\nu-1/2)\,Y_\nu(ay)\times$ $\times S_{\mu-1/2,\,\nu-1}(ay)]$
(5)	$x^{-1/2}(x+\alpha)^{-1},$ $\lvert\arg\alpha\rvert<\pi, \quad -1<\operatorname{Re}\nu<1$ $\nu\neq0, \quad \pm 1/2$	$\dfrac{\pi y^{1/2}\,[\mathbf{E}_\nu(\alpha y)+Y_\nu(\alpha y)]}{\sin(\nu\pi)}+$ $+2\operatorname{ctg}(\nu\pi)\,[\mathbf{J}_\nu(\alpha y)-J_\nu(\alpha y)]$
(6)	$x^{\nu-1/2}(x+\alpha)^{-1}, \quad \lvert\arg\alpha\rvert<\pi$ $-1/2<\operatorname{Re}\nu<3/2$	$-2^{\nu+1}\pi^{-1}\alpha^\nu y^{1/2}\,\Gamma(\nu+1)\times$ $\times S_{-\nu-1,\,\nu}(\alpha y)$
(7)	$x^{-\nu-1/2}(x+\alpha)^{-1}, \quad \lvert\arg\alpha\rvert<\pi$ $-3/2<\operatorname{Re}\nu<1/2$	$\alpha^{-\nu}y^{1/2}\,\{2^{-1}\pi\operatorname{tg}(\nu\pi)\,[Y_\nu(\alpha y)-$ $-\mathbf{H}_\nu(\alpha y)]-2^{1-\nu}\pi^{-1}\cos(\nu\pi)\times$ $\times\Gamma(1-\nu)\,S_{\nu-1,\,\nu}(\alpha y)\}$
(8)	$x^{\mu-1/2}(x-\alpha)^{-1}, \quad \lvert\arg\alpha\rvert<\pi$ $\operatorname{Re}(\mu\pm\nu)>-1, \quad \operatorname{Re}\mu<3/2$	$(2\alpha)^\mu\pi^{-1}y^{1/2}\times$ $\times\{\sin[2^{-1}\pi(\mu-\nu)]\times$ $\times\Gamma(1/2+\mu/2+\nu/2)\times$ $\times\Gamma(1/2+\mu/2-\nu/2)\,S_{-\mu,\,\nu}(\alpha y)-$ $-2\cos[2^{-1}\pi(\mu-\nu)]\times$ $\times\Gamma(1+\mu/2+\nu/2)\times$ $\times\Gamma(1+\mu/2-\nu/2)\,S_{-\mu-1,\,\nu}(\alpha y)\}$
(9)	$x^{-1/2}(x-a)^{-1},$ $a>0, \quad -1<\operatorname{Re}\nu<1$	$\pi y^{1/2}\,\{\operatorname{ctg}(\nu\pi)\,[Y_\nu(ay)+\mathbf{E}_\nu(ay)]+$ $+\mathbf{J}_\nu(ay)+2\,[\operatorname{ctg}(\nu\pi)]^2\times$ $\times[\mathbf{J}_\nu(ay)-J_\nu(ay)]\}$ Интеграл понимается в смысле главного значения по Коши.
(10)	$x^{\nu-1/2}(x-a)^{-1},$ $a>0, \quad -1/2<\operatorname{Re}\nu<3/2$	$a^\nu y^{1/2}\,[\pi J_\nu(ay)-2^{\nu+1}\pi^{-1}\Gamma(\nu+1)\times$ $\times S_{-\nu-1,\,\nu}(ay)]$ Интеграл понимается в смысле главного значения по Коши.
(11)	$x^{-\nu-1/2}(x-a)^{-1},$ $a>0, \quad -3/2<\operatorname{Re}\nu<1/2$	$a^{-\nu}y^{1/2}\,\{2^{-1}\pi\operatorname{tg}(\nu\pi)\,[\mathbf{H}_\nu(ay)-$ $-Y_\nu(ay)]+\pi J_\nu(ay)-$ $-2^{1-\nu}\pi^{-1}\cos(\nu\pi)\,\Gamma(1-\nu)\times$ $\times S_{\nu-1,\,\nu}(ay)\}$ Интеграл понимается в смысле главного значения по Коши.

	$f(x)$	$\int_0^\infty f(x)\, Y_\nu(xy)\,(xy)^{1/2}\,dx, \quad y>0$
(12)	$x^{\mu-1/2}(x-a)^{-1}, \quad a>0$ $\operatorname{Re}(\mu\pm\nu)>-1, \quad \operatorname{Re}\mu>3/2$	$\pi a^\mu y^{1/2} J_\nu(ay)-(2a)^\mu \pi^{-1} y^{1/2}\times$ $\times\{\sin[2^{-1}\pi(\mu-\nu)]\times$ $\times\Gamma(1/2+\mu/2+\nu/2)\times$ $\times\Gamma(1/2+\mu/2-\nu/2)\,S_{-\mu,\nu}(ay)+$ $+2\cos[2^{-1}\pi(\mu-\nu)]\times$ $\times\Gamma(1+\mu/2+\nu/2)\times$ $\times\Gamma(1+\mu/2-\nu/2)\,S_{-\mu-1,\nu}(ay)\}$ Интеграл понимается в смысле главного значения по Коши.
(13)	$x^{-1/2}(x^2+\alpha^2)^{-1},$ $\operatorname{Re}\alpha>0, \quad -1<\operatorname{Re}\nu<1$	$\frac{y^{1/2}}{\cos(\nu\pi/2)}\left[-\frac{\pi}{2\alpha}\operatorname{tg}\left(\frac{\nu\pi}{2}\right)I_\nu(\alpha y)-\right.$ $-\frac{1}{\alpha}K_\nu(\alpha y)+\frac{y\sin(\nu\pi/2)}{1-\nu^2}\times$ $\left.\times{}_1F_2\left(1;\frac{3-\nu}{2},\frac{3+\nu}{2};\frac{\alpha^2y^2}{4}\right)\right]$
(14)	$x^{\nu-1/2}(x^2+\alpha^2)^{-1},$ $\operatorname{Re}\alpha>0, \quad -1/2<\operatorname{Re}\nu<5/2$	$-\alpha^{\nu-1}y^{1/2}K_\nu(\alpha y)$
(15)	$x^{\nu+3/2}(x^2+\alpha^2)^{-1},$ $\operatorname{Re}\alpha>0, \quad -3/2<\operatorname{Re}\nu<1/2$	$\alpha^{\nu+1}y^{1/2}K_\nu(\alpha y)$
(16)	$x^{-\nu-1/2}(x^2+\alpha^2)^{-1}, \quad \operatorname{Re}\alpha>0$ $-5/2<\operatorname{Re}\nu<1/2$	$\alpha^{-\nu-1}y^{1/2}\{2^{-1}\pi\operatorname{tg}(\nu\pi)[\mathbf{L}_\nu(\alpha y)-$ $-I_\nu(\alpha y)]-\frac{K_\nu(\alpha y)}{\cos(\nu\pi)}\}$
(17)	$x^{\mu-3/2}(x^2+\alpha^2)^{-1}, \quad \operatorname{Re}\alpha>0$ $\lvert\operatorname{Re}\nu\rvert<\operatorname{Re}\mu<7/2$	$2^{\mu-3}\pi^{-1}y^{5/2-\mu}\cos[2^{-1}\pi(\mu-\nu)]\times$ $\times\Gamma(\mu/2+\nu/2-1)\,\Gamma(\mu/2-\nu/2-1)\times$ $\times{}_1F_2\left(1;2-\frac{\mu+\nu}{2},2-\frac{\mu-\nu}{2};\frac{\alpha^2y^2}{4}\right)-$ $-\frac{\pi\alpha^{\mu-2}y^{1/2}\operatorname{ctg}[2^{-1}\pi(\mu-\nu)]\,I_\nu(\alpha y)}{2\sin[2^{-1}\pi(\mu+\nu)]}-$ $-\frac{\alpha^{\mu-2}y^{1/2}K_\nu(\alpha y)}{\sin[2^{-1}\pi(\mu-\nu)]}$
(18)	$x^{-1/2}(x^2+\alpha^2)^{-1/2},$ $\operatorname{Re}\alpha>0, \quad -1<\operatorname{Re}\nu<1$	$-\frac{y^{1/2}K_{\nu/2}(\alpha y/2)}{\pi\cos(\nu\pi/2)}\times$ $\times[K_{\nu/2}(\alpha y/2)+\pi\sin(\nu\pi/2)\times$ $\times I_{\nu/2}(\alpha y/2)]$

	$f(x)$	$\int\limits_0^\infty f(x)\, Y_\nu(xy)\,(xy)^{1/2}\,dx, \quad y>0$
(19)	$x^{1/2+\nu}(x^2+\alpha^2)^{\mu}$, $\operatorname{Re}\alpha>0$, $-1<\operatorname{Re}\nu<-2\operatorname{Re}\mu$	$2^{\nu-1}\pi^{-1}\alpha^{2\mu+2}(1+\mu)^{-1}\Gamma(\nu)\,y^{1/2-\nu}\times$ $\times\,{}_1F_2(1;\,1-\nu,\,2+\mu;\,2^{-2}\alpha^2y^2)-$ $-2^{\mu}\alpha^{\mu+\nu+1}(\sin\nu\pi)^{-1}\,\Gamma(\mu+1)\times$ $\times\,y^{-1/2-\mu}\,[I_{\mu+\nu+1}(\alpha y)-$ $-2\cos(\mu\pi)\,K_{\mu+\nu+1}(\alpha y)]$
(20)	$x^{1/2-\nu}(x^2+\alpha^2)^{\mu}$, $\operatorname{Re}\alpha>0$, $1/2+2\operatorname{Re}\mu<\operatorname{Re}\nu<1$	$2^{\mu}\alpha^{\mu-\nu+1}y^{-1/2-\mu}\Big[\pi^{-1}\cos(\nu\pi)\times$ $\times\,\Gamma(\mu+1)\,\Gamma(\nu)\,I_{\nu-\mu-1}(\alpha y)-$ $-\dfrac{2K_{\nu-\mu-1}(\alpha y)}{\sin(\nu\pi)\,\Gamma(-\mu)}\Big]-$ $-\dfrac{\alpha^{2\mu+2}\operatorname{ctg}(\nu\pi)\,y^{1/2+\nu}}{2^{\nu+1}(\mu+1)\,\Gamma(\nu+1)}\times$ $\times\,{}_1F_2(1;\,\nu+1,\,\mu+2;\,2^{-2}\alpha^2y^2)$
(21)	$x^{-1/2}(x^2-a^2)^{-1}$, $a>0$, $-1<\operatorname{Re}\nu<1$	$2^{-1}\pi a^{-1}y^{1/2}\,[J_\nu(ay)+$ $+\operatorname{tg}(\nu\pi/2)\,\{\operatorname{tg}(\nu\pi/2)\,[\mathbf{J}_\nu(ay)-$ $-J_\nu(ay)]-\mathbf{E}_\nu(ay)-Y_\nu(ay)\}]$ Интеграл понимается в смысле главного значения по Коши.
(22)	$x^{\nu-1/2}(x^2-a^2)^{-1}$, $a>0$, $-1/2<\operatorname{Re}\nu<5/2$	$2^{-1}\pi a^{\nu-1}y^{1/2}\,J_\nu(ay)$ Интеграл понимается в смысле главного значения по Коши.
(23)	$x^{\nu+1/2}(x^2-a^2)^{-1}$, $a>0$, $-1<\operatorname{Re}\nu<3/2$	$2^{-1}\pi a^{\nu}y^{1/2}\,J_\nu(ay)-$ $-2^{\nu+1}\pi^{-1}\Gamma(\nu+1)\,a^{\nu}y^{1/2}\times$ $\times\,S_{-\nu-1,\,\nu}(ay)$ Интеграл понимается в смысле главного значения по Коши.
(24)	$x^{-\nu-1/2}(x^2-a^2)^{-1}$, $a>0$, $-5/2<\operatorname{Re}\nu<1/2$	$\dfrac{\pi y^{1/2}}{2a^{\nu+1}\cos(\nu\pi)}\times$ $\times\,[J_{-\nu}(ay)+\sin(\nu\pi)\,\mathbf{H}_\nu(ay)]$ Интеграл понимается в смысле главного значения по Коши.
(25)	$x^{\mu-3/2}(x^2-a^2)^{-1}$, $a>0$, $\lvert\operatorname{Re}\nu\rvert<\operatorname{Re}\mu<7/2$	$2^{-1}\pi a^{\mu-2}y^{1/2}\,J_\nu(ay)+$ $+2^{\mu-1}\pi^{-1}a^{\mu-2}y^{1/2}\times$ $\times\cos[2^{-1}\pi(\mu-\nu)]\times$ $\times\,\Gamma\left(\dfrac{\mu-\nu}{2}\right)\Gamma\left(\dfrac{\mu+\nu}{2}\right)S_{1-\mu,\,\nu}(ay)$ Интеграл понимается в смысле главного значения по Коши.

	$f(x)$	$\int\limits_0^\infty f(x)\, Y_\nu(xy)\,(xy)^{1/2}\,dx, \quad y>0$
(26)	$x^{-1/2}(a^2-x^2)^{-1/2}, \quad 0<x<a$ $0, \quad a<x<\infty$ $\nu=0$	$2^{-1}\pi y^{-1/2} J_0(ay/2)\, Y_0(ay/2)$
(27)	$0, \quad 0<x<a$ $x^{-1/2}(x^2-a^2)^{-1/2}, \quad a<x<\infty$	$2^{-2}\pi y^{1/2}\{[J_{\nu/2}(ay/2)]^2-$ $-[Y_{\nu/2}(ay/2)]^2\}$
(28)	$x^{\nu+1/2}(a^2-x^2)^{-1/2}, \quad 0<x<a$ $0, \quad a<x<\infty$ $\operatorname{Re}\nu>-1$	$\dfrac{(\pi/2)^{1/2}a^{\nu+1/2}}{\sin(\nu\pi)}\times$ $\times[\cos(\nu\pi)\, J_{\nu+1/2}(ay)-\mathbf{H}_{-\nu-1/2}(y)]$
(29)	$0, \quad 0<x<a$ $x^{\nu+1/2}(x^2-a^2)^{-1/2}, \quad a<x<\infty$ $\operatorname{Re}\nu<1/2$	$(\pi/2)^{1/2}a^{\nu+1/2}J_{\nu+1/2}(ay)$
(30)	$x^{1/2-\nu}(a^2-x^2)^{-1/2}, \quad 0<x<a$ $0, \quad a<x<\infty$ $\operatorname{Re}\nu<1$	$(\pi/2)^{1/2}a^{1/2-\nu}\{\operatorname{ctg}(\nu\pi)\,[\mathbf{H}_{\nu-1/2}(ay)-$ $-Y_{\nu-1/2}(ay)]-J_{\nu-1/2}(y)\}$
(31)	$x^{\nu-1/2}(a^2-x^2)^{\nu-1/2}, \quad 0<x<a$ $0, \quad a<x<\infty$ $\operatorname{Re}\nu>-1/2$	$2^{\nu-1}\pi^{1/2}a^{2\nu}y^{1/2-\nu}\,\Gamma(\nu+1/2)\times$ $\times J_\nu(ay/2)\, Y_\nu(ay/2)$
(32)	$0, \quad 0<x<a$ $x^{\nu-1/2}(x^2-a^2)^{\nu-1/2}, \quad a<x<\infty$ $-1/2<\operatorname{Re}\nu<1/2$	$2^{\nu-2}\pi^{1/2}a^{2\nu}y^{1/2-\nu}\,\Gamma(\nu+1/2)\times$ $\times[J_\nu(ay/2)\, J_{-\nu}(ay/2)-$ $-Y_\nu(ay/2)\, Y_{-\nu}(ay/2)]$
(33)	$0, \quad 0<x<a$ $x^{1/2-\nu}(x^2-a^2)^{\nu-1/2}, \quad a<x<\infty$ $-1/2<\operatorname{Re}\nu<1/2$	$2^\nu\pi^{-1/2}y^{-1/2-\nu}\,\Gamma(\nu+1/2)\sin(ay)$
(34)	$0, \quad 0<x<a$ $x^{-\nu-1/2}(x^2-a^2)^{-\nu-1/2},$ $a<x<\infty$ $-1/2<\operatorname{Re}\nu<1/2$	$2^{-\nu-2}\pi^{1/2}a^{-2\nu}y^{\nu+1/2}\,\Gamma(1/2-\nu)\times$ $\times\{[J_\nu(ay/2)]^2-[Y_\nu(ay/2)]^2\}$
(35)	$x^{\nu+1/2}(a^2-x^2)^\mu, \quad 0<x<a$ $0, \quad a<x<\infty$ $\operatorname{Re}\mu>-1, \quad \operatorname{Re}\nu>-1$	$a^{\mu+\nu+1}y^{-\mu-1/2}[2^\mu\Gamma(\mu+1)\times$ $\times Y_{\mu+\nu+1}(ay)+2^{\nu+1}\pi^{-1}\Gamma(\nu+1)\times$ $\times S_{\mu-\nu,\,\mu+\nu+1}(ay)]$

	$f(x)$	$\int_0^\infty f(x) Y_\nu(xy)(xy)^{1/2}\,dx, \quad y>0$
(36)	$0, \quad 0<x<a$ $x^{\nu+1/2}(x^2-a^2)^\mu, \quad a<x<\infty$ $-2<2\operatorname{Re}\mu<-1/2-\operatorname{Re}\nu$	$-2^\mu a^{\mu+\nu+1} y^{-\mu-1/2}\Gamma(\mu+1)\times$ $\times[\sin(\mu\pi) J_{\mu+\nu+1}(ay)+$ $+\cos(\mu\pi) Y_{\mu+\nu+1}(ay)]$
(37)	$x^{1/2-\nu}(a^2-x^2)^\mu, \quad 0<x<a$ $0, \quad a<x<\infty$ $\operatorname{Re}\mu>-1, \quad \operatorname{Re}\nu<1$	$a^{\mu-\nu+1}y^{-\mu-1/2}[2^{1-\nu}\pi^{-1}\cos(\nu\pi)\times$ $\times\Gamma(1-\nu) s_{\mu+\nu,\,\mu-\nu+1}(ay)-$ $-\dfrac{2^\mu\Gamma(\mu+1) J_{\mu-\nu+1}(ay)}{\sin(\nu\pi)}\Big]$
(38)	$0, \quad 0<x<a$ $x^{1/2-\nu}(x^2-a^2)^\mu, \quad a<x<\infty$ $-1<\operatorname{Re}\mu<2^{-1}\operatorname{Re}\nu-1/4$	$2^\mu a^{\mu-\nu+1}y^{-\mu-1/2}\Gamma(\mu+1)\times$ $\times Y_{\nu-\mu-1}(ay)$
(39)	$x^{2n+\nu+4\mu-1/2}(x^4+a^4)^{-\mu-1}$	См. Ватсон Г. Н., 1949, стр. 474.
(40)	$x^{-1/2}(x^2+\alpha^2)^{-1/2}\times$ $\times[(x^2+\alpha^2)^{1/2}-x]^\mu,$ $\operatorname{Re}\alpha>0, \quad \operatorname{Re}\mu>-3/2$ $-1<\operatorname{Re}\nu<1$	$\alpha^\mu y^{1/2}[\operatorname{ctg}(\nu\pi) I_{\mu/2+\nu/2}(\alpha y/2)\times$ $\times K_{\mu/2-\nu/2}(\alpha y/2)-$ $-\dfrac{I_{\mu/2-\nu/2}(\alpha y/2) K_{\mu/2+\nu/2}(\alpha y/2)}{\sin(\nu\pi)}\Big]$
(41)	$x^{-1/2}(x^2+\alpha^2)^{-1/2}\times$ $\times\{[(x^2+\alpha^2)^{1/2}+x]^\mu+$ $+[(x^2+\alpha^2)^{1/2}-x]^\mu\}, \quad \nu=0$ $-3/2<\operatorname{Re}\mu<3/2$	$-2\pi^{-1}\alpha^\mu y^{1/2}\cos(\mu\pi/2)\times$ $\times[K_{\mu/2}(\alpha y/2)]^2$
(42)	$x^{-1/2}(x^2+\alpha^2)^{-1/2}\times$ $\times[(x^2+\alpha^2)^{1/2}-\alpha]^{2k},$ $\operatorname{Re}\alpha>0, \quad \lvert\operatorname{Re}\nu\rvert<1/2+\operatorname{Re}k$	$-\alpha^{-1}y^{-1/2}W_{-k,\,\nu/2}(\alpha y)\times$ $\times\Big\{\dfrac{\Gamma(1/2+\nu/2+k)}{\Gamma(\nu+1)}\operatorname{tg}[(\nu/2-k)\pi]\times$ $\times M_{k,\,\nu/2}(\alpha y)+\dfrac{W_{k,\,\nu/2}(\alpha y)}{\cos[(\nu/2-k)\pi]}\Big\}$
(43)	$0, \quad 0<x<a$ $x^{-1/2}(x^2-a^2)^{-1/2}\times$ $\times\{[x+(x^2-a^2)^{1/2}]^\mu+$ $+[x-(x^2-a^2)^{1/2}]^\mu\},$ $a<x<\infty$ $-3/2<\operatorname{Re}\mu<3/2$	$2^{-1}\pi a^\mu y^{1/2}[J_{\nu/2+\mu/2}(ay/2)\times$ $\times J_{\nu/2-\mu/2}(ay/2)-Y_{\nu/2+\mu/2}(ay/2)\times$ $\times Y_{\nu/2-\mu/2}(ay/2)]$

9.3. Другие элементарные функции

	$f(x)$	$\int_0^\infty f(x)\, Y_\nu(xy)\,(xy)^{1/2}\,dx, \quad y>0$
(1)	$x^{-1/2}e^{-\alpha x}$, $\operatorname{Re}\alpha>0,\ -1<\operatorname{Re}\nu<1$	$\frac{y^{1/2}}{(y^2+\alpha^2)^{1/2}\sin(\nu\pi)}\times \times\left\{y^\nu\left[(y^2+\alpha^2)^{1/2}+\alpha\right]^{-\nu}\cos(\nu\pi)- - y^{-\nu}\left[(y^2+\alpha^2)^{1/2}+\alpha\right]^\nu\right\}$
(2)	$x^{\mu-3/2}e^{-ax}$, $a>0,\ \operatorname{Re}\mu>\lvert\operatorname{Re}\nu\rvert$	$-2\pi^{-1}\Gamma(\mu+\nu)\,y^{1/2}(y^2+a^2)^{-\mu/2}\times \times Q_{\mu-1}^{-\nu}\left[a(y^2+a^2)^{-1/2}\right]$
(3)	$x^{-1/2}\exp(-\alpha x^2)$, $\operatorname{Re}\alpha>0,\ -1<\operatorname{Re}\nu<1$	$-\frac{1}{2}\left(\frac{\pi y}{\alpha}\right)^{1/2}\exp\left(-\frac{y^2}{8\alpha}\right)\times \times\left[\operatorname{tg}\frac{\nu\pi}{2}\, I_{\nu/2}\left(\frac{y^2}{8\alpha}\right)+ +\frac{1}{\pi\cos(\nu\pi/2)}K_{\nu/2}\left(\frac{y^2}{8\alpha}\right)\right]$
(4)	$x^{\mu-1/2}\exp(-\alpha x^2)$, $\operatorname{Re}\alpha>0,\ \operatorname{Re}\mu>\lvert\operatorname{Re}\nu\rvert-1$	$-\frac{1}{\alpha^{\mu/2}y^{1/2}\cos\left[2^{-1}(\nu-\mu)\pi\right]}\times \times\exp\left(-\frac{y^2}{8\alpha}\right)\left\{\frac{\Gamma(1/2+\mu/2+\nu/2)}{\Gamma(1+\nu)}\times \times\sin\left[2^{-1}(\nu-\mu)\pi\right]\times \times M_{\mu/2,\,\nu/2}\left(\frac{y^2}{4\alpha}\right)+W_{\mu/2,\,\nu/2}\left(\frac{y^2}{4\alpha}\right)\right\}$
(5)	$x^{-3/2}e^{-\alpha/x}$, $\operatorname{Re}\alpha>0$	$2y^{1/2}\,Y_\nu\left[(2\alpha y)^{1/2}\right]K_\nu\left[(2\alpha y)^{1/2}\right]$
(6)	$x^{-1/2}(x^2+\beta^2)^{-1/2}\times \times\exp\left[-\alpha(x^2+\beta^2)^{-1/2}\right]$, $\operatorname{Re}\alpha>0,\ \operatorname{Re}\beta>0$ $-1<\operatorname{Re}\nu<1$	$-\frac{y^{1/2}}{\cos(\nu\pi/2)}\times \times K_{\nu/2}\left\{2^{-1}\beta\left[(y^2+\alpha^2)^{1/2}+\alpha\right]\right\}\times \times\left(\frac{1}{\pi}K_{\nu/2}\left\{\frac{\beta}{2}\left[(y^2+\alpha^2)^{1/2}-\alpha\right]\right\}+ +\sin(\nu\pi/2)\times \times I_{\nu/2}\left\{2^{-1}\beta\left[(y^2+\alpha^2)^{1/2}-\alpha\right]\right\}\right)$
(7)	$x^{-1/2}\sin ax^2$, $a>0,\ -3<\operatorname{Re}\nu<3$	$-\frac{(\pi y/a)^{1/2}}{4\cos(\nu\pi/2)}\left[\cos\left(\frac{y^2}{8a}-\frac{3\nu+1}{4}\pi\right)\times +J_{\nu/2}\left(\frac{y^2}{8a}\right)-\sin\left(\frac{y^2}{8a}+\frac{\nu-1}{4}\pi\right)\times \times Y_{\nu/2}\left(\frac{y^2}{8a}\right)\right]$

	$f(x)$	$\int\limits_0^\infty f(x)\, Y_\nu(xy)\,(xy)^{1/2}\,dx, \quad y>0$
(8)	$x^{-1/2}\cos ax^2$, $a>0,\ -1<\operatorname{Re}\nu<1$	$\frac{(\pi y/a)^{1/2}}{4\cos(\nu\pi/2)}\times$ $\times\left[\sin\left(\frac{y^2}{8a}-\frac{3\nu+1}{4}\pi\right)J_{\nu/2}\left(\frac{y^2}{8a}\right)+\right.$ $\left.+\cos\left(\frac{y^2}{8a}+\frac{\nu-1}{4}\pi\right)Y_{\nu/2}\left(\frac{y^2}{8a}\right)\right]$
(9)	$x^{-1/2}(a^2-x^2)^{-1/2}\times$ $\times\sin\left[b(a^2-x^2)^{1/2}\right]$, $0<x<a$ $-x^{-1/2}(x^2-a^2)^{-1/2}\times$ $\times\exp\left[-b^2(x^2-a^2)^{1/2}\right]$, $a<x<\infty$ $b>0,\ -1<\operatorname{Re}\nu<1$	$2^{-2}\pi y^{1/2}\times$ $\times Y^2_{\nu/2}\{2^{-1}a[(y^2+b^2)^{1/2}+b]\}-$ $-2^{-2}\pi y^{1/2}\times$ $\times J^2_{\nu/2}\{2^{-1}a[(y^2+b^2)^{1/2}-b]\}$

Относительно других выражений, содержащих тригонометрические функции, см. таблицы преобразований Фурье (гл. I—III).

9.4. Высшие трансцендентные функции

(1)	$x^{1/2}P_n(1-2x^2)$, $0<x<1$ 0, $1<x<\infty$ $n=0,1,2,\ldots,\ \nu=0$	$\pi^{-1}y^{-1/2}\left[S_{2n+1}(y)+\pi Y_{2n+1}(y)\right]$
(2)	$x^{\nu-2\mu+2n+3/2}\exp(x^2)\,\Gamma(\mu,x^2)$, n — целое $\operatorname{Re}(\nu-\mu+n)>-3/2$ $\operatorname{Re}(-\mu+n)>-3/2$ $\operatorname{Re}\nu<1/2-2n$	$(-1)^n\frac{\Gamma(3/2-\mu+\nu+n)\,\Gamma(3/2-\mu+n)}{y^{1/2}\,\Gamma(1-\mu)}\times$ $\times\exp\left(\frac{y^2}{8}\right)W_{\mu-\nu/2-n-1,\,\nu/2}\left(\frac{y^2}{4}\right)$
(3)	0, $0<x<a$ $P_{\nu-1/2}(a^{-1}x)$, $a<x<\infty$ $\operatorname{Re}\nu<1/2$	$\left(\frac{a}{2y}\right)^{1/2}\left[\cos(ay/2)\,J_\nu(ay/2)-\right.$ $\left.-\sin(ay/2)\,Y_\nu(ay/2)\right]$
(4)	0, $0<x<a$ $x^{-\mu}(x^2-a^2)^{-\mu/2}P^{\mu}_{\nu-1/2}(x/a)$, $a<x<\infty$ $-1/4<\operatorname{Re}\mu<1$ $\operatorname{Re}(2\mu-\nu)>-1/2$	$2^{-3/2}\pi^{1/2}a^{1-\mu}y^{\mu}\times$ $\times\left[J_\nu(ay/2)\,J_{\mu-1/2}(ay/2)-\right.$ $\left.-Y_\nu(ay/2)\,Y_{\mu-1/2}(ay/2)\right]$

	$f(x)$	$\int\limits_0^\infty f(x)\, Y_\nu(xy)\,(xy)^{1/2}\,dx, \quad y>0$
(5)	$0, \quad 0<x<a$ $(x^2-a^2)^{\nu/2-1/4} P_\mu^{1/2-\nu}(2a^{-2}x^2-1), \quad a<x<\infty$ $\operatorname{Re}\nu>-1/2$ $\operatorname{Re}\nu+\lvert 2\operatorname{Re}\mu+1\rvert<3/2$	$\pi^{1/2}2^{\nu-2}ay^{1/2-\nu}\times$ $\times[J_{\mu+1/2}(ay/2)\,J_{-\mu-1/2}(ay/2)-$ $-Y_{\mu+1/2}(ay/2)\,Y_{-\mu-1/2}(ay/2)]$
(6)	$x^\lambda J_\mu(ax)$	См. 6.8 (37).
(7)	$\sin ax\, J_{\nu+1/2}(ax),$ $a>0,\ \operatorname{Re}\nu>-3/2$	$(\pi\cos\theta)^{-1/2}(2a\sin\theta)^{-1}\times$ $\times\cos[(\nu+1)\theta],$ $y=2a\cos\theta, \quad 0<\theta<\pi/2$ $0, \quad 2a<y<\infty$
(8)	$x^{\nu+1/2}[J_\nu(ax)]^2,$ $a>0,\ -1/2<\operatorname{Re}\nu<1/2$	$0, \quad 0<y<2a$ $\dfrac{2^{3\nu+1}a^{2\nu}}{\pi^{1/2}\Gamma(1/2-\nu)}\,y^{-\nu-1/2}(y^2-4a^2)^{-\nu-1/2},$ $2a<y<\infty$
(9)	$x^{1/2}J_{\nu/2}(ax^2),$ $a>0,\ \operatorname{Re}\nu>-1$	$\dfrac{y^{1/2}}{4a}\left[Y_{\nu/2}\left(\dfrac{y^2}{4a}\right)-\operatorname{tg}\left(\dfrac{\nu\pi}{2}\right)J_{\nu/2}\left(\dfrac{y^2}{4a}\right)+\right.$ $\left.+\dfrac{1}{\cos(\nu\pi/2)}\mathbf{H}_{-\nu/2}\left(\dfrac{y^2}{4a}\right)\right]$
(10)	$x^{5/2}J_{\nu/2-1/2}(ax^2),$ $a>0,\ \operatorname{Re}\nu>-3/2$	$a^{-2}y^{1/2}J_{\nu/2+1/2}\left(\dfrac{y^2}{4a}\right)$
(11)	$x^{1/2}J_{\nu/4}(ax^2)\,J_{-\nu/4}(ax^2),$ $a>0,\ -2<\operatorname{Re}\nu<2$	$\dfrac{y^{1/2}}{16a\cos(\nu\pi/4)}\Big\{[1+2\cos(\nu\pi/2)]\times$ $\times\left[J_{\nu/4}\left(\dfrac{y^2}{16a}\right)\right]^2+2\sin(\nu\pi/2)\times$ $\times J_{\nu/4}\left(\dfrac{y^2}{16a}\right)Y_{\nu/4}\left(\dfrac{y^2}{16a}\right)-$ $-\left[Y_{\nu/4}\left(\dfrac{y^2}{16a}\right)\right]^2\Big\}$
(12)	$x^{-1/2}J_\nu(a^2x^{-1}),$ $a>0,\ -1/2<\operatorname{Re}\nu<3/2$	$y^{-1/2}[Y_{2\nu}(2ay^{1/2})+$ $+2\pi^{-1}K_{2\nu}(2ay^{1/2})]$
(13)	$x^{-5/2}J_\nu(a^2x^{-1}),$ $a>0,\ -1/2<\operatorname{Re}\nu<1/2$	$a^{-2}y^{1/2}[Y_{2\nu}(2ay^{1/2})-$ $-2\pi^{-1}K_{2\nu}(2ay^{1/2})]$

	$f(x)$	$\int\limits_0^\infty f(x)\, Y_\nu(xy)\,(xy)^{1/2}\,dx,\quad y>0$
(14)	$x^{-1/2}\, Y_\nu(a^2x^{-1}),$ $a>0,\ -1/2<\operatorname{Re}\nu<1/2$	$-\, y^{-1/2}\, J_{2\nu}(2ay^{1/2})$
(15)	$x^{-5/2}\, Y_\nu(a^2x^{-1}),$ $a>0,\ -1/2<\operatorname{Re}\nu<1/2$	$-\, a^{-2}y^{1/2}\, J_{2\nu}(2ay^{1/2})$
(16)	$x^{-3/2}\, Y_{\nu+1}(a^2x^{-1}),$ $a>0,\ -3/2<\operatorname{Re}\nu<1/2$	$-\, a^{-1}\, J_{2\nu+1}(2ay^{1/2})$
(17)	$J_{2\nu-1}(ax^{1/2}),$ $a>0,\ \operatorname{Re}\nu>-1/2$	$-\dfrac{a}{2y^{3/2}}\, \mathbf{H}_{\nu-1}\left(\dfrac{a^2}{4y}\right)$
(18)	$x^{-1/2}\, J_{2\nu}(ax^{1/2}),$ $a>0,\ \operatorname{Re}\nu>-1/2$	$-\, y^{-1/2}\, \mathbf{H}_\nu\left(\dfrac{a^2}{4y}\right)$
(19)	$x^{-1/2}\, Y_{2\nu}(ax^{1/2}),$ $a>0,\ -1/2<\operatorname{Re}\nu<1/2$	$\dfrac{1}{2y^{1/2}}\left[\dfrac{1}{\cos(\nu\pi)}\, J_{-\nu}\left(\dfrac{a^2}{4y}\right)+\right.$ $+\dfrac{1}{\sin(\nu\pi)}\, \mathbf{H}_{-\nu}\left(\dfrac{a^2}{4y}\right)-$ $\left.-\,2\operatorname{ctg}(2\nu\pi)\, \mathbf{H}_\nu\left(\dfrac{a^2}{4y}\right)\right]$
(20)	$x^{\nu+2n-1/2}\,(x^2+\lambda^2)^{-1}\,(x^2+a^2)^{-\mu/2}\times$ $\times\, J_\mu[b\,(x^2+a^2)^{1/2}],$ $b>0$ $\operatorname{Re}\lambda>0,\ n=0,\ 1,\ 2,\ \ldots$ $-1/2-n<\operatorname{Re}\mu<3-2n+\operatorname{Re}\nu$	$(-1)^{n+1}\,\lambda^{\nu+2n-1}y^{1/2}\,K_\nu(\lambda y)\times$ $\times\,(\lambda^2-a^2)^{-\mu/2}\,I_\mu[b\,(\lambda^2-a^2)^{1/2}],$ $y>b$
(21)	$0,\quad 0<x<a$ $x^{\nu+1/2}\,(x^2-a^2)^{\mu/2}\,J_\mu[b\,(x^2-a^2)^{1/2}],$ $a<x<\infty$ $b>0$ $-1<\operatorname{Re}\mu<-\operatorname{Re}\nu$	$-\,2\pi^{-1}a^{\mu+\nu+1}b^{\mu}y^{\nu+1/2}\cos(\nu\pi)\times$ $\times\,(b^2-y^2)^{-(\mu+\nu+1)/2}\times$ $\times\, K_{\mu+\nu+1}[a\,(b^2-y^2)^{1/2}],$ $0<y<b$
(22)	$0,\quad 0<x<a$ $x^{\nu+1/2}\,(x^2-a^2)^{\mu/2}\times$ $\times\, J_\mu[b\,(x^2-a^2)^{1/2}],\quad a<x<\infty$ $b>0$ $-1<\operatorname{Re}\mu<-\operatorname{Re}\nu$	$-\,a^{\mu+\nu+1}b^{\mu}y^{\nu+1/2}\times$ $\times\,(y^2-b^2)^{-(\mu+\nu+1)/2}\times$ $\times\{\sin(\mu\pi)\, J_{\mu+\nu+1}[a\,(y^2-b^2)^{1/2}]+$ $+\cos(\mu\pi)\, Y_{\mu+\nu+1}[a\,(y^2-b^2)^{1/2}]\},$ $b<y<\infty$

	$f(x)$	$\int_0^\infty f(x)\,Y_\nu(xy)\,(xy)^{1/2}\,dx,\quad y>0$
(23)	$0,\qquad 0<x<a$ $x^{1/2-\nu}(x^2-a^2)^{\mu/2}\times$ $\times J_\mu\left[b(x^2-a^2)^{1/2}\right],\quad a<x<\infty$ $b>0,\ -1<\operatorname{Re}\mu<\operatorname{Re}\nu$	$-2\pi^{-1}a^{\mu-\nu+1}b^\mu y^{1/2-\nu}\times$ $\times(b^2-y^2)^{-(\mu-\nu+1)/2}\times$ $\times K_{\mu-\nu+1}\left[a(b^2-y^2)^{1/2}\right],$ $0<y<b$
(24)	$0,\qquad 0<x<a$ $x^{1/2-\nu}(x^2-a^2)^{\mu/2}\times$ $\times J_\mu\left[b(x^2-a^2)^{1/2}\right],\quad a<x<\infty$ $b>0,\quad -1<\operatorname{Re}\mu<\operatorname{Re}\nu$	$a^{\mu-\nu+1}b^\mu y^{1/2-\nu}(y^2-b^2)^{(\nu-\mu-1)/2}\times$ $\times Y_{\nu-\mu-1}\left[a(y^2-b^2)\right],\ b<y<\infty$
(25)	$x^{1/2}K_{\nu/2}(\alpha x^2),$ $\operatorname{Re}\alpha>0,\ -1<\operatorname{Re}\nu<1$	$\frac{\pi y^{1/2}}{4\alpha}\left[\frac{1}{\sin(\nu\pi)}\mathbf{L}_{-\nu/2}\left(\frac{y^2}{4\alpha}\right)-\right.$ $-\operatorname{ctg}(\nu\pi)\,\mathbf{L}_{\nu/2}\left(\frac{y^2}{4\alpha}\right)-$ $-\operatorname{tg}(\nu\pi/2)\,I_{\nu/2}\left(\frac{y^2}{4\alpha}\right)-$ $\left.-\frac{1}{\pi\cos(\nu\pi/2)}K_{\nu/2}\left(\frac{y^2}{4\alpha}\right)\right]$
(26)	$x^{-1/2}\exp\left(2^{-1}\alpha x^2\right)K_0\left(2^{-1}\alpha x^2\right),$ $\nu=0$	$-2^{-1}\pi\alpha^{-1/2}y^{1/2}\exp\left(\frac{y^2}{8\alpha}\right)K_0\left(\frac{y^2}{8\alpha}\right)$
(27)	$x^{-1/2-2\mu}\exp\left(2^{-1}\alpha x^2\right)K_\mu\left(2^{-1}\alpha x^2\right),$ $\nu=0,\quad -3/4<\operatorname{Re}\mu<1/4$	$-\frac{\alpha^\mu\pi^{1/2}\left[\Gamma(1/2-2\mu)\right]^2}{y^{1/2}\,\Gamma(1-2\mu)}\times$ $\times\exp\left(\frac{y^2}{8\alpha}\right)W_{2\mu,0}\left(\frac{y^2}{4\alpha}\right)$
(28)	$x^{-1/2}K_\nu(\alpha x^{-1}),$ $\operatorname{Re}\alpha>0,\ -1/2<\operatorname{Re}\nu<1/2$	$-2y^{-1/2}\left[\sin(3\nu\pi/2)\operatorname{ker}_{2\nu}\left(2\alpha^{1/2}y^{1/2}\right)+\right.$ $\left.+\cos(3\nu\pi/2)\operatorname{kei}_{2\nu}\left(2\alpha^{1/2}y^{1/2}\right)\right]$
(29)	$x^{-5/2}K_\nu(\alpha x^{-1}),$ $\operatorname{Re}\alpha>0,\ -5/2<\operatorname{Re}\nu<5/2$	$2\alpha^{-1}y^{1/2}\times$ $\times\left[\sin(3\pi\nu/2)\operatorname{kei}_{2\nu}\left(2\alpha^{1/2}y^{1/2}\right)-\right.$ $\left.-\cos(3\pi\nu/2)\operatorname{ker}_{2\nu}\left(2\alpha^{1/2}y^{1/2}\right)\right]$
(30)	$x^{-2\nu}K_{\nu-1/2}(\alpha x^{-1}),$ $\operatorname{Re}\alpha>0,\ \operatorname{Re}\nu>1/6$	$(2\pi)^{1/2}\alpha^{1/2-\nu}y^{\nu-1/2}Y_{2\nu-1}\left[(2\alpha y)^{1/2}\right]\times$ $K_{2\nu-1}\left[(2\alpha y)^{1/2}\right]$
(31)	$x^{-2\nu-2}K_{\nu-1/2}(\alpha x^{-1}),$ $\operatorname{Re}\alpha>0,\ \operatorname{Re}\nu>-1/2$	$(2\pi)^{1/2}\alpha^{-1/2-\nu}y^{1/2+\nu}Y_{2\nu}\left[(2\alpha y)^{1/2}\right]\times$ $\times K_{2\nu}\left[(2\alpha y)^{1/2}\right]$

	$f(x)$	$\int\limits_0^\infty f(x)\, Y_\nu(xy)\,(xy)^{1/2}\,dx, \quad y>0$
(32)	$x^{2\nu-2}K_{\nu+1/2}(\alpha x^{-1})$, $\operatorname{Re}\alpha>0$, $\operatorname{Re}\nu<1/2$	$\frac{(\pi/2)^{1/2}\alpha^{\nu-1/2}y^{1/2-\nu}}{\sin(\nu\pi)}\times$ $\times K_{2\nu}[(2\alpha y)^{1/2}]\{J_{2\nu}[(2\alpha y)^{1/2}]-$ $-J_{-2\nu}[(2\alpha y)^{1/2}]\}$
(33)	$x^{-1/2}[K_\mu(\alpha^2x^{-1})]^2$, $\lvert\arg\alpha\rvert<\pi/4$ $-1/4<\operatorname{Re}\mu<1/4$, $\nu=0$	$2\pi y^{-1/2}[\cos(\mu\pi)\,J_{2\mu}(2\alpha y^{1/2})-$ $-\sin(\mu\pi)\,Y_{2\mu}(2\alpha y^{1/2})]$
(34)	$x^{-1/2}K_{2\nu}(\alpha x^{1/2})$, $\operatorname{Re}\alpha>0$, $-1/2<\operatorname{Re}\nu<1/2$	$-2^{-2}\pi y^{-1/2}\left[\frac{1}{\cos(\nu\pi)}J_{-\nu}\left(\frac{\alpha^2}{4y}\right)-\right.$ $-\frac{1}{\sin(\nu\pi)}\mathbf{H}_{-\nu}\left(\frac{\alpha^2}{4y}\right)+$ $\left.+\frac{2}{\sin(2\nu\pi)}\mathbf{H}_\nu\left(\frac{\alpha^2}{4y}\right)\right]$
(35)	$x^{\nu-1/2}J_{2\nu-1}(\alpha x^{1/2})\,K_{2\nu-1}(\alpha x^{1/2})$ $\lvert\arg\alpha\rvert<\pi/4$, $\operatorname{Re}\nu>0$	$\frac{2^{-\nu-1}\pi^{1/2}\alpha^{2\nu-1}}{y^{2\nu}\sin(\nu\pi)}\times$ $\times\left[\mathbf{L}_{1/2-\nu}\left(\frac{\alpha^2}{2y}\right)-I_{\nu-1/2}\left(\frac{\alpha^2}{2y}\right)\right]$
(36)	$x^{-1/2}\mathbf{H}_{\nu-1}(ax)$, $a>0$, $-1/2<\operatorname{Re}\nu<1/2$	$-a^{\nu-1}y^{1/2-\nu}$, $\quad 0<y<a$ 0, $\quad a<y<\infty$
(37)	$x^{\nu-\mu+1/2}\mathbf{H}_\mu(ax)$, $a>0$, $\operatorname{Re}\mu>\operatorname{Re}\nu$ $-3/2<\operatorname{Re}\nu<1/2$	$\frac{2^{\nu+1-\mu}y^{\nu+1/2}}{a^\mu\,\Gamma(\mu-\nu)}(a^2-y^2)^{\mu-\nu-1}$, $\quad 0<y<a$ 0, $\quad a<y<\infty$
(38)	$x^{-5/2}S_{-\nu-3,\,\nu}(\alpha^2x^{-1})$, $\operatorname{Re}\alpha>0$ $-3/2<\operatorname{Re}\nu<1/2$	$\pi 2^{-\nu-2}\alpha^{-2}y^{1/2}K_{2\nu}(2\alpha y^{1/2})\times$ $\times[\Gamma(\nu+2)]^{-1}$
(39)	$x^{\nu-1/2}\exp(2^{-2}\alpha^2x^2)\,D_{\nu/2-1/2}(\alpha x)$, $\lvert\arg\alpha\rvert<3\pi/4$ $-1/2<\operatorname{Re}\nu<2/3$	$-\pi^{-1}2^{3\nu/4+3/4}\alpha^{-\nu}y^{-1/2}\Gamma(\nu+1)\times$ $\times\exp\left(\frac{y^2}{4\alpha^2}\right)W_{-\nu/2-1/2,\,\nu/2}\left(\frac{y^2}{2\alpha^2}\right)$
(40)	$D_{\nu-1/2}(\alpha x^{-1/2})\,D_{-\nu-1/2}(\alpha x^{-1/2})$, $\lvert\arg\alpha\rvert<\pi/4$	$y^{-1}\exp(-\alpha y^{1/2})\times$ $\times\sin[\alpha y^{1/2}-2^{-1}(\nu-1/2)\pi]$

	$f(x)$	$\int\limits_0^\infty f(x)\, Y_\nu(xy)\,(xy)^{1/2}\,dx, \quad y>0$
(41)	$x^{\nu-m}\exp(-2^{-2}x^2)\times$ $\times M_{\varkappa,\, 1/4-m/2}(2^{-1}x^2)$, m — целое $\operatorname{Re}(2\varkappa-\nu)>-m\geqslant -1$, $\operatorname{Re}\nu>m-3/2$	$\frac{(-1)^m\, 2^{\nu/2-m/2}\,\Gamma(3/2-m)}{\Gamma(3/4+\varkappa-m/2)}(2^{-1}y^2)^\lambda\times$ $\times\exp(-2^{-2}y^2)\, W_{\alpha,\beta}(2^{-1}y^2)$, $\alpha=\varkappa/2+m/4+\nu/2+5/8$ $\beta=\varkappa/2+m/4-\nu/2-3/8$ $\lambda=\varkappa/2+m/4-5/8$
(42)	$x^{-m}\exp(-2^{-2}x^2)\times$ $\times M_{\varkappa,\,\nu/2-m/2+1/4}(2^{-1}x^2)$, m — целое, $2\operatorname{Re}\varkappa>-m\geqslant -1$ $\operatorname{Re}\nu>m-3/2$	$\frac{(-1)^m\,\Gamma(\nu-m+3/2)\,2^{-m/2}}{\Gamma(\varkappa+\nu/2-m/2+3/4)}(2^{-1}y^2)^\lambda\times$ $\times\exp(-2^{-2}y^2)\, W_{\alpha,\beta}(2^{-1}y^2)$, $\alpha=\varkappa/2-3m/4+\nu/4+5/8$ $\beta=\varkappa/2+m/4+\nu/4-3/8$ $\lambda=\varkappa/2+m/4-\nu/4-5/8$
(43)	$x^{2\mu+\nu-1/2}\exp(-2^{-2}x^2)\times$ $\times M_{\varkappa,\mu}(2^{-1}x^2)$, $-1<2\operatorname{Re}\mu<\operatorname{Re}(2\varkappa-\nu)+1/2$ $\operatorname{Re}(2\mu+\nu)>-1$	$\pi^{-1}2^{\mu+\beta}y^{\varkappa-\mu-1}\,\Gamma(2\mu+1)\times$ $\times\Gamma(1/2-\mu-\varkappa)\exp(-2^{-2}y^2)\times$ $\times\Big\{\cos(2\mu\pi)\frac{\Gamma(2\mu+\nu+1)}{\Gamma(\mu+\nu-\varkappa+3/2)}\times$ $\times M_{\alpha,\beta}(2^{-1}y^2)+$ $+\sin[(\mu-\varkappa)\pi]\, W_{\alpha,\beta}(2^{-1}y^2)\Big\}$, $2\alpha=3\mu+\nu+\varkappa+1/2$ $2\beta=\mu+\nu-\varkappa+1/2$
(44)	$x^{2\mu-\nu-1/2}\exp(-2^{-2}x^2)\times$ $\times M_{\varkappa,\mu}(2^{-1}x^2)$, $-1<2\operatorname{Re}\mu<\operatorname{Re}(2\varkappa+\nu)+1/2$ $\operatorname{Re}(2\mu-\nu)>-1$	$\pi^{-1}2^{\mu+\beta}y^{\varkappa-\mu-1}\exp(-2^{-2}y^2)\times$ $\times\Gamma(2\mu+1)\,\Gamma(1/2-\varkappa-\mu)\times$ $\times\Big\{\cos[(\nu-2\mu)\pi]\frac{\Gamma(2\mu-\nu-1)}{\Gamma(2\beta+1)}\times$ $\times M_{\alpha,\beta}(2^{-1}y^2)-$ $-\sin[(\nu+\varkappa-\mu)\pi]\, W_{\alpha,\beta}(2^{-1}y^2)\Big\}$, $2\alpha=3\mu-\nu+\varkappa+1/2$ $2\beta=\mu-\nu-\varkappa+1/2$
(45)	$x^{2\lambda}\exp(-2^{-2}x^2)\, M_{\varkappa,\mu}(2^{-1}x^2)$, $\operatorname{Re}(\varkappa-\lambda)>0$ $\operatorname{Re}(2\lambda+2\mu\pm\nu)>-5/2$	$\frac{2^\lambda\,\Gamma(2\mu+1)}{\Gamma(1/2+\varkappa+\mu)}\times$ $\times G^{31}_{34}\left(\frac{y^2}{2}\,\middle\vert\,\begin{matrix}-\mu-\lambda,\ \mu-\lambda,\ l\\ h,\ k,\ \varkappa-\lambda-1/2,\ l\end{matrix}\right)$, $h=1/4+\nu/2,\ k=1/4-\nu/2$ $l=-1/4-\nu/2$

	$f(x)$	$\int\limits_0^\infty f(x)\,Y_\nu(xy)\,(xy)^{1/2}\,dx,\quad y>0$
(46)	$x^{2\lambda}\exp(-2^{-2}x^2)\,W_{\varkappa,\mu}(2^{-1}x^2)$, $\operatorname{Re}(2\lambda\pm 2\mu\pm\nu)>-5/2$	$(-1)^m\,2^\lambda\times$ $\times G_{34}^{22}\left(\frac{y^2}{2}\,\middle\vert\,\begin{matrix}-\mu-\lambda,\ \mu-\lambda,\ l\\ h,\ k,\ \varkappa-\lambda-1/2,\ l\end{matrix}\right)$, $h=1/4+\nu/2,\ k=1/4-\nu/2$ $l=-1/4-\nu/2$
(47)	$x^{2\lambda}\exp(2^{-2}x^2)\,W_{\varkappa,\mu}(2^{-1}x^2)$, $\operatorname{Re}(\varkappa+\lambda)<0$ $\operatorname{Re}(2\lambda\pm 2\mu\pm\nu)>-5/2$	$2^\lambda\,[\Gamma(1/2-\varkappa+\mu)\,\Gamma(1/2-\varkappa-\mu)]^{-1}\times$ $\times G_{34}^{32}\left(\frac{y^2}{2}\,\middle\vert\,\begin{matrix}-\mu-\lambda,\ \mu-\lambda,\ l\\ h,\ k,\ -1/2-\varkappa-\lambda,\ l\end{matrix}\right)$, $h=1/4+\nu/2,\ k=1/4-\nu/2$ $l=-1/4-\nu/2$
(48)	$x^{1/2}\,W_{\nu/2,\,\mu}(2/x)\,W_{-\nu/2,\,\mu}(2/x)$, $-1/4<\operatorname{Re}\mu<1/4$	$4y^{-1/2}\,K_{2\mu}(2y^{1/2})\times$ $\times\{\cos[(\mu-\nu/2)\pi]\,J_{2\mu}(2y^{1/2})-$ $-\sin[(\mu-\nu/2)\pi]\,Y_{2\mu}(2y^{1/2})\}$
(49)	$x^{\nu+3/2}\times$ $\times{}_2F_1\left(\frac{1}{2},\ \frac{1}{2}-\nu;\ \frac{3}{2};\ -\alpha^2x^2\right)$, $\operatorname{Re}\alpha>0$ $-3/2<\operatorname{Re}\nu<-1/2$	$\frac{2^\nu y^{-\nu-1/2}}{\pi^{1/2}\alpha^2\,\Gamma(1/2-\nu)}\,K_\nu\left(\frac{y}{2\alpha}\right)K_{\nu+1}\left(\frac{y}{2\alpha}\right)$
(50)	$x^{\nu+3/2}\times$ $\times{}_2F_1(1,\ 2\nu+3/2;\ \nu+2;\ -\alpha^2x^2)$, $\operatorname{Re}\alpha>0,\ -1/2<\operatorname{Re}\nu<1/2$	$\pi^{-1/2}2^{-\nu}\alpha^{-2\nu-3}\,\frac{\Gamma(\nu+2)}{\Gamma(2\nu+3/2)}\times$ $\times y^{\nu+1/2}\left[K_\nu\left(\frac{y}{2\alpha}\right)\right]^2$
(51)	$x^{\nu+3/2}\times$ $\times{}_2F_1(1,\ \mu+\nu+3/2;\ 3/2;\ -\alpha^2x^2)$, $\operatorname{Re}\alpha>0,\ -3/2<\operatorname{Re}\nu<1/2$ $\operatorname{Re}(2\mu+\nu)>-3/2$	$\frac{\pi^{1/2}2^{-\mu-\nu-1}\alpha^{-\mu-2\nu-3}}{\Gamma(\mu+\nu+3/2)}\times$ $\times y^{\mu+\nu+1/2}\,K_\mu\left(\frac{y}{\alpha}\right)$
(52)	$x^\sigma\,{}_2F_1(\alpha,\ \beta;\ \gamma;\ -\lambda^2x^2)$, $\operatorname{Re}\lambda>0,\quad \operatorname{Re}\sigma>\lvert\operatorname{Re}\nu\rvert-3/2$ $\operatorname{Re}\sigma<2\operatorname{Re}\alpha,\quad \operatorname{Re}\sigma<2\operatorname{Re}\beta$	$\frac{\lambda^{-\sigma-1}\,\Gamma(\gamma)}{2^{1/2}\,\Gamma(\alpha)\,\Gamma(\beta)}\times$ $\times G_{35}^{41}\left(\frac{y^2}{4\lambda^2}\,\middle\vert\,\begin{matrix}1-p,\ \gamma-p,\ l\\ h,\ k,\ \alpha-p,\ \beta-p,\ l\end{matrix}\right)$ $h=1/4+\nu/2,\quad k=1/4-\nu/2$ $l=-1/4-\nu/2,\ p=1/2+\sigma/2$

	$f(x)$	$\int\limits_0^\infty f(x)\, Y_\nu(xy)\,(xy)^{1/2}\,dx, \quad y>0$
(53)	$x^{\sigma-3/2}\,{}_pF_{p-1}(\alpha_1, \ldots, \alpha_p;$ $\beta_1, \ldots, \beta_{p-1}; -\lambda x^2)$, $\lvert\arg\lambda\rvert<\pi$, $\operatorname{Re}\sigma>\lvert\operatorname{Re}\nu\rvert$ $\operatorname{Re}\alpha_j>2^{-1}\operatorname{Re}\sigma-3/4$	$\frac{y^{1/2}\,\Gamma(\beta_1)\ldots\Gamma(\beta_{p-1})}{2\lambda^{\sigma/2}\,\Gamma(\alpha_1)\ldots\Gamma(\alpha_p)}\times$ $\times\, G^{p+2,\,1}_{p+2,\,p+3}\left(\frac{y^2}{4\lambda}\,\middle\vert\,\begin{matrix}\beta_0^*, \ldots, \beta_{p-1}^*, l\\ h, k, \alpha_1^*, \ldots, \alpha_p^*, l\end{matrix}\right)$, $\alpha_j^*=\alpha_j-\frac{\sigma}{2},\ j=1, \ldots, p$ $\beta_0^*=1-\frac{\sigma}{2}$, $\beta_j^*=\beta_j-\frac{\sigma}{2},\ j=1, \ldots, p-1$ $h=\frac{\nu}{2},\ k=-\frac{\nu}{2},\ l=-\frac{1+\nu}{2}$
(54)	$x^{\sigma-3/2}\times$ $\times\,{}_pF_p(\alpha_1, \ldots, \alpha_p; \beta_1, \ldots, \beta_p; -\lambda x^2)$, $\operatorname{Re}\lambda>0$, $\operatorname{Re}\sigma>\lvert\operatorname{Re}\nu\rvert$ $\operatorname{Re}\alpha_j>2^{-1}\operatorname{Re}\sigma-3/4$ $j=1, \ldots, p$	$\frac{y^{1/2}}{2\lambda^{\sigma/2}}\,\frac{\Gamma(\beta_1)\ldots\Gamma(\beta_p)}{\Gamma(\alpha_1)\ldots\Gamma(\alpha_p)}\times$ $\times\, G^{p+2,\,1}_{p+2,\,p+3}\left(\frac{y^2}{4\lambda}\,\middle\vert\,\begin{matrix}\beta_0^*, \ldots, \beta_p^*, l\\ h, k, \alpha_1^*, \ldots, \alpha_p^*, l\end{matrix}\right)$, $\beta_0^*=1-\frac{\sigma}{2},\ \alpha_j^*=\alpha_j-\frac{\sigma}{2}$, $\beta_j^*=\beta_j-\frac{\sigma}{2},\ j=1, \ldots, p$ $h=\frac{\nu}{2},\ k=-\frac{\nu}{2},\ l=-\frac{1+\nu}{2}$
(55)	$x^{\sigma-3/2}\times$ $\times\,{}_pF_q(\alpha_1, \ldots, \alpha_p; \beta_1, \ldots, \beta_q; -\lambda x^2)$ $p\leqslant q-1$, $\operatorname{Re}\sigma>\lvert\operatorname{Re}\nu\rvert$	$-\pi^{-1}2^{\sigma-1}y^{1/2-\sigma}\cos[2^{-1}\pi(\sigma-\nu)]\times$ $\times\,\Gamma\left(\frac{\sigma+\nu}{2}\right)\Gamma\left(\frac{\sigma-\nu}{2}\right)\times$ $\times\,{}_{p+2}F_q\left(\alpha_1, \ldots, \alpha_p, \frac{\sigma+\nu}{2}, \frac{\sigma-\nu}{2};\right.$ $\left.\beta_1, \ldots, \beta_q; -\frac{4\lambda}{y^2}\right)$
(56)	$G^{mn}_{pq}\left(\lambda x^2\,\middle\vert\,\begin{matrix}\alpha_1, \ldots, \alpha_p\\ \beta_1, \ldots, \beta_q\end{matrix}\right)$, $p+q<2(m+n)$ $\lvert\arg\lambda\rvert<(m+n-p/2-q/2)\pi$ $\operatorname{Re}\alpha_j<1,\ j=1, \ldots, n$ $\operatorname{Re}(\beta_j\pm\nu/2)>-3/4$ $j=1, \ldots, m$	$(2\lambda)^{-1/2}\times$ $\times\, G^{n+2,\,m}_{q+1,\,p+3}\left(\frac{y^2}{4\lambda}\,\middle\vert\,\begin{matrix}\beta_1', \ldots, \beta_q', l\\ h, k, \alpha_1', \ldots, \alpha_p', l\end{matrix}\right)$, $h=1/4+\nu/2,\ k=1/4-\nu/2$ $l=-1/4-\nu/2$ $\beta_j'=\frac{1}{2}-\beta_j,\quad \alpha_j'=\frac{1}{2}-\alpha_j$

ГЛАВА X

К-ПРЕОБРАЗОВАНИЯ

Мы называем *К-преобразованием порядка* ν функции $f(x)$ функцию

$$\mathfrak{K}_\nu \{f(x);\ y\} = \int_0^\infty f(x)\, K_\nu(xy)\,(xy)^{1/2}\, dx$$

комплексного переменного y. Это преобразование было введено Мейером (C. S. Meijer, 1940), который дал формулу обращения 10.1 (1) и теоремы представления. Преобразование было далее изучено Боасом (Boas, 1942a, 1942b) и Эрдейи (Erdélyi, 1950—51).

В силу соотношений между функциями Бесселя первого и второго рода, с одной стороны, и модифицированными функциями Бесселя K_ν, с другой, *К*-преобразование может быть выражено как линейная комбинация любых двух из преобразований $\mathfrak{H}_\nu$, $\mathfrak{H}_{-\nu}$, $\mathfrak{Y}_\nu$, $\mathfrak{Y}_{-\nu}$. Однако переменное y в преобразованиях Ганкеля и *Y*-преобразованиях, встречающихся в этих выражениях, отрицательно, а лишь немногие преобразования Ганкеля и *Y*-преобразования сходятся для отрицательных (или комплексных) значений y. Обратно,

$$\pi\,\mathfrak{H}_\nu \{f(x);\ y\} =$$
$$= \exp[(\nu + 1/2)\pi i/2]\,\mathfrak{K}_\nu\{f(x);\ iy\} + \exp[-(\nu + 1/2)\pi i/2]\,\mathfrak{K}_\nu\{f(x);\ -iy\},$$

$$\pi\,\mathfrak{Y}_\nu \{f(x);\ y\} =$$
$$= -\exp[(\nu + 1/2)\pi i/2]\,\mathfrak{K}_\nu\{f(x);\ iy\} - \exp[-(\nu - 1/2)\pi i/2]\,\mathfrak{K}_\nu\{f(x);\ -iy\}.$$

С помощью этих соотношений можно вычислять преобразования Ганкеля и *Y*-преобразования, используя таблицы *К*-преобразований. Однако во многих случаях приходится рассматривать *К*-преобразования на границе полуплоскости сходимости и, чтобы обеспечить сходимость, приходится накладывать дополнительные ограничения на параметры. Если $\nu = \pm 1/2$, то *К*-преобразование сводится к преобразованию Лапласа

$$\mathfrak{K}_{\pm 1/2}\{f(x);\ y\} = (\pi/2)^{1/2}\,\mathfrak{L}\{f(x);\ y\}$$

и указанные выше соотношения сводятся к выражениям синус- и косинус-преобразований Фурье через преобразования Лапласа.

Из приведенных в этой главе пар преобразований можно получать новые пары с помощью методов, указанных во введении к первому тому, а также с помощью общих формул из п. 10.1. Связь с преобразованием Лапласа в любой из двух форм

$$\mathfrak{K}_\nu\{f(x);\ y\} = \frac{\pi^{1/2} 2^{-\nu} y^{\nu+1/2}}{\Gamma(\nu+1/2)} \mathfrak{L}\left\{\int_0^x (x^2-t^2)^{\nu-1/2} t^{1/2-\nu} f(t)\,dt;\ y\right\}, \quad \operatorname{Re}\nu > -1/2,$$

$$\mathfrak{K}_\nu\{f(x);\ y\} = \frac{\pi^{1/2} 2^{-\nu} y^{1/2-\nu}}{\Gamma(\nu+1/2)} \int_y^\infty (t^2-y^2)^{\nu-1/2} \mathfrak{L}\{x^{1/2+\nu} f(x);\ t\}\,dt, \quad \operatorname{Re}\nu > -1/2.$$

может быть использована для вычисления K-преобразований с помощью таблиц преобразований Лапласа, данных в главе IV.

10.1. Общие формулы

	$f(x)$	$\int_0^\infty f(x) K_\nu(xy)(xy)^{1/2}\,dx = g(y;\ \nu)$
(1)	$\frac{1}{\pi i}\int_{c-i\infty}^{c+i\infty} g(y) I_\nu(xy)(xy)^{1/2}\,dy$	$g(y)$
(2)	$f(ax),\quad a>0$	$a^{-1} g(y/a;\ \nu)$
(3)	$x^m f(x),\quad m=0, 1, 2, \ldots$	$y^{1/2-\nu} \times \left(-\frac{d}{y\,dy}\right)^m y^{m+\nu-1/2} g(y;\ \nu+m)]$
(4)	$x^m f(x),\quad m=0, 1, 2, \ldots$	$y^{1/2+\nu} \times \left(-\frac{d}{y\,dy}\right)^m [y^{m-\nu-1/2} g(y;\ \nu-m)]$
(5)	$2\nu x^{-1} f(x)$	$y\,g(y;\ \nu+1) - y\,g(y;\ \nu-1)$
(6)	$x^{-1} f(x)$	$y^{\nu+1/2}\int_y^\infty \eta^{-\nu-1/2} g(\eta;\ \nu+1)\,d\eta$
(7)	$x^{-\mu} f(x),\quad \operatorname{Re}\mu>0$	$2^{1-\mu}[\Gamma(\mu)]^{-1} y^{\nu+1/2} \times \int_y^\infty \eta^{1/2-\mu-\nu}(\eta^2-y^2)^{\mu-1} \times g(\eta;\ \nu+\mu)\,d\eta$
(8)	$2\nu f'(x)$	$(\nu-1/2)\,y\,g(y;\ \nu+1) + (\nu+1/2)\,y\,g(y;\ \nu-1)$

	$f(x)$	$\int\limits_0^\infty f(x)\,K_\nu(xy)\,(xy)^{1/2}\,dx = g(y;\nu)$
(9)	$x^{1/2-\nu}\left(\frac{d}{x\,dx}\right)^m [x^{m+\nu-1/2} f(x)]$, $m=0,1,2,\ldots$	$y^m g(y;\nu+m)$
(10)	$x^{1/2+\nu}\left(\frac{d}{x\,dx}\right)^m [x^{m-\nu-1/2} f(x)]$, $m=0,1,2,\ldots$	$y^m g(y;\nu-m)$
(11)	$x^{1/2-\nu}\int\limits_0^x \xi^{\nu-\mu+1/2}(x^2-\xi^2)^{\mu-1}\times$ $\times f(\xi)\,d\xi$, $\operatorname{Re}\mu>0$	$2^{\mu-1}\Gamma(\mu)\,y^{-\mu} g(y;\nu-\mu)$

10.2. Элементарные функции

(1)	$x^{\rho-1}$, $\operatorname{Re}\rho > \lvert\operatorname{Re}\nu\rvert - 1/2$	$2^{\rho-3/2} y^{-\rho}\,\Gamma\left(\frac{\rho}{2}+\frac{\nu}{2}+\frac{1}{4}\right)\times$ $\times\Gamma\left(\frac{\rho}{2}-\frac{\nu}{2}+\frac{1}{4}\right)$, $\operatorname{Re} y>0$
(2)	0, $0<x<a$ $x^{\nu+1/2}$, $a<x<\infty$	$a^{\nu+1} y^{-1/2} K_{\nu+1}(ay)$, $\operatorname{Re} y>0$
(3)	0, $0<x<a$ $x^{\sigma+1/2}$, $a<x<\infty$	$ay^{-1/2-\sigma}\exp(-\pi\sigma i/2)\times$ $\times[K_{\nu-1}(ay)\,S_{\sigma+1,\,\nu}(iay)+$ $+i(\nu+\sigma)\,K_\nu(ay)\,S_{\sigma,\,\nu-1}(iay)]$, $\operatorname{Re} y>0$
(4)	$x^{\mu-1/2}(x+\alpha)^{-1}$, $\lvert\arg\alpha\rvert<\pi$ $\operatorname{Re}\mu>\lvert\operatorname{Re}\nu\rvert-1$	$2^{\mu-2}\Gamma\left(\frac{\mu}{2}+\frac{\nu}{2}\right)\Gamma\left(\frac{\mu}{2}-\frac{\nu}{2}\right)y^{1/2-\mu}\times$ $\times\,{}_1F_2\left(1;\,1-\frac{\mu}{2}-\frac{\nu}{2},\right.$ $\left.1-\frac{\mu}{\nu}+\frac{\nu}{2};\,\frac{\alpha^2y^2}{4}\right)-$ $-2^{\mu-3}\Gamma\left(\frac{\mu}{2}-\frac{\nu}{2}-\frac{1}{2}\right)\times$ $\times\Gamma\left(\frac{\mu}{2}+\frac{\nu}{2}-\frac{1}{2}\right)\alpha y^{3/2-\mu}\times$ $\times\,{}_1F_2\left(1;\,\frac{3}{2}-\frac{\mu}{2}-\frac{\nu}{2},\right.$ $\left.\frac{3}{2}-\frac{\mu}{2}+\frac{\nu}{2};\,\frac{\alpha^2y^2}{4}\right)-\frac{\pi\alpha^\mu y^{1/2}}{\sin[\pi(\mu-\nu)]}\times$ $\times\left\{K_\nu(\alpha y)+\frac{\pi\cos(\mu\pi)\,I_\nu(\alpha y)}{\sin[\pi(\nu+\mu)]}\right\}$, $\operatorname{Re} y>0$

	$f(x)$	$\int\limits_0^\infty f(x)\,K_\nu(xy)\,(xy)^{1/2}\,dx$
(5)	$x^{-1/2}(x+\alpha)^{-1}$, $\lvert\arg\alpha\rvert<\pi,\quad -1<\operatorname{Re}\nu<1$	$\frac{\pi^2 y^{1/2}}{2[\sin(\nu\pi)]^2}\times$ $\times[I_\nu(\alpha y)+I_{-\nu}(\alpha y)-$ $-\exp(-i\nu\pi/2)\,\mathbf{J}_\nu(i\alpha y)-$ $-\exp(i\nu\pi/2)\,\mathbf{J}_{-\nu}(i\alpha y)]$, $\operatorname{Re}y>0$
(6)	$x^{-1/2}(\alpha^2+x^2)^{-1/2}$, $\operatorname{Re}\alpha>0,\quad -1<\operatorname{Re}\nu<1$	$\frac{\pi^2 y^{1/2}}{8\cos(\nu\pi/2)}\times$ $\times\{[J_{\nu/2}(\alpha y/2)]^2+[Y_{\nu/2}(\alpha y/2)]^2\}$, $\operatorname{Re}y>0$
(7)	$x^{-1/2-\nu}(x^2+\alpha^2)^{-1}$, $\operatorname{Re}\alpha>0,\ \operatorname{Re}\nu<1/2$	$\frac{\pi^2 y^{1/2}[\mathbf{H}_\nu(\alpha y)-Y_\nu(\alpha y)]}{4\alpha^{\nu+1}\cos(\nu\pi)}$, $\operatorname{Re}y>0$
(8)	$x^{1/2+\nu}(x^2+\alpha^2)^{\mu}$, $\operatorname{Re}\alpha>0,\ \operatorname{Re}\nu>-1$	$2^\nu\Gamma(\nu+1)\,\alpha^{\nu+\mu+1}y^{-1/2-\mu}\times$ $\times S_{\mu-\nu,\,\mu+\nu+1}(\alpha y),\quad \operatorname{Re}y>0$
(9)	$x^{\rho-3/2}(x^2+\alpha^2)^{\mu}$, $\operatorname{Re}\alpha>0,\ \operatorname{Re}\rho>\lvert\operatorname{Re}\nu\rvert$	$\frac{\alpha^{\rho+2\mu}y^{1/2}}{4\Gamma(-\mu)}[f(\nu)+f(-\nu)]+$ $+2^{2\mu+\rho-2}\,\Gamma\left(\frac{\rho}{2}+\mu-\frac{\nu}{2}\right)\times$ $\times\Gamma\left(\frac{\rho}{2}+\mu+\frac{\nu}{2}\right)y^{1/2-\rho-2\mu}\times$ $\times{}_1F_2\Bigl(-\mu;\ 1-\mu-\frac{\rho}{2}-\frac{\nu}{2},$ $1-\mu-\frac{\rho}{2}+\frac{\nu}{2};\ -\frac{\alpha^2y^2}{4}\Bigr)$, $f(\nu)=(\alpha/2)^\nu\,\Gamma(-\nu)\,\Gamma\left(\frac{\rho}{2}+\frac{\nu}{2}\right)\times$ $\times\Gamma\left(-\frac{\nu}{2}-\frac{\rho}{2}-\mu\right)y^\nu\times$ $\times{}_1F_2\Bigl(\frac{\rho}{2}+\frac{\nu}{2};\ \frac{\rho}{2}+\mu+1+\frac{\nu}{2},$ $1+\nu;\ -\frac{\alpha^2y^2}{4}\Bigr),\quad \operatorname{Re}y>0$
(10)	$[x(a^2-x^2)]^{\nu-1/2}$, $\quad 0<x<a$ 0, $\quad a<x<\infty$ $\operatorname{Re}\nu>-1/2$	$\pi^{1/2}2^{\nu-1}a^{2\nu}y^{1/2-\nu}\,\Gamma(\nu+1/2)\times$ $\times I_\nu(ay/2)\,K_\nu(ay/2)$
(11)	0, $\quad 0<x<a$ $[x(x^2-a^2)]^{\nu-1/2}$, $\quad a<x<\infty$ $\operatorname{Re}\nu>-1/2$	$\pi^{-1/2}2^{\nu-1}a^{2\nu}y^{1/2-\nu}\,\Gamma(\nu+1/2)\times$ $\times[K_\nu(ay/2)]^2,\quad \operatorname{Re}y>0$

	$f(x)$	$\int\limits_0^\infty f(x)\,K_\nu(xy)\,(xy)^{1/2}\,dx$
(12)	$x^{1/2-\nu}(a^2-x^2)^\mu,\quad 0<x<a$ $0,\quad a<x<\infty$ $\operatorname{Re}\mu>-1,\ \operatorname{Re}\nu<1$	$2^{-\nu-2}a^{2\mu+2}y^{\nu+1/2}(\mu+1)^{-1}\times$ $\times\Gamma(-\nu)\,{}_1F_2\Big(1;\ \nu+1,\ \mu+2;$ $2^{-2}a^2y^2\Big)+\dfrac{\pi 2^{\mu-1}a^{\mu-\nu+1}y^{-\mu-1/2}}{\sin(\nu\pi)}\times$ $\times\Gamma(\mu+1)\,I_{\mu-\nu+1}(ay)$
(13)	$0,\quad 0<x<a$ $x^{1/2-\nu}(x^2-a^2)^\mu,\quad a<x<\infty$ $\operatorname{Re}\mu>-1$	$2^\mu a^{\mu-\nu+1}y^{-\mu-1/2}\,\Gamma(\mu+1)\times$ $\times K_{\mu-\nu+1}(ay),\quad \operatorname{Re}y>0$
(14)	$x^{-1/2}(x^2+\alpha^2)^{-1/2}\times$ $\times\left[(x^2+\alpha^2)^{1/2}+x\right]^{-2\mu},$ $\operatorname{Re}\alpha>0,\ \nu=0$	$-2^{-2}\pi\alpha^{-2\mu}y^{1/2}\times$ $\times\Big\{J_\mu(2^{-1}\alpha y)\dfrac{\partial}{\partial\mu}\left[Y_\mu(2^{-1}\alpha y)\right]-$ $-Y_\mu(2^{-1}\alpha y)\dfrac{\partial}{\partial\mu}\left[J_\mu(2^{-1}\alpha y)\right]\Big\},$ $\operatorname{Re}y>0$
(15)	$x^{-1/2}(x^2+\alpha^2)^{-1/2}\times$ $\times\left[(x^2+\alpha^2)^{1/2}+x\right]^{-2\mu},$ $\operatorname{Re}\alpha>0$	$\dfrac{\pi^2y^{1/2}}{4\alpha^{2\mu}\sin(\nu\pi)}\times$ $\times\Big[J_{\mu+\nu/2}(2^{-1}\alpha y)\,Y_{\mu-\nu/2}(2^{-1}\alpha y)-$ $-Y_{\mu+\nu/2}(2^{-1}\alpha y)\,J_{\mu-\nu/2}(2^{-1}\alpha y)\Big],$ $\operatorname{Re}y>0$
(16)	$x^{-1/2}(x^2+\alpha^2)^{-1/2}\times$ $\times\Big\{\left[(x^2+\alpha^2)^{1/2}+x\right]^{2\mu}+$ $+\left[(x^2+\alpha^2)^{1/2}-x\right]^{2\mu}\Big\},$ $\operatorname{Re}\alpha>0,\ \nu=0$	$2^{-2}\pi^2\alpha^{2\mu}y^{1/2}\{[J_\mu(2^{-1}\alpha y)]^2+$ $+[Y_\mu(2^{-1}\alpha y)]^2\},\quad \operatorname{Re}y>0$
(17)	$x^{-1/2}(x^2+\alpha^2)^{-1/2}\times$ $\times\Big\{\left[(x^2+\alpha^2)^{1/2}+x\right]^{2\mu}\times$ $\times\cos[(\nu/2-\mu)\pi]+$ $+\left[(x^2+\alpha^2)^{1/2}-x\right]^{2\mu}\times$ $\times\cos[(\nu/2+\mu)\pi]\Big\},\quad \operatorname{Re}\alpha>0$	$2^{-2}\pi^2\alpha^{2\mu}y^{1/2}\Big[J_{\nu/2+\mu}(2^{-1}\alpha y)\times$ $\times J_{\nu/2-\mu}(2^{-1}\alpha y)+$ $\times Y_{\nu/2+\mu}(2^{-1}\alpha y)\,Y_{\nu/2-\mu}(2^{-1}\alpha y)\Big],$ $\operatorname{Re}y>0$
(18)	$x^{-1/2-2\mu}(x^2+\alpha^2)^{-1/2}\times$ $\times\left[(x^2+\alpha^2)^{1/2}+\alpha\right]^{2\mu},\quad \operatorname{Re}\alpha>0$ $2\operatorname{Re}\mu+\lvert\operatorname{Re}\nu\rvert<1$	$2^{-1}\alpha^{-1}y^{-1/2}\,\Gamma\left(\dfrac{1+\nu}{2}-\mu\right)\times$ $\times\Gamma\left(\dfrac{1-\nu}{2}-\mu\right)W_{\mu,\nu/2}(i\alpha y)\times$ $\times W_{\mu,\nu/2}(-i\alpha y),$ $\operatorname{Re}y>0$

	$f(x)$	$\int\limits_0^\infty f(x)\, K_\nu(xy)\,(xy)^{1/2}\,dx$
(19)	$0, \quad 0 < x < a$ $x^{-1/2}(x^2-a^2)^{-1/2}\times$ $\times\left\{\left[x+(x^2-a^2)^{1/2}\right]^{2\mu}+\right.$ $\left.+\left[x-(x^2-a^2)^{1/2}\right]^{2\mu}\right\},$ $a < x < \infty$	$a^{2\mu}y^{1/2}K_{\nu/2+\mu}(2^{-1}ay)\times$ $\times K_{\nu/2-\mu}(2^{-1}ay), \quad \operatorname{Re} y > 0$
(20)	$0, \quad 0 < x < a$ $x^{-1/2-2\mu}(x^2-a^2)^{-1/2}\times$ $\times\left\{\left[a+i(x^2-a^2)^{1/2}\right]^{2\mu}+\right.$ $\left.+\left[a-i(x^2-a^2)^{1/2}\right]^{2\mu}\right\},$ $a < x < \infty$	$\pi a^{-1}y^{-1/2}W_{\mu,\,\nu/2}(ay)\,W_{-\mu,\,\nu/2}(ay),$ $\operatorname{Re} y > 0$
(21)	$x^{-1/2}e^{-\alpha x}, \quad \nu = 0$	$y^{1/2}(y^2-\alpha^2)^{-1/2}\arccos(\alpha/y),$ $\operatorname{Re}(\alpha+y) > 0$ $y(y^2-\alpha^2)^{-1/2}\arccos(\alpha/y)\to\pi/2$ при $y\to\infty$
(22)	$x^{-1/2}e^{-\alpha x}, \quad -1 < \operatorname{Re}\nu < 1$	$\dfrac{\pi y^{-1/2}}{\sin(\nu\pi)}\,\dfrac{\sin(\nu\theta)}{\sin\theta}, \quad \operatorname{Re}(\alpha+y) > 0$ $\cos\theta = \alpha/y, \quad \theta\to\pi/2$ при $y\to\infty$
(23)	$x^{\mu-1}e^{-\alpha x}, \quad \operatorname{Re}\mu > \lvert\operatorname{Re}\nu\rvert - 1/2$	$\dfrac{\pi^{1/2}2^\nu y^{\nu+1/2}}{(\alpha+y)^{\mu+\nu+1/2}}\times$ $\times\dfrac{\Gamma(\mu+\nu+1/2)\,\Gamma(\mu-\nu+1/2)}{\Gamma(\mu+1)}\times$ $\times\,{}_2F_1\left(\mu+\nu+1/2,\ \nu+1/2;\ \mu+1;\right.$ $\left.\dfrac{\alpha-y}{\alpha+y}\right), \quad \operatorname{Re}(\alpha+y) > 0$
(24)	$x^{-1/2}\exp(-\alpha x^2),$ $\operatorname{Re}\alpha > 0, \quad -1 < \operatorname{Re}\nu < 1$	$\dfrac{1}{4\cos(\nu\pi/2)}\left(\dfrac{\pi y}{\alpha}\right)^{1/2}\times$ $\times\exp\left(\dfrac{y^2}{8\alpha}\right)K_{\nu/2}\left(\dfrac{y^2}{8\alpha}\right)$
(25)	$x^{-1/2-2\mu}\exp(-\alpha x^2), \quad \operatorname{Re}\alpha > 0$ $2\operatorname{Re}\mu < 1 - \lvert\operatorname{Re}\nu\rvert$	$2^{-1}\alpha^\mu y^{-1/2}\,\Gamma\left(\dfrac{1+\nu}{2}-\mu\right)\times$ $\times\Gamma\left(\dfrac{1-\nu}{2}-\mu\right)\exp\left(\dfrac{y^2}{8\alpha}\right)\times$ $\times W_{\mu,\,\nu/2}\left(\dfrac{y^2}{4\alpha}\right)$

	$f(x)$	$\int\limits_0^\infty f(x)\,K_\nu(xy)\,(xy)^{1/2}\,dx$
(26)	$x^{-1/2}(x^2+\alpha^2)^{-1/2}\times$ $\times\exp[-\beta(x^2+\alpha^2)^{1/2}]$, $\operatorname{Re}\alpha>0$, $\operatorname{Re}\beta>0$ $-1<\operatorname{Re}\nu<1$	$\dfrac{y^{1/2}}{2\cos(\nu\pi/2)}\times$ $\times K_{\nu/2}\{2^{-1}\alpha[\beta+(\beta^2-y^2)^{1/2}]\}\times$ $\times K_{\nu/2}\{2^{-1}\alpha[\beta-(\beta^2-y^2)^{1/2}]\}$, $\operatorname{Re}(y+\beta)>0$
(27)	$x^{-1}\cos(\alpha x^{1/2})$, $-1/2<\operatorname{Re}\nu<1/2$	$\dfrac{\pi}{2\cos(\nu\pi)}\Big[D_{\nu-1/2}\Big(\dfrac{\alpha}{2^{1/2}y^{1/2}}\Big)\times$ $\times D_{-\nu-1/2}\Big(-\dfrac{\alpha}{2^{1/2}y^{1/2}}\Big)+$ $+D_{\nu-1/2}\Big(-\dfrac{\alpha}{2^{1/2}y^{1/2}}\Big)\times$ $\times D_{-\nu-1/2}\Big(\dfrac{\alpha}{2^{1/2}y^{1/2}}\Big)\Big]$, $\operatorname{Re}y>0$
(28)	$x^{-1}\exp(-\alpha x^{1/2})\times$ $\times\cos(\alpha x^{1/2}+\pi/4-\nu\pi/2)$, $-1/2<\operatorname{Re}\nu<1/2$	$(\pi/2)^{1/2}\,\Gamma(1/2-\nu)\times$ $\times D_{\nu-1/2}(\alpha y^{-1/2}e^{\pi i/4})\times$ $\times D_{\nu-1/2}(\alpha y^{-1/2}e^{-\pi i/4})$, $\operatorname{Re}y>0$
(29)	0, $0<x<a$ $x^{1/2-\nu}\sin[\beta(x^2-a^2)^{1/2}]$, $a<x<\infty$	$(\pi/2)^{1/2}a^{3/2-\nu}\times$ $\times\beta y^{1/2-\nu}(y^2+\beta^2)^{\nu/2-3/4}\times$ $\times K_{\nu-3/2}[a(y^2+\beta^2)^{1/2}]$, $\operatorname{Re}y>\lvert\operatorname{Im}\beta\rvert$
(30)	$x^{-1/2}(a^2-x^2)^{-1/2}\times$ $\times\cos[\beta(a^2-x^2)^{-1/2}]$, $0<x<a$ 0, $a<x<\infty$ $-1<\operatorname{Re}\nu<1$	$-\dfrac{\pi^2y^{1/2}}{4\sin(\nu\pi)}[J_{\nu/2}(u)\,J_{\nu/2}(v)-$ $-J_{-\nu/2}(u)\,J_{-\nu/2}(v)]$, $u=2^{-1}a[\beta+(\beta^2-y^2)^{1/2}]$, $v=2^{-1}a[\beta-(\beta^2-y^2)^{1/2}]$
(31)	0, $0<x<a$ $x^{1/2-\nu}(x^2-a^2)^{-1/2}\times$ $\times\cos[\beta(x^2-a^2)^{1/2}]$, $a<x<\infty$	$(\pi/2)^{1/2}a^{1/2-\nu}y^{1/2-\nu}(y^2+\beta^2)^{\nu/2-1/4}\times$ $\times K_{\nu-1/2}[a(y^2+\beta^2)^{1/2}]$, $\operatorname{Re}y>\lvert\operatorname{Im}\beta\rvert$
(32)	$x^{-1/2}\operatorname{sh}(\alpha x)$, $-2<\operatorname{Re}\nu<2$	$\dfrac{\pi y^{1/2}\sin[\nu\arcsin(\alpha/y)]}{2(y^2-\alpha^2)^{1/2}\sin(\nu\pi/2)}$, $\operatorname{Re}y>\lvert\operatorname{Re}\alpha\rvert$

	$f(x)$	$\int\limits_0^\infty f(x) K_\nu(xy)(xy)^{1/2}\,dx$
(33)	$x^{-1/2}\operatorname{ch}(\alpha x)$, $\quad -1 < \operatorname{Re}\nu < 1$	$\dfrac{\pi y^{1/2}\cos[\nu\arcsin(\alpha/y)]}{2(y^2-\alpha^2)^{1/2}\cos(\nu\pi/2)}$, $\quad \operatorname{Re} y > \lvert\operatorname{Re}\alpha\rvert$
(34)	$x^{-3/2}\operatorname{sh}(\alpha x)$, $\quad -1 < \operatorname{Re}\nu < 1$	$\dfrac{\pi\sin[\nu\arcsin(\alpha/y)]}{2\nu y^{1/2}\cos(\nu\pi/2)}$, $\quad \operatorname{Re} y > \lvert\operatorname{Re}\alpha\rvert$
(35)	$x^{-1/2}(a^2-x^2)^{-1/2}\operatorname{ch}[\beta(a^2-x^2)^{1/2}]$, $\quad 0<x<a$ 0, $\quad a<x<\infty$ $-1 < \operatorname{Re}\nu < 1$	$\dfrac{\pi^2 y^{1/2}}{4\sin(\nu\pi/2)}[I_{-\nu/2}(u)I_{-\nu/2}(v) - I_{\nu/2}(u)I_{\nu/2}(v)]$, $u = 2^{-1}a[(\beta^2+y^2)^{1/2}+\beta]$, $v = 2^{-1}a[(\beta^2+y^2)^{1/2}-\beta]$ $\operatorname{Re} y > \lvert\operatorname{Im}\beta\rvert$

10.3. Высшие трансцендентные функции

(1)	$x^{1/2}P_n(1-2x^2)$, $\quad 0<x<1$ 0, $\quad 1<x<\infty$ $\nu=0,\ n=0, 1, 2, \ldots$	$y^{-1/2}[(-1)^{n+1}K_{2n+1}(y) + 2^{-1}iS_{2n+1}(iy)]$
(2)	0, $\quad 0<x<a$ $x^{-1/2}(x^2-a^2)^{-1/2}T_n(a/x)$, $\quad a<x<\infty$ $n=0, 1, 2, \ldots$	$2^{-1}\pi a^{-1}y^{-1/2}W_{n/2,\,\nu/2}(ay)\times W_{-n/2,\,\nu/2}(ay)$, $\quad \operatorname{Re} y>0$
(3)	0, $\quad 0<x<a$ $x^{\mu}(x^2-a^2)^{-\mu/2}P^{\mu}_{\nu-1/2}(x/a)$, $\quad a<x<\infty$ $\operatorname{Re}\mu<1$	$(\pi/2)^{1/2}y^{-1}\exp(-ay/2)W_{\mu,\nu}(ay)$, $\quad \operatorname{Re} y>0$
(4)	0, $\quad 0<x<a$ $x^{\mu-2}(x^2-a^2)^{-\mu/2}P^{\mu}_{\nu-1/2}(x/a)$, $\quad a<x<\infty$ $\operatorname{Re}\mu<1$	$(\pi/2)^{1/2}a^{-1}\exp(-ay/2)W_{\mu-1,\,\nu}(ay)$ $\quad \operatorname{Re} y>0$
(5)	0, $\quad 0<x<a$ $x^{-\mu}(x^2-a^2)^{-\mu/2}P^{\mu}_{\nu-1/2}(x/a)$, $\quad a<x<\infty$ $\operatorname{Re}\mu<1$	$(2\pi)^{-1/2}a^{1-\mu}y^{\mu}K_\nu(2^{-1}ay)\times K_{\mu-1/2}(2^{-1}ay)$, $\quad \operatorname{Re} y>0$

	$f(x)$	$\int_0^\infty f(x) K_\nu(xy)(xy)^{1/2}\,dx$
(6)	$0, \quad 0<x<a$ $x^{\mu-1}(x^2-a^2)^{-\mu/2} P^{\mu}_{\nu-3/2}(x/a), \quad a<x<\infty$ $\operatorname{Re}\mu<1$	$\left(\frac{\pi}{2ay}\right)^{1/2}\exp(-ay/2)\,W_{\mu-1/2,\,\nu-1/2}(y)$
(7)	$x^{1/2}(x^2+\alpha^2)^{\nu/2} P^{\nu}_{\mu}(1+2x^2\alpha^{-2})$, $\operatorname{Re}\alpha>0, \ \operatorname{Re}\nu<1$	$2^{-\nu}\alpha y^{-\nu-1/2} S_{2\nu,\,2\mu+1}(\alpha y)$, $\operatorname{Re}y>0$
(8)	$x^{1/2}(x^2+\alpha^2)^{\nu/2}\times$ $\times\left[(\mu-\nu)P^{\nu}_{\mu}(1+2x^2\alpha^{-2})+\right.$ $\left.+(\mu+\nu)P^{\nu}_{-\mu}(1+2x^2\alpha^{-2})\right]$, $\operatorname{Re}\alpha>0, \ \operatorname{Re}\nu<1$	$2^{1-\nu}\mu y^{-\nu-3/2} S_{2\nu+1,\,2\mu}(\alpha y)$, $\operatorname{Re}y>0$
(9)	$x^{1/2}(x^2+\alpha^2)^{\nu/2-1}\times$ $\times\left[P^{\nu}_{\mu}(1+2x^2\alpha^{-2})+\right.$ $\left.+P^{\nu}_{-\mu}(1+2x^2\alpha^{-2})\right]$, $\operatorname{Re}\alpha>0, \ \operatorname{Re}\nu<1$	$2^{1-\nu}y^{1/2-\nu} S_{2\nu-1,\,2\mu}(\alpha y)$, $\operatorname{Re}y>0$
(10)	$0, \quad 0<x<a$ $x^{1/2}(x^2-a^2)^{-\nu/2} P^{\nu}_{\mu}(2x^2a^{-2}-1)$, $a<x<\infty$ $\operatorname{Re}\nu<1$	$2^{-\nu}ay^{\nu-1/2} K_{\mu+1}(ay), \quad \operatorname{Re}y>0$
(11)	$0, \quad 0<x<a$ $(x^2-a^2)^{\nu/2-1/4} P^{1/2-\nu}_{\mu}(2x^2a^{-2}-1)$, $a<x<\infty$ $\operatorname{Re}\nu>-1/2$	$\pi^{-1/2}2^{\nu-1}ay^{1/2-\nu}\left[K_{\mu+1/2}(2^{-1}ay)\right]^2$, $\operatorname{Re}y>0$
(12)	$x^{-\nu-1/2}(x^2+\alpha^2)^{1/4-\nu/2}\times$ $\times Q^{1/2-\nu}_{-1/2}(1+2\alpha^2x^{-2})$, $\operatorname{Re}\alpha>0, \ \operatorname{Re}\nu<1$	$i\,e^{-i\pi\nu}\pi^{3/2}2^{-\nu-3}\times$ $\times\alpha^{1/2-\nu}y^{\nu-1/2}[\Gamma(1-\nu)]^2\times$ $\times\left\{\left[J_{\nu-1/2}(2^{-1}ay)\right]^2+\right.$ $\left.+\left[Y_{\nu-1/2}(2^{-1}ay)\right]^2\right\}$, $\operatorname{Re}y>0$

	$f(x)$	$\int_0^\infty f(x)\,K_\nu(xy)\,(xy)^{1/2}\,dx$
(13)	$x^{-\nu-1/2}(x^2+\alpha^2)^{1/4-\nu/2}\times$ $\times Q_\mu^{1/2-\nu}(1+2\alpha^2x^{-2})$, $\operatorname{Re}\alpha>0$, $\operatorname{Re}\mu>-3/2$ $\operatorname{Re}(\mu-\nu)>-3/2$	$i\,e^{-i\pi\nu}\pi^{1/2}2^{-\nu-1}\alpha^{-\nu-1/2}\times$ $\times y^{\nu-3/2}[\Gamma(3/2+\mu-\nu)]^2\times$ $\times W_{-\mu-1/2,\,\nu-1/2}(i\alpha y)\times$ $\times W_{-\mu-1/2,\,\nu-1/2}(-i\alpha y)$, $\operatorname{Re}y>0$
(14)	$x^{1/2}P_\mu^\nu[(1+x^2)^{1/2}]$, $\operatorname{Re}\nu<1$	$y^{-1}S_{\nu+1/2,\,\mu+1/2}(y)$, $\operatorname{Re}y>0$
(15)	$x^{1/2}(1+x^2)^{-1/2}P_\mu^\nu[(1+x^2)^{1/2}]$, $\operatorname{Re}\nu<1$	$S_{\nu-1/2,\,\mu+1/2}(y)$, $\operatorname{Re}y>0$
(16)	$x^{\mu+\nu+1/2}J_\mu(\alpha x)$, $\operatorname{Re}\mu>\lvert\operatorname{Re}\nu\rvert-1$	$2^{\mu+\nu}\alpha^\mu y^{\nu+1/2}\Gamma(\mu+\nu+1)\times$ $\times(y^2+\alpha^2)^{-\mu-\nu-1}$, $\operatorname{Re}y>\lvert\operatorname{Im}\alpha\rvert$
(17)	$x^{\sigma+1/2}J_\mu(\alpha x)$, $\operatorname{Re}(\mu+\sigma)>\lvert\operatorname{Re}\nu\rvert-2$	$\frac{2^\sigma\alpha^\mu}{\Gamma(\mu+1)}y^{-\sigma-\mu-3/2}\times$ $\times\Gamma\left(\frac{\mu+\nu+\sigma}{2}+1\right)\Gamma\left(\frac{\mu-\nu+\sigma}{2}+1\right)\times$ $\times{}_2F_1\left(\frac{\mu+\nu+\sigma}{2}+1,\ \frac{\mu-\nu+\sigma}{2}+1;\ \mu+1;\ -\frac{\alpha^2}{y^2}\right)$. $\operatorname{Re}y>\lvert\operatorname{Im}\alpha\rvert$
(18)	$x^{-1/2}[J_\mu(\alpha x)]^2$, $2\operatorname{Re}\mu>\lvert\operatorname{Re}\nu\rvert-1$	$2^{-1}\Gamma(\mu+\nu/2+1/2)\times$ $\times\Gamma(\mu-\nu/2+1/2)\,y^{-1/2}\times$ $\times\left\{P^{-\mu}_{\nu/2-1/2}[(1+4\alpha^2y^{-2})^{1/2}]\right\}^2$, $\operatorname{Re}y>2\lvert\operatorname{Im}\alpha\rvert$
(19)	$x^{1/2}[J_\mu(\alpha x)]^2$, $2\operatorname{Re}\mu>\lvert\operatorname{Re}\nu\rvert-2$	$\Gamma(\mu+\nu/2+1)\,\Gamma(\mu-\nu/2+1)\times$ $\times y^{-3/2}(1+4\alpha^2y^{-2})^{-1/2}\times$ $\times P^{-\mu}_{\nu/2}[(1+4\alpha^2y^{-2})^{1/2}]\times$ $\times P^{-\mu}_{\nu/2-1}[(1+4\alpha^2y^{-2})^{1/2}]$, $\operatorname{Re}y>2\lvert\operatorname{Im}\alpha\rvert$

	$f(x)$	$\int\limits_0^\infty f(x) K_\nu(xy)(xy)^{1/2}\,dx$
(20)	$x^{1/2} J_\mu(\alpha x) J_{\mu+1}(\alpha x)$, $2\operatorname{Re}\mu > \mid\operatorname{Re}\nu\mid - 3$	$\Gamma\left(\mu+\frac{3+\nu}{2}\right)\Gamma\left(\mu+\frac{3-\nu}{2}\right)\times$ $\times\, y^{-3/2}(1+4\alpha^2 y^{-2})^{1/2}\times$ $\times\, P^{-\mu}_{\nu/2-1/2}\left[(1+4\alpha^2 y^{-2})^{1/2}\right]\times$ $\times\, P^{-\mu-1}_{\nu/2-1/2}\left[(1+4\alpha^2 y^{-2})^{1/2}\right]$, $\operatorname{Re} y > 2\mid\operatorname{Im}\alpha\mid$
(21)	$x^{-1/2} J_\mu(\alpha x) J_{-\mu}(\alpha x)$, $-1 < \operatorname{Re}\nu < 1$	$\frac{\pi}{2y^{1/2}\cos(\nu\pi/2)}\times$ $\times\, P^{\mu}_{\nu/2-1/2}\left[(1+4\alpha^2 y^{-2})^{1/2}\right]\times$ $\times\, P^{-\mu}_{\nu/2-1/2}\left[(1+4\alpha^2 y^{-2})^{1/2}\right]$, $\operatorname{Re} y > 2\mid\operatorname{Im}\alpha\mid$
(22)	$x^{1/2} J_\mu(\alpha x) J_{-\mu}(\alpha x)$, $-2 < \operatorname{Re}\nu < 2$	$-\frac{\pi}{2y^{3/2} z\sin(\nu\pi/2)}\times$ $\times\left[(\mu-\nu/2) P^{\mu}_{\nu/2}(z) P^{-\mu}_{\nu/2-1}(z) -\right.$ $\left.-(\nu/2+\mu) P^{\mu}_{\nu/2-1}(z) P^{-\mu}_{\nu/2}(z)\right]$, $z=(1+4\alpha^2 y^{-2})^{1/2}$, $\operatorname{Re} y > 2\mid\operatorname{Im}\alpha\mid$
(23)	$x^{1/2} J_\mu(\alpha x) J_{1-\mu}(\alpha x)$, $-3 < \operatorname{Re}\nu < 3$	$\frac{\alpha\,\Gamma\left(\frac{3+\nu}{2}\right)\Gamma\left(\frac{3-\nu}{2}\right)}{y^{5/2}\,\Gamma(2-\mu)\,\Gamma(1+\mu)}\times$ $\times\, {}_4F_3\left(\frac{3+\nu}{2}, \frac{3-\nu}{2}, 1, \frac{3}{2};\right.$ $\left.2-\mu, 1+\mu, 2; -\frac{4\alpha^2}{y^2}\right)$, $\operatorname{Re} y > 2\mid\operatorname{Im}\alpha\mid$
(24)	$x^{1/2} J_\mu(\alpha x) J_{-\mu-1}(\alpha x)$, $-1 < \operatorname{Re}\nu < 1$	$\frac{\pi}{2y^{3/2} z\cos(\nu\pi/2)}\times$ $\times\left[P^{-\mu}_{\nu/2-1/2}(z) P^{\mu+1}_{\nu/2-1/2}(z) +\right.$ $+(\nu/2-1/2-\mu)(\nu/2+1/2+\mu)\times$ $\left.\times\, P^{-\mu-1}_{\nu/2-1/2}(z) P^{\mu}_{\nu/2-1/2}(z)\right] -$ $-\frac{y^{1/2}\sin(\mu\pi)}{4\pi\cos(\nu\pi/2)}$, $z=(1+4\alpha^2 y^{-2})^{1/2}$, $\operatorname{Re} y > 2\mid\operatorname{Im}\alpha\mid$

	$f(x)$	$\int_0^\infty f(x) K_\nu(xy)(xy)^{1/2}\,dx$
(25)	$x^{\sigma+1/2} J_\mu(\alpha x) J_\lambda(\alpha x)$, $\operatorname{Re}(\sigma+\mu+\lambda) > \lvert \operatorname{Re}\nu \rvert - 2$	$\frac{2^\sigma \alpha^{\mu+\lambda} y^{-\mu-\lambda-\sigma-3/2}}{\Gamma(1+\mu)\Gamma(1+\lambda)} \times \Gamma\left(\frac{\mu+\lambda+\nu+\sigma}{2}+1\right) \times \Gamma\left(\frac{\mu+\lambda+\sigma-\nu}{2}+1\right) \times {}_4F_5\left(\frac{\mu+\lambda+1}{2}, \frac{\mu+\lambda}{2}+1, \frac{\mu+\lambda+\nu+\sigma}{2}+1, \frac{\mu+\lambda-\nu+\sigma}{2}+1; 1+\mu, 1+\lambda, 1+\mu+\lambda; -\frac{4\alpha^2}{y^2}\right)$, $\operatorname{Re} y > \lvert \operatorname{Im}\alpha \rvert$
(26)	$x^{\sigma+1/2} J_\mu(\alpha x) J_\lambda(\beta x)$	См. Bailey W. N., 1936: Proc. London Math. Soc., 40, 37—48; J. London Math. Soc. 11, 16—20.
(27)	$x^{1/2} J_{\nu/2}(ax^2)$, $a>0$, $\operatorname{Re}\nu > -1$	$\frac{\pi y^{1/2}}{8a\cos(\nu\pi/2)} \times \left[\mathbf{H}_{-\nu/2}\left(\frac{y^2}{4a}\right) - Y_{-\nu/2}\left(\frac{y^2}{4a}\right)\right]$, $\operatorname{Re} y > 0$
(28)	$x^{1/2} Y_{\nu/2}(ax^2)$, $a>0$, $-1 < \operatorname{Re}\nu < 1$	$\frac{\pi y^{1/2}}{4a\sin(\nu\pi)} \times \left[\cos(\nu\pi/2)\,\mathbf{H}_{-\nu/2}\left(\frac{y^2}{4a}\right) - \sin(\nu\pi/2)\,J_{-\nu/2}\left(\frac{y^2}{4a}\right) - \mathbf{H}_{\nu/2}\left(\frac{y^2}{4a}\right)\right]$, $\operatorname{Re} y > 0$
(29)	$x^{1/2} J_{\nu/4}(ax^2) J_{-\nu/4}(ax^2)$, $a>0$, $-2 < \operatorname{Re}\nu < 2$	$\frac{\pi y^{1/2}}{32a\cos(\nu\pi/4)} \times \left\{\left[J_{\nu/4}\left(\frac{y^2}{16a}\right)\right]^2 + \left[Y_{\nu/4}\left(\frac{y^2}{16a}\right)\right]^2\right\}$, $\operatorname{Re} y > 0$
(30)	$x^{1/2} J_{\mu+\nu/4}(ax^2) J_{\mu-\nu/4}(ax^2)$, $a>0$, $4\operatorname{Re}\mu > \lvert \operatorname{Re}\nu \rvert - 2$	$\pi^{-1} y^{-3/2} \times \Gamma(\mu+\nu/4+1/2)\,\Gamma(\mu-\nu/4+1/2) \times W_{-\mu,\,\nu/4}\left(\frac{y^2}{8a}e^{i\pi/2}\right) \times W_{-\mu,\,\nu/4}\left(\frac{y^2}{8a}e^{-i\pi/2}\right)$, $\operatorname{Re} y > 0$

	$f(x)$	$\int\limits_0^\infty f(x)\,K_\nu(xy)\,(xy)^{1/2}\,dx$
(31)	$x^{-1/2}J_\nu(a/x)$, $a>0$, $-5/2<\operatorname{Re}\nu<5/2$	$y^{-1/2}\exp[2^{-1}i(\nu+1)\pi]\times K_{2\nu}[2(ay)^{1/2}e^{i\pi/4}]+y^{-1/2}\exp[-2^{-1}i(\nu+1)\pi]\times K_{2\nu}[2(ay)^{1/2}e^{-i\pi/4}]$, $\operatorname{Re}y>0$
(32)	$x^{-5/2}J_\nu(a/x)$, $a>0$, $-1/2<\operatorname{Re}\nu<1/2$	$a^{-1}y^{1/2}\exp(2^{-1}i\nu\pi)\times K_{2\nu}[2(ay)^{1/2}e^{i\pi/4}]+a^{-1}y^{1/2}\exp(-2^{-1}i\nu\pi)\times K_{2\nu}[2(ay)^{1/2}e^{-i\pi/4}]$, $\operatorname{Re}y>0$
(33)	$x^{2\nu-2}J_{\nu+1/2}(a/x)$, $a>0$, $\operatorname{Re}\nu>-1/3$	$(2\pi)^{1/2}(y/a)^{-\nu+1/2}J_{2\nu}[(2ay)^{1/2}]\times K_{2\nu}[(2ay)^{1/2}]$, $\operatorname{Re}y>0$
(34)	$x^{-2\nu}J_{\nu-1/2}(a/x)$, $a>0$, $\operatorname{Re}\nu<1$	$(2\pi)^{1/2}(y/a)^{\nu-1/2}K_{2\nu-1}[(2ay)^{1/2}]\times\{\sin(\nu\pi)J_{2\nu-1}[(2ay)^{1/2}]+\cos(\nu\pi)Y_{2\nu-1}[(2ay)^{1/2}]\}$, $\operatorname{Re}y>0$
(35)	$x^{2\nu}J_{1/2+\nu}(a/x)$, $a>0$, $\operatorname{Re}\nu>-1$	$(2\pi)^{1/2}(y/a)^{-\nu-1/2}J_{1+2\nu}[(2ay)^{1/2}]\times K_{1+2\nu}[(2ay)^{1/2}]$, $\operatorname{Re}y>0$
(36)	$x^{\sigma-1}J_\mu(a/x)$, $a>0$, $\operatorname{Re}\sigma>\lvert\operatorname{Re}\nu\rvert-2$	$2^{-\sigma-3/2}a^\sigma\times G^{30}_{04}\left(\frac{a^2y^2}{4}\middle\vert\frac{\mu-\sigma}{4},\frac{1}{4}+\frac{\nu}{2},\frac{1}{4}-\frac{\nu}{2},-\frac{\mu+\sigma}{2}\right)$, $\operatorname{Re}y>0$
(37)	$x^{-1/2}Y_\nu(a/x)$, $a>0$, $-5/2<\operatorname{Re}\nu<5/2$	$-y^{-1/2}\exp(2^{-1}i\nu\pi)\times K_{2\nu}[2(ay)^{1/2}e^{i\pi/4}]-y^{-1/2}\exp(-2^{-1}i\nu\pi)\times K_{2\nu}[2(ay)^{1/2}e^{-i\pi/4}]$, $\operatorname{Re}y>0$,

	$f(x)$	$\int\limits_0^\infty f(x)\, K_\nu(xy)\,(xy)^{1/2}\, dx$
(38)	$x^{-5/2} Y_\nu(a/x)$, $a>0$, $-1/2<\operatorname{Re}\nu<1/2$	$a^{-1}y^{1/2}\exp[2^{-1}i(\nu+1)\pi]\times K_{2\nu}[2(ay)^{1/2}e^{i\pi/4}]+a^{-1}y^{1/2}\exp[-2^{-1}i(\nu+1)\pi]\times K_{2\nu}[2(ay)^{1/2}e^{-i\pi/4}]$, $\operatorname{Re} y>0$
(39)	$x^{2\nu-2} Y_{\nu+1/2}(a/x)$, $a>0$, $\operatorname{Re}\nu>-1/3$	$(2\pi)^{1/2}(y/a)^{1/2-\nu} Y_{2\nu}[(2ay)^{1/2}]\times K_{2\nu}[(2ay)^{1/2}]$, $\operatorname{Re} y>0$
(40)	$x^{-2\nu} Y_{\nu-1/2}(a/x)$, $a>0$, $\operatorname{Re}\nu<1$	$-\frac{(\pi/2)^{1/2}(y/a)^{\nu-1/2}}{\cos(\nu\pi)}\times K_{2\nu-1}[(2ay)^{1/2}]\{J_{2\nu-1}[(2ay)^{1/2}]-J_{1-2\nu}[(2ay)^{1/2}]\}$
(41)	$x^{2\nu} Y_{\nu+1/2}(a/x)$, $a>0$, $\operatorname{Re}\nu<-1$	$(2\pi)^{1/2}(y/a)^{-\nu-1/2} Y_{2\nu+1}[(2ay)^{1/2}]\times K_{2\nu+1}[(2ay)^{1/2}]$, $\operatorname{Re} y>0$
(42)	$x^{-1/2} J_\mu(a/x)\, Y_\mu(a/x)$, $a>0$, $\nu=0$	$-2y^{-1/2} J_{2\mu}(2a^{1/2}y^{1/2})\times K_{2\mu}(2a^{1/2}y^{1/2})$, $\operatorname{Re} y>0$
(43)	$x^{-1/2}\{[J_\mu(a/x)]^2-[Y_\mu(a/x)]^2\}$, $a>0$, $\nu=0$	$4y^{-1/2} Y_{2\mu}(2a^{1/2}y^{1/2})\, K_{2\mu}(2a^{1/2}y^{1/2})$, $\operatorname{Re} y>0$
(44)	$J_{2\nu-1}(\alpha x^{1/2})$, $\operatorname{Re}\nu>-1/2$	$\frac{\pi\alpha}{4y^{3/2}}\left[I_{\nu-1}\left(\frac{\alpha^2}{4y}\right)-\mathbf{L}_{\nu-1}\left(\frac{\alpha^2}{4y}\right)\right]$, $\operatorname{Re} y>0$
(45)	$x^{-1/2} J_{2\nu}(\alpha x^{1/2})$, $\operatorname{Re}\nu>-1/2$	$\frac{\pi}{2y^{1/2}}\left[I_\nu\left(\frac{\alpha^2}{4y}\right)-\mathbf{L}_\nu\left(\frac{\alpha^2}{4y}\right)\right]$, $\operatorname{Re} y>0$
(46)	$x^{-1/2} Y_{2\nu}(\alpha x^{1/2})$, $-1/2<\operatorname{Re}\nu<1/2$	$\frac{\pi}{2y^{1/2}}\left[\frac{1}{\sin(2\nu\pi)}\mathbf{L}_{-\nu}\left(\frac{\alpha^2}{4y}\right)-\operatorname{ctg}(2\nu\pi)\,\mathbf{L}_\nu\left(\frac{\alpha^2}{4y}\right)-\operatorname{tg}(\nu\pi)\, I_\nu\left(\frac{\alpha^2}{4y}\right)-\frac{1}{\pi\cos(\nu\pi)}K_\nu\left(\frac{\alpha^2}{4y}\right)\right]$, $\operatorname{Re} y>0$

	$f(x)$	$\int_0^\infty f(x) K_\nu(xy)(xy)^{1/2}\,dx$
(47)	$0, \quad 0<x<a$ $x^{1/2-\nu}(x^2-a^2)^{\mu/2}\times$ $\times J_\mu[\beta(x^2-a^2)^{1/2}],$ $a<x<\infty$ $\operatorname{Re}\mu>-1$	$a^{\mu-\nu+1}\beta^\mu y^{1/2-\nu}(y^2+\beta^2)^{(\nu-\mu-1)/2}\times$ $\times K_{\nu-\mu-1}[a(y^2+\beta^2)^{1/2}],$ $\operatorname{Re} y>\operatorname{Im}\beta$
(48)	$x^{1/2}K_\nu(\alpha x), \ -1<\operatorname{Re}\nu<1$	$\dfrac{\pi\alpha^{-\nu}y^{\nu/2}}{2\sin(\nu\pi)}\,\dfrac{\alpha^{2\nu}-y^{2\nu}}{\alpha^2-y^2},$ $\operatorname{Re}(y+\alpha)>0$
(49)	$x^{\sigma-3/2}K_\mu(\alpha x),$ $\operatorname{Re}\sigma>\lvert\operatorname{Re}\mu\rvert+\lvert\operatorname{Re}\nu\rvert$	$\dfrac{2^{\sigma-3}\alpha^{-\nu-\sigma}}{\Gamma(\sigma)}\Gamma\left(\dfrac{\sigma+\mu+\nu}{2}\right)\Gamma\left(\dfrac{\sigma-\mu+\nu}{2}\right)\times$ $\times\Gamma\left(\dfrac{\sigma+\mu-\nu}{2}\right)\Gamma\left(\dfrac{\sigma-\mu-\nu}{2}\right)y^{\nu+1/2}\times$ $\times{}_2F_1\left(\dfrac{\sigma+\mu+\nu}{2},\ \dfrac{\sigma-\mu+\nu}{2};\sigma;1-\dfrac{y^2}{\alpha^2}\right),$ $\operatorname{Re}(y+\alpha)>0$
(50)	$x^{1/2}[2\pi^{-1}K_0(\alpha x)-Y_0(\alpha x)],$ $\nu=0$	$2\pi^{-1}y^{1/2}[(y^2+\alpha^2)^{-1}+(y^2-\alpha^2)^{-1}]\times$ $\times\ln(y/\alpha),$ $\operatorname{Re} y>\lvert\operatorname{Im}\alpha\rvert,\ \operatorname{Re}(y+\alpha)>0$
(51)	$x^{\sigma+1/2}J_\mu(\alpha x)K_\lambda(\beta x),$ $x^{\sigma+1/2}K_\mu(\alpha x)K_\lambda(\beta x)$	См. Bailey W. N., 1936: J. London Math. Soc., 11, 16—20; Proc. London Math. Soc. 40, 37—48.
(52)	$x^{1/2}K_{\nu/2}(\alpha x^2),$ $\operatorname{Re}\alpha>0,\ -1<\operatorname{Re}\nu<1$	$\dfrac{\pi y^{1/2}}{8\alpha}\left\{\dfrac{1}{\cos(\nu\pi/2)}K_{\nu/2}\left(\dfrac{y^2}{4\alpha}\right)+\right.$ $+\dfrac{\pi}{\sin(\nu\pi)}\left[\mathbf{L}_{-\nu/2}\left(\dfrac{y^2}{4\alpha}\right)-\right.$ $\left.\left.-\mathbf{L}_{\nu/2}\left(\dfrac{y^2}{4\alpha}\right)\right]\right\}$
(53)	$x^{2\mu+\nu+1/2}\exp(-2^{-1}\alpha x^2)\times$ $\times I_\mu(2^{-1}\alpha x^2),\ \operatorname{Re}\alpha>0$ $\operatorname{Re}\mu>-1/2,\ \operatorname{Re}(2\mu+\nu)>-1$	$\pi^{-1/2}2^{\mu-1/2}\alpha^{-\mu/2-\nu/2-1/4}y^{-\mu-1}\times$ $\times\Gamma(2\mu+\nu+1)\Gamma(\mu+1/2)\times$ $\times\exp\left(\dfrac{y^2}{8\alpha}\right)W_{k,m}\left(\dfrac{y^2}{4\alpha}\right),$ $2k=-3\mu-\nu-1/2$ $2m=\mu+\nu+1/2$
(54)	$x^{-1/2}K_\nu(\alpha/x), \quad \operatorname{Re}\alpha>0$	$\pi y^{-1/2}K_{2\nu}(2\alpha^{1/2}y^{1/2}), \quad \operatorname{Re} y>0$
(55)	$x^{-5/2}K_\nu(\alpha/x), \quad \operatorname{Re}\alpha>0$	$\pi\alpha^{-1}y^{1/2}K_{2\nu}(2\alpha^{1/2}y^{1/2}), \quad \operatorname{Re} y>0$

	$f(x)$	$\int_0^\infty f(x) K_\nu(xy)(xy)^{1/2}\,dx$
(56)	$x^{2\nu} K_{\nu+1/2}(\alpha/x)$, $\quad\operatorname{Re}\alpha>0$	$(2\pi)^{1/2}(y/\alpha)^{-\nu-1/2}\times$ $\times K_{2\nu+1}[(2\alpha y)^{1/2}e^{i\pi/4}]\times$ $\times K_{2\nu+1}[(2\alpha y)^{1/2}e^{-i\pi/4}]$, $\operatorname{Re}y>0$
(57)	$x^{2\nu-2} K_{\nu+1/2}(\alpha/x)$, $\quad\operatorname{Re}\alpha>0$	$(2\pi)^{1/2}(y/\alpha)^{1/2-\nu}\times$ $\times K_{2\nu}[(2\alpha y)^{1/2}e^{i\pi/4}]\times$ $\times K_{2\nu}[(2\alpha y)^{1/2}e^{-i\pi/4}]$, $\operatorname{Re}y>0$
(58)	$x^{\sigma-1} K_\mu(\alpha/x)$, $\quad\operatorname{Re}\alpha>0$	$2^{-\sigma-5/2}\alpha^\sigma\times$ $\times G_{04}^{40}\left(\frac{\alpha^2y^2}{4}\middle\vert \frac{\mu-\sigma}{2}, \frac{1}{4}+\frac{\nu}{2}, \frac{1}{4}-\frac{\nu}{2}, -\frac{\mu+\sigma}{2}\right)$
(59)	$x^{-1/2}[K_\mu(\alpha/x)]^2$, $\quad\operatorname{Re}\alpha>0,\ \nu=0$	$2\pi y^{-1/2}K_{2\mu}(2\alpha^{1/2}y^{1/2}e^{i\pi/4})\times$ $\times K_{2\mu}(2\alpha^{1/2}y^{1/2}e^{-i\pi/4})$, $\operatorname{Re}y>0$
(60)	$x^{-1/2} I_{2\nu}(\alpha x^{1/2})$, $\quad\operatorname{Re}\nu>-1/2$	$\frac{\pi}{2y^{1/2}}\left[I_\nu\left(\frac{\alpha^2}{4y}\right)+\mathbf{L}_\nu\left(\frac{\alpha^2}{4y}\right)\right]$, $\operatorname{Re}y>0$
(61)	$x^{-1/2}[J_{2\nu}(\alpha x^{1/2})+I_{2\nu}(\alpha x^{1/2})]$, $\quad\operatorname{Re}\nu>-1/2$	$\frac{\pi}{y^{1/2}} I_\nu\left(\frac{\alpha^2}{4y}\right)$, $\quad\operatorname{Re}y>0$
(62)	$x^{-1/2}[I_{2\nu}(\alpha x^{1/2})-J_{2\nu}(\alpha x^{1/2})]$, $\quad\operatorname{Re}\nu>-1/2$	$\frac{\pi}{y^{1/2}}\mathbf{L}_\nu\left(\frac{\alpha^2}{4y}\right)$, $\quad\operatorname{Re}y>0$
(63)	$x^{-1/2}K_{2\nu}(\alpha x^{1/2})$, $\quad -1/2<\operatorname{Re}\nu<1/2$	$\frac{\pi y^{-1/2}}{4\cos(\nu\pi)}\left\{K_\nu\left(\frac{\alpha^2}{4y}\right)+\right.$ $\left.+\frac{\pi}{2\sin(\nu\pi)}\left[\mathbf{L}_{-\nu}\left(\frac{\alpha^2}{4y}\right)-\mathbf{L}_\nu\left(\frac{\alpha^2}{4y}\right)\right]\right\}$, $\operatorname{Re}y>0$
(64)	$x^{\nu+1/2}I_{2\nu}(\alpha x^{1/2})J_{2\nu}(\alpha x^{1/2})$, $\quad\operatorname{Re}\nu>-1/2$	$\pi^{1/2}2^{-\nu-1}\alpha^{2\nu+1}y^{-2\nu-2}\times$ $\times J_{\nu-1/2}\left(\frac{\alpha^2}{2y}\right)$, $\operatorname{Re}y>0$
(65)	$x^{\nu-1/2}I_{2\nu-1}(\alpha x^{1/2})J_{2\nu-1}(\alpha x^{1/2})$, $\quad\operatorname{Re}\nu>0$	$\pi^{1/2}2^{-\nu}\alpha^{2\nu-1}y^{-2\nu}J_{\nu-1/2}\left(\frac{\alpha^2}{2y}\right)$, $\operatorname{Re}y>0$

	$f(x)$	$\int_0^\infty f(x) K_\nu(xy)(xy)^{1/2}\,dx$
(66)	$x^{\nu-1/2} I_{2\nu-1}(\alpha x^{1/2}) Y_{2\nu-1}(\alpha x^{1/2})$, $\operatorname{Re}\nu > 0$	$\frac{\pi^{1/2}\alpha^{2\nu-1}}{2^{\nu+1}y^{2\nu}\sin(\nu\pi)}\left[\mathbf{H}_{1/2-\nu}\left(\frac{\alpha^2}{2y}\right) + \cos(\nu\pi) J_{\nu-1/2}\left(\frac{\alpha^2}{2y}\right) + \sin(\nu\pi) Y_{\nu-1/2}\left(\frac{\alpha^2}{2y}\right)\right]$, $\operatorname{Re} y > 0$
(67)	$x^{\nu-1/2} J_{2\nu-1}(\alpha x^{1/2}) K_{2\nu-1}(\alpha x^{1/2})$, $\operatorname{Re}\nu > 0$	$\frac{\pi^{3/2}\alpha^{2\nu-1}}{2^{\nu+2}y^{2\nu}\sin(\nu\pi)} \times \left[\mathbf{H}_{1/2-\nu}\left(\frac{\alpha^2}{2y}\right) - Y_{1/2-\nu}\left(\frac{\alpha^2}{2y}\right)\right]$, $\operatorname{Re} y > 0$
(68)	$x^{-\nu-1/2} I_{2\nu+1}(\alpha x^{1/2}) \times J_{-2\nu-1}(\alpha x^{1/2})$, $\operatorname{Re}\nu < 1/2$	$-\pi^{1/2} 2^{\nu}\alpha^{-2\nu-1} y^{2\nu} \times \left[\cos(\nu\pi)\,\mathbf{H}_{\nu+1/2}\left(\frac{\alpha^2}{2y}\right) + \sin(\nu\pi) J_{\nu+1/2}\left(\frac{\alpha^2}{2y}\right)\right]$, $\operatorname{Re} y > 0$
(69)	$x^{-\nu-1/2} I_{-2\nu-1}(\alpha x^{1/2}) \times J_{2\nu+1}(\alpha x^{1/2})$, $\operatorname{Re}\nu < 1/2$	$\pi^{1/2} 2^{\nu}\alpha^{-2\nu-1} y^{2\nu} \times \left[\cos(\nu\pi)\,\mathbf{H}_{\nu+1/2}\left(\frac{\alpha^2}{2y}\right) - \sin(\nu\pi) J_{\nu+1/2}\left(\frac{\alpha^2}{2y}\right)\right]$, $\operatorname{Re} y > 0$
(70)	$x^{1/2-\nu}[I_{2\nu}(\alpha x^{1/2}) J_{-2\nu}(\alpha x^{1/2}) - J_{2\nu}(\alpha x^{1/2}) I_{-2\nu}(\alpha x^{1/2})]$, $\operatorname{Re}\nu < 3/2$	$-\pi^{1/2} 2^{\nu}\alpha^{1-2\nu}\sin(\nu\pi)\, y^{2\nu-2} \times J_{\nu+1/2}\left(\frac{\alpha^2}{2y}\right)$, $\operatorname{Re} y > 0$
(71)	$x^{-1/2} K_\mu(\alpha x^{1/2}) \times [\sin(2^{-1}\mu\pi) J_\mu(\alpha x^{1/2}) + \cos(2^{-1}\mu\pi) Y_\mu(\alpha x^{1/2})]$, $-1 < \operatorname{Re}\mu < 1$, $\nu = 0$	$-\frac{\pi^2 y^{-1/2}}{16\cos(2^{-1}\mu\pi)} H^{(1)}_{\mu/2}\left(\frac{\alpha^2}{4y}\right) \times H^{(2)}_{\mu/2}\left(\frac{\alpha^2}{4y}\right)$, $\operatorname{Re} y > 0$
(72)	$x^{-1/2} K_\mu(\alpha x^{1/2}) \times \{\sin[2^{-1}(\mu-\nu)\pi] J_\mu(\alpha x^{1/2}) + \cos[2^{-1}(\mu-\nu)\pi] Y_\mu(\alpha x^{1/2})\}$, $\lvert\operatorname{Re}\mu\rvert + \lvert\operatorname{Re}\nu\rvert < 1$	$-2^{-1}\alpha^{-2} y^{1/2}\Gamma\left(\frac{1+\mu-\nu}{2}\right)\Gamma\left(\frac{1-\mu-\nu}{2}\right) \times W_{\nu/2,\,\mu/2}\left(\frac{\alpha^2}{2y}e^{i\pi/2}\right) \times W_{\nu/2,\,\mu/2}\left(\frac{\alpha^2}{2y}e^{-i\pi/2}\right)$, $\operatorname{Re} y > 0$

	$f(x)$	$\int_0^\infty f(x) K_\nu(xy)(xy)^{1/2}\,dx$
(73)	$x^{1/2}\mathbf{H}_\nu(\alpha x)$, $\quad \operatorname{Re}\nu > -3/2$	$\alpha^{\nu+1} y^{-\nu-1/2}(y^2+\alpha^2)^{-1}$, $\quad \operatorname{Re} y > \lvert\operatorname{Im}\alpha\rvert$
(74)	$x^{\mu+\nu+1/2}\mathbf{H}_\mu(\alpha x)$, $\quad \operatorname{Re}\mu > -3/2$, $\operatorname{Re}(\mu+\nu) > -3/2$	$\pi^{-1/2} 2^{\mu+\nu+1}\alpha^{\mu+1}\times$ $\times\, y^{-2\mu-\nu-5/2}\,\Gamma(\mu+\nu+3/2)\times$ $\times\, {}_2F_1\left(\mu+\nu+\frac{3}{2}, 1; \frac{3}{2}; -\frac{\alpha^2}{y^2}\right)$, $\quad \operatorname{Re} y > \lvert\operatorname{Im}\alpha\rvert$
(75)	$x^{1/2}\mathbf{H}_{\nu/2}(ax^2)$, $a>0$, $\quad \operatorname{Re}\nu > -2$	$\dfrac{y^{1/2}\,\Gamma(1+\nu/2)}{2^{1-\nu/2}a\pi} S_{-\nu/2-1,\,\nu/2}\left(\dfrac{y^2}{4a}\right)$, $\quad \operatorname{Re} y > 0$
(76)	$x^{3/2}\mathbf{H}_{\nu/2+1/2}(ax^2)$, $a>0$, $\quad \operatorname{Re}\nu > -3$	$\dfrac{2^{\nu/2+1/2}y^{3/2}}{a^2\pi}\Gamma\left(\dfrac{3+\nu}{2}\right)\times$ $\times\, S_{-\frac{\nu+5}{2},\,\frac{\nu-1}{2}}\left(\dfrac{y^2}{4a}\right)$, $\quad \operatorname{Re} y > 0$
(77)	$x^{5/2}\mathbf{H}_{\nu/2}(ax^2)$, $a>0$, $\quad \operatorname{Re}\nu > -3$	$\dfrac{y^{5/2}\,\Gamma(2+\nu/2)}{2^{-1-\nu/2}a^3\pi} S_{-3-\nu/2,\,\nu/2}\left(\dfrac{y^2}{4a}\right)$, $\quad \operatorname{Re} y > 0$
(78)	$x^{1/2} s_{\mu,\,\nu/2}(ax^2)$, $\quad a>0$, $\operatorname{Re}\mu > 2^{-1}\lvert\operatorname{Re}\nu\rvert - 2$	$\dfrac{y^{1/2}}{4a}\Gamma(\mu+\nu/2+1)\,\Gamma(\mu-\nu/2+1)\times$ $\times\, S_{-\mu-1,\,\nu/2}\left(\dfrac{y^2}{4a}\right)$, $\quad \operatorname{Re} y > 0$
(79)	$x^{3/2} s_{\mu,\,\nu/2+1/2}(ax^2)$, $\quad a>0$, $2\operatorname{Re}\mu > \lvert\operatorname{Re}\nu\rvert - 5$	$\dfrac{y^{3/2}}{8a^2}\left(\mu+\dfrac{3-\nu}{2}\right)\times$ $\times\,\Gamma\left(\mu+\dfrac{1-\nu}{2}\right)\Gamma\left(\mu+\dfrac{3+\nu}{2}\right)\times$ $\times\, S_{-\mu-2,\,\nu/2-1/2}\left(\dfrac{y^2}{4a}\right)$, $\quad \operatorname{Re} y > 0$
(80)	$x^{5/2} s_{\mu,\,\nu/2}(ax^2)$, $\quad a>0$, $\operatorname{Re}\mu > 2^{-1}\lvert\operatorname{Re}\nu\rvert - 3$	$\dfrac{y^{5/2}}{16a^3}(2+\mu+\nu/2)(2+\mu-\nu/2)\times$ $\times\,\Gamma(1+\mu+\nu/2)\,\Gamma(1+\mu-\nu/2)\times$ $\times\, S_{-\mu-3,\,\nu/2}\left(\dfrac{y^2}{4a}\right)$, $\quad \operatorname{Re} y > 0$

	$f(x)$	$\int_0^\infty f(x)\, K_\nu(xy)\,(xy)^{1/2}\,dx$
(81)	$D_{\nu-1/2}(\alpha x^{-1/2})\,D_{-\nu-1/2}(\alpha x^{-1/2})$, $\lvert\arg\alpha\rvert < \pi/4$	$\frac{\pi}{2y}\exp[-\alpha(2y)^{1/2}]$, $\operatorname{Re} y > 0$
(82)	$x^{2\mu+\nu-1/2}\exp(-2^{-1}\alpha x^2)\times$ $\times M_{\varkappa,\mu}(\alpha x^2)$, $\operatorname{Re}\alpha > 0$ $\operatorname{Re}\mu > -1/2$, $\operatorname{Re}(2\mu+\nu) > -1$	$2^{\mu-\varkappa-1/2}\alpha^{1/2-m-\varkappa}y^{\varkappa-\mu-1}\times$ $\times\Gamma(2\mu+1)\,\Gamma(2\mu+\nu+1)\times$ $\times\exp\left(\frac{y^2}{8\alpha}\right)W_{k,m}\left(\frac{y^2}{4\alpha}\right)$, $2k = -3\mu-\nu-\varkappa-1/2$ $2m = \mu+\nu-\varkappa+1/2$ $\operatorname{Re} y > 0$
(83)	$x^{-3/2}M_{\varkappa,0}(iax^2)\,M_{\varkappa,0}(-iax^2)$, $a > 0$, $\nu = 0$	$\frac{\pi y^{1/2}}{16}\left\{\left[J_\varkappa\left(\frac{y^2}{8a}\right)\right]^2 + \left[Y_\varkappa\left(\frac{y^2}{8a}\right)\right]^2\right\}$
(84)	$x^{-3/2}M_{\varkappa,\mu}(iax^2)M_{\varkappa,\mu}(-iax^2)$, $a > 0$, $\operatorname{Re}\mu > -1/2$, $\nu = 0$	$ay^{-3/2}[\Gamma(2\mu+1)]^2\,W_{-\mu,\varkappa}\left(\frac{iy^2}{4a}\right)\times$ $\times W_{-\mu,\varkappa}\left(-\frac{iy^2}{4a}\right)$, $\operatorname{Re} y > 0$
(85)	$x^{1/2}W_{\nu/2,\mu}(\alpha/x)\,W_{-\nu/2,\mu}(\alpha/x)$, $\operatorname{Re}\alpha > 0$	$2\alpha y^{-1/2}K_{2\mu}\left[(2\alpha y)^{1/2}e^{i\pi/4}\right]\times$ $\times K_{2\mu}\left[(2\alpha y)^{1/2}e^{-i\pi/4}\right]$, $\operatorname{Re} y > 0$
(86)	$x^{\nu+1/2}\,{}_2F_1(\alpha,\beta;\nu+1;-\lambda^2x^2)$, $\operatorname{Re}\lambda > 0$, $\operatorname{Re}\nu > -1$	$2^{\nu+1}\lambda^{-\alpha-\beta}y^{\alpha+\beta-\nu-3/2}\Gamma(\nu+1)\times$ $\times S_{1-\alpha-\beta,\,\alpha-\beta}(y/\lambda)$, $\operatorname{Re} y > 0$
(87)	$x^{\nu+2\gamma-3/2}\times$ $\times\,{}_3F_2(1,\alpha,\beta;\gamma,\gamma+\nu;-\lambda^2x^2)$, $\operatorname{Re}\lambda > 0$, $\operatorname{Re}\gamma > 0$ $\operatorname{Re}(\gamma+\nu) > 0$	$2^{\nu+2\gamma-2}\lambda^{-\alpha-\beta}y^{\alpha+\beta-2\gamma-\nu+1/2}\times$ $\times\Gamma(\gamma)\,\Gamma(\gamma+\nu)\,S_{1-\alpha-\beta,\,\alpha-\beta}(y/\lambda)$, $\operatorname{Re} y > 0$
(88)	$x^{\mu-3/2}\,{}_pF_q(\alpha_1,\ldots,\alpha_p;$ $\beta_1,\ldots,\beta_q;-\lambda x^2)$, $p \leqslant q-1$, $\operatorname{Re}\mu > \lvert\operatorname{Re}\nu\rvert$	$2^{\mu-2}y^{1/2-\mu}\Gamma\left(\frac{\mu+\nu}{2}\right)\Gamma\left(\frac{\mu-\nu}{2}\right)\times$ $\times\,{}_{p+2}F_q\left(\alpha_1,\ldots,\alpha_p,\frac{\mu+\nu}{2},\right.$ $\left.\frac{\mu-\nu}{2};\beta_1\ldots,\beta_q;\frac{4\lambda}{y^2}\right)$, $\operatorname{Re} y > 0$

	$f(x)$	$\int_0^\infty f(x) K_\nu(xy)(xy)^{1/2}\,dx$
(89)	$x^{\mu-3/2}\times$ $\times E(\alpha_1, \ldots, \alpha_p: \rho_1, \ldots, \rho_q: ax^{-2})$, $a>0$, $\operatorname{Re}\mu > \lvert\operatorname{Re}\nu\rvert$	$2^{\mu-2}a^{-\mu}y^{1/2}\times$ $\times E(\alpha_1, \ldots, \alpha_{p+2}: \rho_1, \ldots, \rho_q: 2^{-2}ay^2)$, $\alpha_{p+1}=\frac{\mu+\nu}{2}$, $\alpha_{p+2}=\frac{\mu-\nu}{2}$, $\operatorname{Re} y>0$
(90)	$G_{pq}^{mn}\left(\lambda x^2 \middle\vert \begin{matrix}\alpha_1, \ldots, \alpha_p\\ \beta_1, \ldots, \beta_q\end{matrix}\right)$, $p+q<2(m+n)$ $\lvert\arg\lambda\rvert<(m+n-p/2-q/2)\pi$ $\operatorname{Re}\beta_j > 2^{-1}\lvert\operatorname{Re}\nu\rvert - 3/4$ $j=1, \ldots, m$	$2^{-3/2}\lambda^{-1/2}\times$ $\times G_{q,\,p+2}^{n+2,\,m}\left(\frac{y^2}{4\lambda}\middle\vert \begin{matrix}\beta_1', \ldots, \beta_q'\\ h, k, \alpha_1', \ldots, \alpha_p'\end{matrix}\right)$, $h=1/4+\nu/2$, $k=1/4-\nu/2$ $\operatorname{Re} y>0$ $\alpha_j'=\frac{1}{2}-\alpha_j$, $\beta_j'=\frac{1}{2}-\beta_j$

ГЛАВА XI

Н-ПРЕОБРАЗОВАНИЯ

Мы называем Н-*преобразованием порядка* ν функции $f(x)$ функцию

$$\int_0^\infty f(x)\,\mathbf{H}(xy)\,(xy)^{1/2}\,dx$$

положительного переменного y. Формула обращения 11.1 (1) дана Титчмаршем (1948, стр. 280). Н-преобразование обратно Y-преобразованию (см. главу IX).

Из пар преобразований, указанных в этой главе, можно получать новые пары преобразований, применяя методы, указанные во введении к первому тому, а также общие формулы из п. 11.1. Кроме того, поскольку Н-преобразование при $-^1/_2 < \operatorname{Re}\nu < {}^1/_2$ обратно Y-преобразованию, многие новые формулы могут быть получены из формул, приведенных в главе IX. Очевидно, как с помощью аналитического продолжения получить соответствующие формулы в более широкой области изменения $\operatorname{Re}\nu$.

11.1. Общие формулы

	$f(x)$	$\int_0^\infty f(x)\,\mathbf{H}_\nu(xy)\,(xy)^{1/2}\,dx = g(y;\ \nu),\quad y > 0$
(1)	$\int_0^\infty g(y;\ \nu)\,Y_\nu(xy)\,(xy)^{1/2}\,dy,\quad -^1/_2 < \operatorname{Re}\nu < {}^1/_2$	$g(y;\ \nu)$
(2)	$f(ax),\quad a > 0$	$a^{-1}\,g(a^{-1}y;\ \nu)$
(3)	$x^m f(x),\quad m = 0,\ 1,\ 2,\ \ldots$	$y^{1/2-\nu} \times \left(\dfrac{d}{y\,dy}\right)^m \left[y^{\nu-1/2+m}\,g(y;\ \nu+m)\right]$
(4)	$x^{1/2+\nu}\left(\dfrac{d}{x\,dx}\right)^m \left[x^{m-\nu-1/2} f(x)\right],\quad m = 0,\ 1,\ 2,\ \ldots$	$(-y)^m\,g(y;\ \nu-m)$

	$f(x)$	$\int_0^\infty f(x)\,\mathbf{H}_\nu(xy)\,(xy)^{1/2}\,dx = g(y;\ \nu),\quad y>0$
(5)	$x^{\nu+1/2}\times$ $\times\int_x^\infty \xi^{1/2-\nu-\mu}(\xi^2-x^2)^{\mu-1}f(\xi)\,d\xi,$ $\operatorname{Re}\mu>0,\ \operatorname{Re}\nu>-3/2$	$2^{\mu-1}\Gamma(\mu)\,y^{-\mu}g(y;\ \nu+\mu)$
(6)	$x^{-\mu}f(x),$ $\operatorname{Re}\nu+3/2>\operatorname{Re}\mu>0$	$2^{1-\mu}[\Gamma(\mu)]^{-1}y^{1/2-\nu}\times$ $\times\int_0^y \eta^{1/2-\mu+\nu}(y^2-\eta^2)^{\mu-1}\times$ $\times g(\eta;\ \nu-\mu)\,d\eta$

11.2. Элементарные функции

(1)	$x^{-1/2},\quad -2<\operatorname{Re}\nu<0$	$-\operatorname{ctg}(\nu\pi/2)\,y^{-1/2}$
(2)	$x^{\nu+1/2},\quad 0<x<a$ $0,\quad a<x<\infty$ $\operatorname{Re}\nu>-3/2$	$a^{\nu+1}y^{-1/2}\,\mathbf{H}_{\nu+1}(ay)$
(3)	$x^{1/2-\nu},\quad 0<x<a$ $0,\quad a<x<\infty$	$\dfrac{ay^{\nu-1/2}}{2^{\nu-1}\pi^{1/2}\,\Gamma(\nu+1/2)}-$ $-a^{1-\nu}y^{-1/2}\,\mathbf{H}_{\nu-1}(ay)$
(4)	$x^{\lambda-1/2},\quad \operatorname{Re}\lambda<1/2$ $-2<\operatorname{Re}(\lambda+\nu)<0$	$2^\lambda y^{-\lambda-1/2}\operatorname{tg}[2^{-1}(\lambda+\nu+1)\pi]\times$ $\times\dfrac{\Gamma(1/2+\lambda/2+\nu/2)}{\Gamma(1/2-\lambda/2+\nu/2)}$
(5)	$x^{\lambda-1/2},\quad 0<x<a$ $0,\quad a<x<\infty$ $\operatorname{Re}(\lambda+\nu)>-2$	$\dfrac{a^{\lambda+\nu+2}y^{\nu+3/2}}{2^\nu\pi^{1/2}\,\Gamma(\nu+3/2)(\lambda+\nu+2)}\times$ $\times\,{}_2F_3\left(1,\ \dfrac{\lambda+\nu}{2}+1;\ \dfrac{3}{2},\ \nu+\dfrac{3}{2},\ \dfrac{\lambda+\nu}{2}+2;\ -\dfrac{a^2y^2}{4}\right)$
(6)	$x^{-1/2}(x^2+\alpha^2)^{-1},$ $\operatorname{Re}\alpha>0,\ \nu=1$	$\dfrac{\pi y^{1/2}}{2\alpha}[I_1(\alpha y)-\mathbf{L}_1(\alpha y)]$

	$f(x)$	$\int_0^\infty f(x)\,\mathbf{H}_\nu(xy)\,(xy)^{1/2}dx,\quad y>0$
(7)	$x^{-1/2}(x^2+\alpha^2)^{-1}$, $\operatorname{Re}\alpha>0,\ -2<\operatorname{Re}\nu<2$	$-\dfrac{\pi y^{1/2}}{2\alpha\sin(\nu\pi/2)}\mathbf{L}_\nu(\alpha y)+$ $+\dfrac{y^{3/2}\operatorname{ctg}(\nu\pi/2)}{1-\nu^2}\times$ $\times\,{}_1F_2\left(1;\ \dfrac{3-\nu}{2},\ \dfrac{3+\nu}{2};\ \dfrac{\alpha^2y^2}{4}\right)$
(8)	$x^{\nu+1/2}(x^2+\alpha^2)^{\mu-1}$, $\operatorname{Re}\alpha>0,\ \operatorname{Re}\nu>-3/2$ $\operatorname{Re}(\mu+\nu)<1/2$ $\operatorname{Re}(2\mu+\nu)<3/2$	$\dfrac{2^{\mu-1}\pi\alpha^{\mu+\nu}y^{1/2-\mu}}{\Gamma(1-\mu)\cos[(\mu+\nu)\pi]}\times$ $\times[I_{-\mu-\nu}(\alpha y)-\mathbf{L}_{\mu+\nu}(\alpha y)]$
(9)	$x^{1/2-\nu}(x^2+\alpha^2)^{\mu-1}$, $\operatorname{Re}\alpha>0,\ \operatorname{Re}\mu<1/2$ $\operatorname{Re}(2\mu-\nu)<3/2$	$\dfrac{2^{\mu-1}\pi\alpha^{\mu-\nu}y^{1/2-\mu}}{\Gamma(1-\mu)\cos(\mu\pi)}I_{\nu-\mu}(\alpha y)+$ $+\dfrac{\alpha^{2\mu+1}y^{\nu+3/2}\Gamma(-1/2-\mu)}{2^{\nu+2}\Gamma(1-\mu)\Gamma(\nu+3/2)}\times$ $\times\,{}_1F_2\left(1;\ \mu+\dfrac{3}{2},\ \nu+\dfrac{3}{2};\ \dfrac{\alpha^2y^2}{4}\right)$
(10)	$x^{\lambda-1/2}(x^2+\alpha^2)^{\mu-1}$, $\operatorname{Re}\alpha>0,\ \operatorname{Re}(\lambda+\nu)>-2$ $\operatorname{Re}(\lambda+2\mu)<5/2$ $\operatorname{Re}(\lambda+2\mu+\nu)<2$	$2^{-1/2}[\Gamma(1-\mu)]^{-1}\alpha^{\lambda+2\mu-3/2}\times$ $\times\,G_{24}^{22}\left(\dfrac{\alpha^2y^2}{4}\middle\vert\begin{matrix}l,\ m\\ l,\ m-\mu,\ h,\ k\end{matrix}\right)$, $h=1/4+\nu/2,\ k=1/4-\nu/2$ $l=3/4+\nu/2,\ m=3/4-\lambda/2$
(11)	$(x^2+\alpha^2)^{-1/2}[x+(x^2+\alpha^2)^{1/2}]^{\nu+1}$, $\operatorname{Re}\alpha>0,\ -2<\operatorname{Re}\nu<0$	$\dfrac{\pi^{1/2}\alpha^{\nu+1/2}}{y^{1/2}\sin(\nu\pi)}[\operatorname{sh}(\alpha y/2)\,I_{\nu+1/2}(\alpha y/2)-$ $-\operatorname{ch}(\alpha y/2)\,I_{-\nu-1/2}(\alpha y/2)]$
(12)	$x^{\nu+1/2}(a^2-x^2)^{\mu-1},\quad 0<x<a$ $0,\quad a<x<\infty$ $\operatorname{Re}\mu>0,\ \operatorname{Re}\nu>-3/2$	$2^{\mu-1}a^{\mu+\nu}y^{1/2-\mu}\,\Gamma(\mu)\,\mathbf{H}_{\mu+\nu}(ay)$
(13)	$x^{\lambda-1/2}(a^2-x^2)^{\mu-1},\quad 0<x<a$ $0,\quad a<x<\infty$ $\operatorname{Re}\mu>0,\ \operatorname{Re}(\lambda+\nu)>-2$	$\dfrac{a^{2\mu+\nu+\lambda}y^{\nu+3/2}\,\Gamma(\mu)\,\Gamma\left(\frac{\lambda+\nu}{2}+1\right)}{2^{\nu+1}\pi^{1/2}\Gamma(\nu+3/2)\,\Gamma\left(\frac{\lambda+\nu}{2}+\mu+1\right)}\times$ $\times\,{}_2F_3\left(1,\ \dfrac{\lambda+\nu}{2}+1;\ \dfrac{3}{2},\ \nu+\dfrac{3}{2},\ \dfrac{\lambda+\nu}{2}+\mu+1;\ -\dfrac{a^2y^2}{4}\right)$

	$f(x)$	$\int_0^\infty f(x)\,\mathbf{H}_\nu(xy)\,(xy)^{1/2}\,dx, \quad y>0$
(14)	$0, \quad 0<x<a$ $x^{-\nu-1/2}(x^2-a^2)^{-\nu-1/2}, \quad a<x<\infty$ $-1/2<\operatorname{Re}\nu<1$	$2^{-\nu-1}\pi^{1/2}a^{-2\nu}y^{\nu+1/2}\times$ $\times\Gamma(1/2-\nu)\,[J_\nu(ay/2)]^2$
(15)	$0, \quad 0<x<a$ $x^{\nu+1/2}(x^2-a^2)^m, \quad a<x<\infty$ $m=0,\ 1,\ 2,\ \ldots$ $\operatorname{Re}\nu<-2m-1/2$	$(-1)^{m+1}2^m a^{m+\nu+1}y^{-m-1/2}m!\times$ $\times\mathbf{H}_{\nu+m+1}(ay)$
(16)	$0, \quad 0<x<a$ $x^{\nu+1/2}(x^2-a^2)^{\mu-1}, \quad a<x<\infty$ $\operatorname{Re}\mu>0,\ \operatorname{Re}(\mu+\nu)<1/2$ $\operatorname{Re}(2\mu+\nu)<3/2$	$\dfrac{2^{\mu-1}a^{\mu+\nu}y^{1/2-\mu}\,\Gamma(\mu)}{\cos[(\mu+\nu)\pi]}\times$ $\times[\sin(\mu\pi)\,J_{-\mu-\nu}(ay)+$ $+\cos(\nu\pi)\,\mathbf{H}_{\mu+\nu}(ay)]$
(17)	$0, \quad 0<x<a$ $x^{-\nu-1/2}(x^2-a^2)^\mu, \quad a<x<\infty$ $-1<\operatorname{Re}\mu<0$ $\operatorname{Re}\nu>2\operatorname{Re}\mu-1/2$	$-\pi 2^{2\mu-\nu}y^{\nu-2\mu-1/2}\,[\Gamma(1/2-\mu)]\times$ $\times[\Gamma(1/2+\nu-\mu)\sin(\mu\pi)]^{-1}\times$ $\times{}_1F_2(-\mu;\ 1/2-\mu,\ 1/2+\nu-\mu;$ $-2^{-2}a^2y^2)$
(18)	$0, \quad 0<x<a$ $x^{\lambda-1/2}(x^2-a^2)^{\mu-1}, \quad a<x<\infty$ $\operatorname{Re}\mu>0,\ \operatorname{Re}(\lambda+2\mu)<5/2$ $\operatorname{Re}(\lambda+2\mu+\nu)<2$	$2^{-1/2}\,\Gamma(\mu)\,a^{2\mu+\lambda-1/2}\times$ $\times G_{24}^{21}\left(\frac{a^2y^2}{4}\,\middle\vert\,\begin{matrix}l,\ m\\ l,\ m-\mu,\ h,\ k\end{matrix}\right),$ $h=1/4+\nu/2, \quad k=1/4-\nu/2$ $l=3/4+\nu/2, \quad m=3/4-\lambda/2$
(19)	$x^{\lambda-1/2}e^{-\alpha x},$ $\operatorname{Re}\alpha>0,\ \operatorname{Re}(\lambda+\nu)>-2$	$\dfrac{y^{\nu+3/2}\,\Gamma(\lambda+\nu+2)}{2^\nu\alpha^{\lambda+\nu+2}\pi^{1/2}\,\Gamma(\nu+3/2)}\times$ $\times{}_3F_2\left(1,\ \frac{\lambda+\nu}{2}+1,\ \frac{\lambda+\nu+3}{2};\right.$ $\left.\frac{3}{2},\ \nu+\frac{3}{2};\ -\frac{y^2}{\alpha^2}\right)$
(20)	$x^{\lambda+1/2}\exp(-\alpha x^2),$ $\operatorname{Re}\alpha>0,\ \operatorname{Re}(\lambda+\nu)>-3$	$2^{-\nu-1}\pi^{-1/2}\alpha^{-(\lambda+\nu+3)/2}y^{\nu+3/2}\times$ $\times\dfrac{\Gamma\left(\frac{\lambda+\nu+3}{2}\right)}{\Gamma(\nu+3/2)}\times$ $\times{}_2F_2\left(1,\frac{\lambda+\nu+3}{2};\ \frac{3}{2},\nu+\frac{3}{2};\ -\frac{y^2}{4\alpha}\right)$

	$f(x)$	$\int_0^\infty f(x)\, \mathbf{H}_\nu(xy)\,(xy)^{1/2}\,dx, \quad y>0$
(21)	$x^{-\nu-1/2}\sin(ax)$, $a>0,\ \operatorname{Re}\nu>-1/2$	$0, \quad 0<y<a$ $\pi^{1/2}2^{-\nu}y^{1/2-\nu}[\Gamma(\nu+1/2)]^{-1}\times$ $\times(y^2-a^2)^{\nu-1/2}, \quad a<y<\infty$
(22)	$\dfrac{x^{1/2}\cos[(\nu+1)\theta]}{\sin\theta}, \quad 0<x<a$ $0, \quad a<x<\infty$ $0<\theta<\pi/2,\ x=a\cos\theta$ $\operatorname{Re}\nu>-2$	$\pi^{1/2}a^{1/2}\sin(ay/2)\,J_{\nu+1/2}(ay/2)$

11.3. Высшие трансцендентные функции

(1)	$J_{\nu+1/2}(ax)$, $a>0,\ -3/2<\operatorname{Re}\nu<1$	$0, \quad 0<y<a$ $\left(\frac{2}{\pi}\right)^{1/2}\left(\frac{y}{a}\right)^{\nu+1/2}(y^2-a^2)^{-1/2}$, $a<y<\infty$
(2)	$x^{-1/2}Y_{\nu+1}(ax)$, $a>0,\ -3/2<\operatorname{Re}\nu<3/2$	$0, \quad 0<y<a$ $-a^{-\nu-1}y^{1/2+\nu}, \quad a<y<\infty$
(3)	$x^{\mu-\nu+1/2}Y_\mu(ax)$, $a>0,\ \operatorname{Re}(\nu-\mu)>0$ $-3/2<\operatorname{Re}\mu<1/2$	$0, \quad 0<y<a$ $\dfrac{2^{1+\mu-\nu}a^\mu}{\Gamma(\nu-\mu)}y^{1/2-\nu}(y^2-a^2)^{\nu-\mu-1}$, $a<y<\infty$
(4)	$x^{1/2-\mu}[\sin(\mu\pi)J_{\mu+\nu}(ax)+$ $+\cos(\mu\pi)Y_{\mu+\nu}(ax)]$, $a>0,\ 1<\operatorname{Re}\mu<3/2$ $\operatorname{Re}\nu>-3/2,\ \operatorname{Re}(\nu-\mu)<1/2$	$0, \quad 0<y<a$ $\dfrac{y^{1/2+\nu}(y^2-a^2)^{\mu-1}}{2^{\mu-1}a^{\mu+\nu}\Gamma(\mu)}, \quad a<y<\infty$
(5)	$x^{\nu+1/2}J_\nu(ax)\,Y_\nu(ax)$, $a>0,\ -3/4<\operatorname{Re}\nu<0$	$\dfrac{\Gamma(2\nu+3/2)\,y^{\nu+3/2}}{\pi^{3/2}2^{\nu+2}a^{2\nu+3}\Gamma(\nu+2)}\times$ $\times{}_2F_1\left(1,\ 2\nu+\frac{3}{2};\ \nu+2;\ \frac{y^2}{4a^2}\right)$, $0<y<2a$
(6)	$x^{\nu+1/2}\{[J_\nu(ax)]^2-[Y_\nu(ax)]^2\}$, $a>0,\ -3/4<\operatorname{Re}\nu<0$	$0, \quad 0<y<2a$ $\dfrac{2^{3\nu+2}a^{2\nu}y^{-\nu-1/2}}{\pi^{1/2}\Gamma(1/2-\nu)}(y^2-4a^2)^{-\nu-1/2}$ $2a<y<\infty$

	$f(x)$	$\int_0^\infty f(x)\,\mathbf{H}_\nu(xy)\,(xy)^{1/2}\,dx, \quad y>0$
(7)	$x^{1/2}\{[J_{\nu/2}(ax)]^2-[Y_{\nu/2}(ax)]^2\}$, $a>0,\ {}^3/_2<\operatorname{Re}\nu<0$	$0, \quad 0<y<2a$ $4\pi^{-1}y^{-1/2}(y^2-4a^2)^{-1/2}, \quad 2a<y<\infty$
(8)	$x^{1/2}[J_{\nu/2+\mu/2}(ax)\,J_{\nu/2-\mu/2}(ax)-$ $-Y_{\nu/2+\mu/2}(ax)\,Y_{\nu/2-\mu/2}(ax)]$, $a>0,\ -{}^3/_2<\operatorname{Re}\nu<0$	$0, \quad 0<y<2a$ $4\pi^{-1}y^{-1/2}(y^2-4a^2)^{-1/2}\operatorname{ch}(\mu u)$, $y=2a\operatorname{ch}u,\ u>0$
(9)	$J_{2\nu+1}(ax^{1/2})$, $a>0,\ -{}^3/_2<\operatorname{Re}\nu<{}^1/_4$	$-\dfrac{a}{2y^{3/2}}Y_{\nu+1}\left(\dfrac{a^2}{4y}\right)$
(10)	$x^{-1/2}J_{2\nu}(ax^{1/2})$, $a>0,\ -1<\operatorname{Re}\nu<{}^5/_4$	$-y^{-1/2}Y_\nu\left(\dfrac{a^2}{4y}\right)$
(11)	$x^{1/2}[\{J_{\nu/2}[b(z-\alpha)]\}^2-$ $-\{Y_{\nu/2}[b(z+\alpha)]\}^2]$, $\operatorname{Re}\alpha>0,\ b>0$ $-{}^3/_2<\operatorname{Re}\nu<1$ $z=(x^2+\alpha^2)^{1/2}$	$-\dfrac{4\sin[\alpha(4b^2-y^2)^{1/2}]}{\pi y^{1/2}(4b^2-y^2)^{1/2}}, \quad 0<y<2b$ $\dfrac{4\exp[-\alpha(y^2-4b^2)^{1/2}]}{\pi y^{1/2}(y^2-4b^2)^{1/2}}, \quad 2b<y<\infty$
(12)	$x^{1/2}K_\nu(\alpha x)$, $\operatorname{Re}\alpha>0,\ \operatorname{Re}\nu>-{}^3/_2$	$\alpha^{-\nu-1}y^{\nu+3/2}(y^2+\alpha^2)^{-1}$
(13)	$x^{\mu+\nu+1/2}K_\mu(\alpha x)$, $\operatorname{Re}\alpha>0,\ \operatorname{Re}\nu>-{}^3/_2$ $\operatorname{Re}(\mu+\nu)>-{}^3/_2$	$2^{\mu+\nu+1}\pi^{-1/2}\alpha^{-\mu-2\nu-3}\times$ $\times y^{\nu+3/2}\Gamma(\mu+\nu+{}^3/_2)\times$ $\times {}_2F_1\left(1,\mu+\nu+\dfrac{3}{2};\dfrac{3}{2};-\dfrac{y^2}{\alpha^2}\right)$
(14)	$x^{\mu-\nu+1/2}K_\mu(\alpha x)$, $\operatorname{Re}\alpha>0,\ \operatorname{Re}\mu>-{}^3/_2$	$\dfrac{2^{\mu-\nu}y^{\nu+3/2}\Gamma(\mu+{}^3/_2)}{\alpha^{\mu+3}\Gamma(\nu+{}^3/_2)}\times$ $\times {}_2F_1\left(1,\mu+\dfrac{3}{2};\nu+\dfrac{3}{2};-\dfrac{y^2}{\alpha^2}\right)$
(15)	$x^{\sigma-1/2}K_\mu(\alpha x)$, $\operatorname{Re}\alpha>0$ $\operatorname{Re}(\sigma+\nu)>\|\operatorname{Re}\mu\|-2$	$2^\sigma\pi^{-1/2}\alpha^{-\nu-\sigma-2}y^{\nu+3/2}\times$ $\times\dfrac{\Gamma\left(1+\dfrac{\nu+\sigma+\mu}{2}\right)\Gamma\left(1+\dfrac{\nu+\sigma-\mu}{2}\right)}{\Gamma(\nu+{}^3/_2)}\times$ $\times {}_3F_2\left(1,\ 1+\dfrac{\nu+\sigma+\mu}{2},1+\dfrac{\nu+\sigma-\mu}{2};\right.$ $\left.\dfrac{3}{2},\ \nu+\dfrac{3}{2};-\dfrac{y^2}{\alpha^2}\right)$

	$f(x)$	$\int_0^\infty f(x)\,\mathbf{H}_\nu(xy)\,(xy)^{1/2}\,dx, \quad y>0$
(16)	$x^{\sigma-1/2}e^{-\alpha x}K_\nu(\alpha x)$	См. Mohan Brij, 1942: Bull. Calcutta Math. Soc., 34, 55—59.
(17)	$x^{\nu+1/2}[K_\nu(\alpha x)]^2$, $\operatorname{Re}\alpha>0,\ \operatorname{Re}\nu>-{}^3/_4$	$\pi^{1/2}2^{-\nu-3}\alpha^{-2\nu-3}y^{\nu+3/2}\times$ $\times\,\Gamma(2\nu+{}^3/_2)\,[\Gamma(\nu+2)]^{-1}\times$ $\times\,{}_2F_1\left(1,\ 2\nu+\frac{3}{2};\ \nu+2;\ -\frac{y^2}{4\alpha^2}\right)$
(18)	$x^{1/2}[K_\mu(\alpha x)]^2$, $\nu=0$ $\operatorname{Re}\alpha>0,\ -{}^3/_2<\operatorname{Re}\mu<{}^3/_2$	$-\frac{\pi}{2^{\mu+1}\alpha^{2\mu}y^{1/2}z\cos(\mu\pi)}\times$ $\times\,[(z+y)^{2\mu}+(z-y)^{2\mu}]$, $z=(y^2+4\alpha^2)^{1/2}$
(19)	$x^{-\nu-1/2}K_0(\alpha x)\,K_1(\alpha x)$, $\operatorname{Re}\alpha>0$	$\frac{\pi^{3/2}y^{\nu+3/2}}{2^{\nu+3}\alpha^2\,\Gamma(\nu+3/2)}\times$ $\times\,{}_2F_1\left(\frac{1}{2},\ \frac{1}{2};\ \nu+\frac{3}{2};\ -\frac{y^2}{4\alpha^2}\right)$
(20)	$x^{-\nu-1/2}K_\nu(\alpha x)\,K_{\nu+1}(\alpha x)$, $\operatorname{Re}\alpha>0,\ \operatorname{Re}\nu<{}^1/_2$	$\pi^{1/2}2^{-\nu-2}\alpha^{-2}y^{\nu+3/2}\,\Gamma({}^1/_2-\nu)\times$ $\times\,{}_2F_1\left(\frac{1}{2},\ \frac{1}{2}-\nu;\ \frac{3}{2};\ -\frac{y^2}{4\alpha^2}\right)$
(21)	$x^{\sigma-3/2}K_\lambda(\alpha x)\,K_\mu(\alpha x)$, $\operatorname{Re}\alpha>0$ $\operatorname{Re}(\sigma+\nu)>\|\operatorname{Re}\lambda\|+\|\operatorname{Re}\mu\|$	$\frac{2^{\sigma-3}y^{\nu+3/2}\,\Gamma\left(\frac{\sigma+\nu+\lambda+\mu}{2}\right)}{\pi^{1/2}\alpha^{\sigma+\nu}\,\Gamma(\nu+3/2)\,\Gamma(\sigma+\nu)}\times$ $\times\,\Gamma\left(\frac{\sigma+\nu+\lambda-\mu}{2}\right)\Gamma\left(\frac{\sigma+\nu-\lambda+\mu}{2}\right)\times$ $\times\,\Gamma\left(\frac{\sigma+\nu-\lambda-\mu}{2}\right)\times$ $\times\,{}_5F_4\left(1,\ \frac{\sigma+\nu+\lambda+\mu}{2},\ \frac{\sigma+\nu+\lambda-\mu}{2},\right.$ $\frac{\sigma+\nu-\lambda+\mu}{2},\ \frac{\sigma+\nu-\lambda-\mu}{2};$ $\left.\frac{3}{2},\ \nu+\frac{3}{2},\ \frac{\sigma+\nu}{2},\ \frac{\sigma+\nu+1}{2};\ -\frac{y^2}{4\alpha^2}\right)$
(22)	$x^{\nu+3/2}K_\nu(\alpha x)\,K_{\nu+1}(\alpha x)$, $\operatorname{Re}\alpha>0,\ \operatorname{Re}\nu>-{}^5/_4$	$\pi^{1/2}2^{-\nu-3}\alpha^{-2\nu-4}y^{\nu+3/2}\times$ $\times\,\Gamma(2\nu+{}^5/_2)\,[\Gamma(\nu+2)]^{-1}\times$ $\times\,{}_2F_1\left(1,\ 2\nu+\frac{5}{2};\ \nu+2;\ -\frac{y^2}{4\alpha^2}\right)$

	$f(x)$	$\int_0^\infty f(x)\, \mathbf{H}_\nu(xy)(xy)^{1/2}\,dx, \quad y>0$
(23)	$x^{\sigma-5/2}\exp(-2^{-1}\alpha^2x^2)\times$ $\times K_\mu(2^{-1}\alpha^2x^2)$, $\lvert\arg\alpha\rvert<\pi/4$ $\mathrm{Re}(\sigma+\nu)>2\lvert\mathrm{Re}\,\mu\rvert$	$2^{-\nu-1}\alpha^{-\nu-\sigma}y^{\nu+3/2}\times$ $\times\dfrac{\Gamma\left(\frac{\nu+\sigma}{2}+\mu\right)\Gamma\left(\frac{\nu+\sigma}{2}-\mu\right)}{\Gamma\left(\nu+\frac{3}{2}\right)\Gamma\left(\frac{\nu+\sigma}{2}\right)}\times$ $\times{}_3F_3\left(1, \frac{\nu+\sigma}{2}+\mu, \frac{\nu+\sigma}{2}-\mu; \frac{3}{2}, \nu+\frac{3}{2}, \frac{\nu+\sigma}{2}; -\frac{y^2}{4\alpha^2}\right)$
(24)	$x^{1/2}\exp\left(\frac{\alpha^2x^2}{8}\right)K_{\nu/2}\left(\frac{\alpha^2x^2}{8}\right)$, $\lvert\arg\alpha\rvert<3\pi/4$ $-3/2<\mathrm{Re}\,\nu<0$	$2\pi^{-1/2}\alpha^{-\nu/2-1}y^{\nu/2-1/2}\times$ $\times\cos(\nu\pi/2)\,\Gamma(-\nu/2)\times$ $\times\exp\left(\frac{y^2}{2\alpha^2}\right)W_{k,m}\left(\frac{y^2}{\alpha^2}\right)$, $k=\nu/4,\ m=1/2+\nu/4$
(25)	$x^\sigma\exp(\alpha x^2)K_\mu(\alpha x^2)$, $\lvert\arg\alpha\rvert<3\pi/2,\ \mathrm{Re}\,\sigma<1$ $\lvert\mathrm{Re}\,\mu\rvert-5/2<\mathrm{Re}(\sigma+\nu)<1/2$	$\pi^{-1/2}2^{-1-\sigma/2}\alpha^{-1/2-\sigma/2}\cos(\mu\pi)\times$ $\times G_{34}^{23}\left(\frac{y^2}{8\alpha}\middle\vert \begin{matrix} l, \frac{1-\sigma}{2}+\mu, \frac{1-\sigma}{2}-\mu \\ l, -\frac{\sigma}{2}, h, k\end{matrix}\right)$, $h=1/4+\nu/2,\ k=1/4-\nu/2,$ $l=3/4+\nu/2$
(26)	$K_{2\nu-1}(2\alpha x^{1/2})$, $\mathrm{Re}\,\alpha>0,\ \mathrm{Re}\,\nu>-1$	$2^{\nu+1}\pi^{-1}\alpha y^{-3/2}\Gamma(\nu+1)\times$ $\times S_{-\nu-2,\,\nu-1}(\alpha^2/y)$
(27)	$x^{-1/2}K_{2\nu}(2\alpha x^{1/2})$, $\mathrm{Re}\,\alpha>0,\ \mathrm{Re}\,\nu>-1$	$2^\nu\pi^{-1}y^{-1/2}\Gamma(\nu+1)\times$ $\times S_{-\nu-1,\,\nu}(\alpha^2/y)$
(28)	$x^{1/2}K_{2\nu}(2\alpha x^{1/2})$, $\mathrm{Re}\,\alpha>0,\ \mathrm{Re}\,\nu>-2$	$2^\nu\pi^{-1}\alpha^2y^{-5/2}\Gamma(\nu+2)\times$ $\times S_{-\nu-3,\,\nu}(\alpha^2/y)$
(29)	$x^\sigma K_\mu(2\alpha x^{1/2})$, $\mathrm{Re}\,\alpha>0$ $2\,\mathrm{Re}(\sigma+\nu)>\lvert\mathrm{Re}\,\mu\rvert-5$	$2^{2\sigma-1/2}\alpha^{-2\sigma-2}\pi^{-1}\times$ $\times G_{53}^{15}\left(\frac{4y^2}{\alpha^4}\middle\vert \begin{matrix} l, \beta_1, \beta_2, \beta_3, \beta_4 \\ l, h, k\end{matrix}\right)$, $h=1/4+\nu/2,\ k=1/4-\nu/2,$ $l=3/4+\nu/2$ $2\beta_1=1-\sigma+\mu/2,\ 2\beta_2=1-\sigma-\mu/2$ $2\beta_3=-\sigma+\mu/2,$ $2\beta_4=-\sigma-\mu/2$

	$f(x)$	$\int_0^\infty f(x)\,\mathbf{H}_\nu(xy)\,(xy)^{1/2}\,dx,\quad y>0$
(30)	$x^{-1/2}[2\pi^{-1}K_{2\nu}(2ax^{1/2})+$ $+Y_{2\nu}(2ax^{1/2})]$, $a>0,\ -1/2<\operatorname{Re}\nu<1/2$	$y^{-1/2}J_\nu(a^2/y)$
(31)	$x^{1/2-\nu}[J_{2\nu}(\alpha x^{1/2})-J_{-2\nu}(\alpha x^{1/2})]\times$ $\times K_{2\nu}(\alpha x^{1/2})$, $\lvert\arg\alpha\rvert<\pi/4$ $-3/2<\operatorname{Re}\nu<3/2$	$2^\nu\pi^{-1/2}\alpha^{1-2\nu}y^{2\nu-2}\times$ $\times\sin(\nu\pi)\,K_{\nu+1/2}\left(\frac{\alpha^2}{2y}\right)$
(32)	$x^{1/2}Y_\nu(\alpha x^{1/2})K_\nu(\alpha x^{1/2})$, $\lvert\arg\alpha\rvert<\pi/4,\ \operatorname{Re}\nu>-3/2$	$\frac{1}{2y^{3/2}}\exp\left(-\frac{\alpha^2}{2y}\right)$
(33)	$x^{\nu-1/2}Y_{2\nu-1}(\alpha x^{1/2})K_{2\nu-1}(\alpha x^{1/2})$, $\lvert\arg\alpha\rvert<\pi/4,\ \operatorname{Re}\nu>-1/4$	$\frac{\alpha^{2\nu-1}}{\pi^{1/2}2^\nu y^{2\nu}}K_{\nu-1/2}\left(\frac{\alpha^2}{2y}\right)$
(34)	$x^{\nu+1/2}Y_{2\nu}(\alpha x^{1/2})K_{2\nu}(\alpha x^{1/2})$, $\lvert\arg\alpha\rvert<\pi/4,\ \operatorname{Re}\nu>-3/4$	$\frac{\alpha^{2\nu+1}}{\pi^{1/2}2^{\nu+1}y^{2\nu+2}}K_{\nu-1/2}\left(\frac{\alpha^2}{2y}\right)$
(35)	$x^{-1/2}\{\cos[2^{-1}(\mu-\nu)\pi]\,J_\mu(\alpha x^{1/2})-$ $-\sin[2^{-1}(\mu-\nu)\pi]\,Y_\mu(\alpha x^{1/2})\}\times$ $\times K_\mu(\alpha x^{1/2})$, $\lvert\arg\alpha\rvert<\pi/4$ $\operatorname{Re}\nu>\lvert\operatorname{Re}\mu\rvert-2$	$\alpha^{-2}y^{1/2}W_{\nu/2,\,\mu/2}\left(\frac{\alpha^2}{2y}\right)\times$ $\times W_{-\nu/2,\,\mu/2}\left(\frac{\alpha^2}{2y}\right)$
(36)	$x^{\nu-1/2}K_{2\nu-1}(\alpha x^{1/2}e^{i\pi/4})\times$ $\times K_{2\nu-1}(\alpha x^{1/2}e^{-i\pi/4})$, $\operatorname{Re}\alpha>0,\ \operatorname{Re}\nu>-1/4$	$\pi^{-1/2}2^{3\nu-1}y^{-\nu-1/2}\times$ $\times\Gamma(\nu+1)\,\Gamma(2\nu+1/2)\times$ $\times S_{-3\nu-1/2,\,\nu-1/2}\left(\frac{\alpha}{2^{1/2}y^{1/2}}\right)$
(37)	$x^{-1/2}\mathbf{H}_\nu(a^2/x)$, $a>0,\ \operatorname{Re}\nu>-3/2$	$-y^{-1/2}J_{2\nu}(2ay^{1/2})$
(38)	$x^{-3/2}\mathbf{H}_{\nu-1}(a^2/x)$, $a>0,\ \operatorname{Re}\nu>-1/2$	$-a^{-1}J_{2\nu-1}(2ay^{1/2})$
(39)	$x^{-1/2}[J_{-\nu}(a^2/x)+$ $+\sin(\nu\pi)\,\mathbf{H}_\nu(a^2/x)]$, $a>0,\ -3/2<\operatorname{Re}\nu<0$	$y^{-1/2}[2\pi^{-1}K_{2\nu}(2ay^{1/2})-$ $-Y_{2\nu}(2ay^{1/2})]$

	$f(x)$	$\int\limits_0^\infty f(x)\,\mathbf{H}_\nu(xy)\,(xy)^{1/2}\,dx, \quad y>0$
(40)	$x^\sigma S_{\lambda,\mu}(\alpha x)$, $\lvert\arg\alpha\rvert<\pi$, $\operatorname{Re}(\lambda+\sigma)<1$, $\operatorname{Re}(\sigma+\nu)>\lvert\operatorname{Re}\mu\rvert-5/2$, $-3/2<\operatorname{Re}(\lambda+\nu+\sigma)<1/2$	$\dfrac{2^{\lambda+\sigma-1/2}\alpha^{-\sigma-1}}{\Gamma(1/2-\lambda/2-\mu/2)\,\Gamma(1/2-\lambda/2+\mu/2)}\times$ $\times G_{44}^{24}\left(\dfrac{y^2}{\alpha^2}\,\middle\vert\,\begin{matrix} l, -\frac{\lambda+\sigma}{2}, \frac{1-\sigma+\mu}{2}, \frac{1-\sigma-\mu}{2} \\ l, -\frac{\lambda+\sigma}{2}, h, k\end{matrix}\right)$, $h=1/4+\nu/2$, $k=1/4-\nu/2$, $l=3/4+\nu/2$
(41)	$x^{-\nu-1/2}\exp(-2^{-2}x^2)\times$ $\times[D_\mu(x)-D_\mu(-x)]$, $\operatorname{Re}(\mu+\nu)>-3/2$, $\operatorname{Re}>-1$	$\dfrac{2^{3/2}\,\Gamma(\mu/2+1/2)}{\Gamma(\mu/2+\nu+1)}\,y^{\mu+\nu+1/2}\sin(\mu\pi/2)\times$ $\times{}_1F_1\left(\dfrac{\mu}{2}+\dfrac{1}{2};\dfrac{\mu}{2}+\nu+1;-\dfrac{y^2}{2}\right)$
(42)	$x^{2\lambda}\exp(-2^{-2}x^2)\,M_{\varkappa,\mu}(2^{-1}x^2)$, $\operatorname{Re}(2\lambda+2\mu+\nu)>-7/2$, $\operatorname{Re}(\varkappa-\lambda)>0$, $\operatorname{Re}(2\lambda-2\varkappa+\nu)<-1/2$	$\dfrac{2^{-\lambda}\,\Gamma(2\mu+1)}{\Gamma(1/2+\varkappa+\mu)}\times$ $\times G_{34}^{22}\left(\dfrac{y^2}{2}\,\middle\vert\,\begin{matrix} l, -\mu-\lambda, \mu-\lambda \\ l, \varkappa-\lambda-1/2, h, k\end{matrix}\right)$, $h=1/4+\nu/2$, $k=1/4-\nu/2$, $l=3/4+\nu/2$
(43)	$x^{2\lambda}\exp(-2^{-2}x^2)\,W_{\varkappa,\mu}(2^{-1}x^2)$, $\operatorname{Re}(2\lambda+\nu)>2\lvert\operatorname{Re}\mu\rvert-7/2$	$2^{1/4-\lambda-\nu/2}\pi^{-1/2}y^{\nu+3/2}\times$ $\times\dfrac{\Gamma(7/4+\nu/2+\lambda+\mu)\,\Gamma(7/4+\nu/2+\lambda-\mu)}{\Gamma(\nu+3/2)\,\Gamma(9/4+\lambda-\varkappa-\nu/2)}\times$ $\times{}_3F_3(1, 7/4+\nu/2+\lambda+\mu,$ $7/4+\nu/2+\lambda-\mu;$ $3/2, \nu+3/2, 9/4+\lambda-\varkappa+\nu/2; -y^2/2)$
(44)	$x^{-1/2}\exp(2^{-1}x^2)\times$ $\times W_{-\nu/2-1/2,\,\nu/2}(x^2)$, $\operatorname{Re}\nu>-1$	$2^{-\nu-1}y^{\nu+1/2}\pi\exp(2^{-2}y^2)\operatorname{Erfc}(y/2)$
(45)	$x^{-1/2}\exp(2^{-2}x^2)\,W_{\varkappa,\,\nu/2}(2^{-1}x^2)$, $-3/2<\operatorname{Re}\nu<-2\operatorname{Re}\varkappa$, $\operatorname{Re}\varkappa<1/4$	$\dfrac{2^{\varkappa/2-\nu/4}\,y^{\nu/2-\varkappa-1/2}\,\Gamma(-\varkappa-\nu/2)}{\Gamma(1/2-\varkappa+\nu/2)\,\Gamma(1/2-\varkappa-\nu/2)}\times$ $\times\exp(2^{-2}y^2)\,W_{k,m}(2^{-1}y^2)$, $2k=\varkappa+\nu/2$, $2m=\varkappa+\nu/2+1$
(46)	$x^{2\lambda}\exp(2^{-2}x^2)\,W_{\varkappa,\lambda}(2^{-1}x^2)$, $\operatorname{Re}(2\lambda+\nu)>2\lvert\operatorname{Re}\mu\rvert-7/2$, $\operatorname{Re}(2\varkappa+2\lambda+\nu)<-1/2$, $\operatorname{Re}(\varkappa+\lambda)<0$	$[2^\lambda\,\Gamma(1/2-\varkappa+\mu)\,\Gamma(1/2-\varkappa-\mu)]^{-1}\times$ $\times G_{34}^{23}\left(\dfrac{y^2}{2}\,\middle\vert\,\begin{matrix} l, -\mu-\lambda, \mu-\lambda \\ l, -\varkappa-\lambda-1/2, h, k\end{matrix}\right)$, $h=1/4+\nu/2$, $k=1/4-\nu/2$, $l=3/4+\nu/2$

	$f(x)$	$\int\limits_0^\infty f(x)\,\mathbf{H}_\nu(xy)\,(xy)^{1/2}\,dx,\quad y>0$
(47)	$G_{pq}^{mn}\left(\lambda x^2\left\vert\begin{matrix}\alpha_1,\ \ldots,\ \alpha_p\\ \beta_1,\ \ldots,\ \beta_q\end{matrix}\right.\right)$, $p+q<2(m+n)$, $\lvert\arg\lambda\rvert<(m+n-p/2-q/2)\pi$ $\operatorname{Re}\alpha_j<\min(1,\ 3/4-\nu/2)$, $j=1,\ \ldots,\ n$ $\operatorname{Re}(2\beta_j+\nu)>-5/2$, $j=1,\ \ldots,\ m$	$(2\lambda)^{-1/2}\times$ $\times\, G_{q+1,\ p+3}^{n+1,\ m+1}\left(z\left\vert\begin{matrix}l,\ \beta_1',\ \ldots,\ \beta_q'\\ l,\ \alpha_1',\ \ldots,\ \alpha_p',\ h,\ k\end{matrix}\right.\right)$ $h=1/4+\nu/2,\ k=1/4-\nu/2,$ $l=3/4+\nu/2$ $z=\dfrac{y^2}{4\lambda},\ \alpha_j'=\dfrac{1}{2}-\alpha_j,\ \beta_j'=\dfrac{1}{2}-\beta_j$

ГЛАВА XII

ПРЕОБРАЗОВАНИЯ КОНТОРОВИЧА — ЛЕБЕДЕВА

М. И. Конторович и Н. Н. Лебедев (1938, 1939) вывели пару взаимнообратных формул

$$g(y) = \int_0^\infty f(x) K_{ix}(y)\, dx,$$

$$f(x) = 2\pi^{-2} x \operatorname{sh}(\pi x) \int_0^\infty g(y) K_{ix}(y)\, y^{-1}\, dy$$

и использовали их для решения некоторых краевых задач. Дальнейшие приложения к краевым задачам были даны Лебедевым и Конторовичем, а математическая теория была развита Лебедевым (1946, 1949). Следует отметить, что функция $K_{ix}(y)$ вещественна, если x вещественно и y положительно. Иная форма соотношений была установлена в отмеченных выше работах. См. также ВТФ, т. II, стр. 87.

В этой главе мы даем краткий список интегралов, соответствующих первой из указанных формул. Интегралы, соответствующие второй формуле, могут быть вычислены с помощью таблицы, приведенной в главе X. Мы считаем y положительным, хотя некоторые из приведенных ниже интегралов справедливы и при комплексных значениях y.

12.1. Формулы

	$f(x)$	$\int_0^\infty f(x) K_{ix}(y)\, dx, \quad y > 0$
(1)	$x \sin(\alpha x), \qquad \lvert \operatorname{Im} \alpha \rvert < \pi/2$	$2^{-1}\pi y \operatorname{sh} \alpha \exp(-y \operatorname{ch} \alpha)$
(2)	$\cos \alpha x, \qquad \lvert \operatorname{Im} \alpha \rvert < \pi/2$	$2^{-1}\pi \exp(-y \operatorname{ch} \alpha)$
(3)	$x \operatorname{th}(\pi x) P_{-1/2+ix}(z)$	$(2^{-1}\pi y)^{1/2} e^{-zy}$
(4)	$x \operatorname{th}(\pi x) K_{ix}(\beta), \qquad \lvert \arg \beta \rvert < \pi$	$2^{-1}\pi (\beta y)^{1/2} (\beta + y)^{-1} \exp(-\beta - y)$

	$f(x)$	$\int\limits_0^\infty f(x)\, K_{ix}(y)\, dx, \quad y>0$
(5)	$x\,\mathrm{sh}(\pi x)\, K_{2ix}(\alpha)$ $\lvert\arg\alpha\rvert<\pi/4$	$\frac{\pi^{3/2}\alpha}{2^{5/2}y^{1/2}} \exp\left(-y-\frac{\alpha^2}{8y}\right)$
(6)	$x\sin(\pi x/2)\, K_{ix/2}(\alpha)$ $\lvert\arg\alpha\rvert<\pi/2$	$\frac{\pi^{3/2}y}{2^{1/2}\alpha^{1/2}} \exp\left(-\alpha-\frac{y^2}{8\alpha}\right)$
(7)	$\mathrm{ch}(\alpha x)\, K_{ix}(\beta)$, $\lvert\mathrm{Re}\,\alpha\rvert+\lvert\arg\beta\rvert<\pi$	$2^{-1}\pi\, K_0\left[(y^2+\beta^2+2\beta y\cos\alpha)^{1/2}\right]$
(8)	$x(x^2+n^2)^{-1}\,\mathrm{sh}(\pi x)\, K_{ix}(a)$ $a>0,\ n=0,\ 1,\ 2,\ 3,\ \ldots$	$2^{-1}\pi^2 I_n(y)\, K_n(a), \quad 0<y<a$ $2^{-1}\pi^2 I_n(a)\, K_n(y), \quad a<y<\infty$
(9)	$x\,\mathrm{sh}(\pi x)\, K_{ix}(\alpha)\, K_{ix}(\beta)$, $\lvert\arg\alpha\rvert+\lvert\arg\beta\rvert<\pi/2$	$\frac{\pi^2}{4}\exp\left[-\frac{y}{2}\left(\frac{\alpha}{\beta}+\frac{\beta}{\alpha}+\frac{\alpha\beta}{y^2}\right)\right]$
(10)	$x\,\mathrm{sh}(\pi x/2)\, K_{ix/2}(\alpha)\, K_{ix/2}(\beta)$, $\lvert\arg\alpha\rvert+\lvert\arg\beta\rvert<\pi$	$\frac{\pi^2 y}{2z}\exp\left[-\frac{(\alpha+\beta)\,z}{2(\alpha\beta)^{1/2}}\right]$, $z=(y^2+4\alpha\beta)^{1/2}$
(11)	$x\,\mathrm{sh}(\pi x)\, K_{ix/2+\lambda}(a)\, K_{ix/2-\lambda}(a)$, $a>0$	$0, \quad 0<y<2a$ $\frac{\pi^2 y}{2^{2\lambda+1}a^{2\lambda}z}\left[(y+z)^{2\lambda}+(y-z)^{2\lambda}\right]$, $2a<y<\infty$ $z=(y^2-4a^2)^{1/2}$
(12)	$x\,\mathrm{sh}(\pi x)\,\Gamma(\lambda+ix)\,\times$ $\times\,\Gamma(\lambda-ix)\, K_{ix}(\alpha)$, $\lvert\arg\alpha\rvert<\pi,\ \mathrm{Re}\,\lambda>0$	$2^{\nu-1}\pi^{3/2}(\alpha y)^{\lambda}(y+\alpha)^{-\lambda}\,\times$ $\times\,\Gamma(\lambda+1/2)\, K_\lambda(y+\alpha)$
(13)	$x\,\mathrm{sh}(2\pi x)\,\Gamma(\lambda+ix)\,\times$ $\times\,\Gamma(\lambda-ix)\, K_{ix}(a)$, $a>0,\ 0<\mathrm{Re}\,\lambda<1/2$	$\frac{2^{\lambda}\pi^{5/2}}{\Gamma(1/2-\lambda)}\left(\frac{ay}{\lvert y-a\rvert}\right)^{\lambda} K_\lambda(\lvert y-a\rvert)$
(14)	$x\,\mathrm{sh}(\pi x)\,\Gamma(\lambda+ix/2)\,\times$ $\times\,\Gamma(\lambda-ix/2)\, K_{ix}(\alpha)$, $\lvert\arg\alpha\rvert<\pi/2,\ \mathrm{Re}\,\lambda>0$	$2\pi^2\left(\frac{\alpha y}{2z}\right)^{2\lambda} K_{2\lambda}(z)$, $z=(y^2+\alpha^2)^{1/2}$

	$f(x)$	$\int_0^\infty f(x) K_{ix}(y)\,dx, \quad y>0$
(15)	$\dfrac{x \operatorname{th}(\pi x) K_{ix}(\alpha)}{\Gamma(3/4 + ix/2)\,\Gamma(3/4 - ix/2)}$ $\lvert\arg\alpha\rvert < \pi/2$	$\frac{1}{2}\left(\frac{\pi\alpha y}{y^2+\alpha^2}\right)^{1/2} \exp[-(y^2+\alpha^2)^{1/2}]$
(16)	$x \operatorname{sh}(\pi x)\, \Gamma(\lambda + ix)\, \Gamma(\lambda - ix) \times$ $\times P^{1/2-\lambda}_{-1/2+ix}(\beta)\, K_{ix}(\alpha),$ $\lvert\arg\alpha\rvert < \pi/2$ $\lvert\arg(\beta - 1)\rvert < \pi$ $\operatorname{Re}\lambda > 0$	$2^{-1/2}\pi^{3/2}(\alpha y/z)^{\lambda}(\beta^2-1)^{\lambda/2-1/4} K_\lambda(z),$ $z = (y^2+\alpha^2+2\alpha\beta y)^{1/2}$

ГЛАВА XIII

ИНТЕГРАЛЫ ДРОБНОГО ПОРЯДКА

Мы называем

$$g(y;\ \mu) = \mathfrak{R}_\mu\{f(x);\ y\} = \frac{1}{\Gamma(\mu)} \int_0^y f(x)\,(y-x)^{\mu-1}\,dx \qquad (*)$$

интегралом Римана—Лиувилля (дробного) *порядка* μ от функции $f(x)$, а

$$h(y;\ \mu) = \mathfrak{W}_\mu\{f(x);\ y\} = \frac{1}{\Gamma(\mu)} \int_y^\infty f(x)\,(x-y)^{\mu-1}\,dx \qquad (**)$$

интегралом Вейля (дробного) *порядка* μ от функции $f(x)$. Вообще μ и y рассматриваются как комплексные переменные, причем путем интегрирования для преобразования (*) является отрезок $x = yt$, $0 < t < 1$, а для преобразования (**) — луч $x = yt$, $t > 1$, или $x = y + t$, $t > 0$.

Многие авторы обозначают $g(y;\ \mu)$ через $I^\mu f$ или $I^\mu_+ f$, а $h(y;\ \mu)$ через $K^\mu f$ или $K^\mu_- f$. Интеграл

$$\frac{1}{\Gamma(\mu)} \int_y^a f(x)\,(x-y)^{\mu-1}\,dx$$

иногда обозначают $I^\mu_- f$. Путем замены переменного его можно выразить в виде $\mathfrak{R}_\mu\{f(a-x);\ a-y\}$. С другой стороны, если положить $f(x) = 0$ при $x > a$, его можно записать в виде $\mathfrak{W}_\mu\{f(x);\ y\}$.

Производные дробного порядка α могут быть определены формулами

$$D^\alpha_0 f(x) = \frac{d^n}{dx^n} \mathfrak{R}_{n-\alpha}\{f(t);\ x\}, \qquad n-1 < \operatorname{Re}\alpha < n,$$

$$D^\alpha_\infty f(x) = \frac{d^n}{dx^n} \mathfrak{W}_{n-\alpha}\{f(t);\ x\}, \qquad n-1 < \operatorname{Re}\alpha < n,$$

так что таблицы интегралов дробного порядка можно использовать для вычисления производных дробного порядка.

В списке литературы указаны избранные книги и работы, содержащие информацию по теории интегралов и производных дробного порядка. В работе Харди и Литтлвуда (Hardy and Littlewood, 1928) содержатся дальнейшие ссылки. Насколько нам известно, не существует достаточно обширных таблиц интегралов дробного порядка, хотя почти во всех таблицах интегралов встречается много интегралов такого вида.

Расширение операторов $\mathfrak{R}_\mu$ и $\mathfrak{W}_\mu$ было введено Кобером (Kober, 1940) и Эрдейи (Erdélyi, 1940). Кобер (Kober, 1941b) рассмотрел также интегралы комплексного порядка. Интегрирование по частям дробного порядка по конечному отрезку выражается формулой

$$\int_0^a g_1(x;\ \mu)\, f_2(a-x)\, dx = \int_0^a f_1(a-x)\, g_2(x;\ \mu)\, dx$$

и было изучено Янгом и Лоувом (Young and Love, 1938). Для бесконечного промежутка формула имеет вид

$$\int_0^\infty f_1(x)\, g_2(x;\ \mu)\, dx = \int_0^\infty h_1(x;\ \mu)\, f_2(x)\, dx$$

и была изучена Кобером (Kober, 1940). В этих формулах $g_{1,2} = \mathfrak{R}f_{1,2}$ и $h_1 = \mathfrak{W}f_1$.

Операторы $\mathfrak{R}_\mu$ и $\mathfrak{W}_\mu$ связаны соотношениями с операторами дифференцирования и интегрирования, а также друг с другом. Мы укажем некоторые из этих соотношений. Остальные будут перечислены в списке общих формул в пп. 13.1 и 13.2.

$$g(x;\ 1) = \int_0^x f(t)\, dt, \qquad h(x;\ 1) = \int_x^\infty f(t)\, dt,$$

$$\frac{d}{dx}\, g(x;\ \mu) = g(x;\ \mu-1), \qquad -\frac{d}{dx}\, h(x;\ \mu) = h(x;\ \mu-1),$$

$$\mathfrak{R}_\mu \mathfrak{R}_\nu = \mathfrak{R}_{\mu+\nu}, \qquad \mathfrak{W}_\mu \mathfrak{W}_\nu = \mathfrak{W}_{\mu+\nu}.$$

Функции $g(x;\ \mu)$ и $h(x;\ \mu)$ можно рассматривать как μ раз взятый интеграл с постоянным пределом 0 в случае g и ∞ в случае h и переменным вторым пределом.

Связи между интегралами дробного порядка и другими интегральными преобразованиями выражаются следующими формулами:

$$\mathfrak{L}\{g(t;\ \mu);\ p\} = p^{-\mu}\,\mathfrak{L}\{f(t);\ p\},$$

$$\mathfrak{F}_e\{h(x;\ \mu);\ y\} = e^{\mu\pi i/2} y^{-\mu}\,\mathfrak{F}_e\{f(x);\ y\},$$

$$\mathfrak{M}\{g(x;\ \mu);\ s\} = \frac{\Gamma(1-s-\mu)}{\Gamma(1-s)}\,\mathfrak{M}\{f(x);\ s+\mu\},$$

$$\mathfrak{M}\{h(x;\ \mu);\ s\} = \frac{\Gamma(s)}{\Gamma(s+\mu)}\,\mathfrak{M}\{f(x);\ s+\mu\}.$$

Эти формулы могут быть использованы для вычисления интегралов дробного порядка при помощи таблиц преобразований Фурье, Лапласа, Меллина и их обращений. Эти формулы могут быть также использованы для того, чтобы вывести из известной пары, например преобразований Фурье, новую пару преобразований с помощью интегрирования дробного порядка.

Связь между интегралами дробного порядка и преобразованиями Лапласа была изучена Дёчем (Doetsch, 1937, стр. 293—305) и Уиддером (Widder,

1941. стр. 70—75). Дёч изучил также интегральное уравнение Абеля $g = \mathfrak{R}_\mu f$. Относительно связи между интегралами дробного порядка и преобразованиями Фурье см. Кобер (Kober, 1941a, лемма 3). Относительно связи между интегралами дробного порядка и преобразованиями Меллина см. Кобер (Kober, 1940). Относительно связи между интегралами дробного порядка и преобразованиями Ганкеля см. Эрдейи и Кобер (Erdélyi and Kober, 1940) и Эрдейи (Erdélyi, 1940); см. также 8.1 (13)—8.1 (16). Относительно приложений интегрирования дробного порядка к теории тригонометрических рядов см. Зигмунд (1965, т. II, стр. 200 и сл.).

Интегралы дробного порядка встречаются в выражениях решений линейных дифференциальных уравнений через определенные интегралы. В этом контексте интегралы дробного порядка часто называют *преобразованиями Эйлера* (см., например, Айнс, 1939, стр. 258 и далее). $\mathfrak{R}_\mu f$ является преобразованием Эйлера первого рода, а $\mathfrak{W}_\mu f$ преобразованием Эйлера второго рода функции f.

М. Рисс (M. Riesz, 1949) развил теорию интегрирования дробного порядка для функций многих переменных; она была применена Риссом и другими для решения дифференциальных уравнений в частных производных (см., например, Baker and Copson, 1950, гл. I, § 7).

Из формул для интегралов дробного порядка, приведенных в этой главе, могут быть получены новые формулы для таких интегралов с помощью общих методов, перечисленных во введении к первому тому, с помощью общих формул, установленных выше и в пп. 13.1, 13.2, а также с помощью указанных выше связей между интегрированием дробного порядка и другими интегральными преобразованиями.

13.1. Интегралы Римана — Лиувилля

	$f(x)$	$[\Gamma(\mu)]^{-1} \int_0^y f(x)(y-x)^{\mu-1}\,dx = g(y;\mu)$
(1)	$f(\alpha x)$	$\alpha^{-\mu} g(\alpha y;\mu)$
(2)	$f(\alpha/x)$	$\alpha y^{\mu-1}\mathfrak{W}_\mu\{t^{-\mu-1}f(t);\ \alpha/y\}$ Относительно таблиц см. п. 13.2.
(3)	$f'(x)$	$g(y;\mu-1) - \frac{f(0)\,y^{\mu-1}}{\Gamma(\mu)}$
(4)	$\int_0^x f(t)\,dt$	$g(y;\mu+1)$
(5)	$g(x;\nu)$	$g(y;\mu+\nu)$
(6)	$1, \qquad \operatorname{Re}\mu>0$	$\frac{y^\mu}{\Gamma(\mu+1)}$

	$f(x)$	$[\Gamma(\mu)]^{-1} \int\limits_0^y f(x)(y-x)^{\mu-1}\,dx$
(7)	$x^{\nu-1}$, $\quad \operatorname{Re}\mu>0,\ \operatorname{Re}\nu>0$	$\dfrac{\Gamma(\nu)}{\Gamma(\mu+\nu)} y^{\mu+\nu-1}$
(8)	$(x+\alpha)^{\nu}$, $\quad \operatorname{Re}\mu>0$	$\dfrac{\alpha^{\nu} y^{\mu}}{\Gamma(\mu+1)}\, {}_2F_1(1,\ -\nu;\ 1+\mu;\ -y/\alpha)$, $\quad \lvert\arg(y/\alpha)\rvert<\pi$
(9)	$x^{\nu-1}(x+\alpha)^{\lambda}$, $\quad \operatorname{Re}\mu>0,\ \operatorname{Re}\nu>0$	$\dfrac{\alpha^{\lambda} y^{\mu+\nu-1}\Gamma(\nu)}{\Gamma(\mu+\nu)}\times$ $\times\, {}_2F_1(-\lambda,\ \nu;\ \mu+\nu;\ -y/\alpha)$, $\quad \lvert\arg y/\alpha\rvert<\pi$
(10)	$x^{\nu-1}(x^2+\alpha^2)^{\lambda}$, $\quad \operatorname{Re}\mu>0,\ \operatorname{Re}\nu>0$	$\dfrac{\alpha^{2\lambda} y^{\mu+\nu-1}\Gamma(\nu)}{\Gamma(\mu+\nu)}\times$ $\times\, {}_3F_2\left(-\lambda,\ \dfrac{\nu}{2},\ \dfrac{\nu+1}{2};\ \dfrac{\mu+\nu}{2},\ \dfrac{\mu+\nu+1}{2};\ -\dfrac{y^2}{\alpha^2}\right)$, $\quad \operatorname{Re}(y/\alpha)>0$
(11)	$x^{\nu-1}(x^k+\alpha^k)^{\lambda}$, $\quad k=1,\ 2,\ \ldots$ $\quad \operatorname{Re}\mu>0,\ \operatorname{Re}\nu>0$	$\dfrac{\alpha^{k\lambda} y^{\mu+\nu+1}\Gamma(\nu)}{\Gamma(\mu+\nu)}\times$ $\times\, {}_{k+1}F_k\left(-\lambda,\ \dfrac{\nu}{k},\ \dfrac{\nu+1}{k},\ \ldots,\ \dfrac{\nu+k-1}{k};\ \dfrac{\mu+\nu}{k},\ \dfrac{\mu+\nu+1}{k},\ \ldots,\ \dfrac{\mu+\nu+k-1}{k};\ -\dfrac{y^k}{\alpha^k}\right)$, $\quad \lvert\arg(y/\alpha)\rvert<\pi/k$
(12)	$x^{-1/2}(x+2)^{-1/2}\left\{\left[(x+2)^{1/2}+x^{1/2}\right]^{2\nu}+\right.$ $\left.+\left[(x+2)^{1/2}-x^{1/2}\right]^{2\nu}\right\}$, $\quad \operatorname{Re}\mu>0$	$2^{\mu+1/2}\pi^{1/2}\,[y(y+2)]^{\mu/2-1/4}\times$ $\times\, P^{1/2-\mu}_{\nu-1/2}(y+1)$, $\quad \lvert\arg y\rvert<\pi$
(13)	$x^{\mu-1}e^{\alpha x}$, $\quad \operatorname{Re}\mu>0$	$\pi^{1/2}(y/\alpha)^{\mu-1/2}\exp(\alpha y/2)\times$ $\times\, I_{\mu-1/2}(\alpha y/2)$
(14)	$x^{\nu-1}e^{\alpha x}$, $\quad \operatorname{Re}\mu>0,\ \operatorname{Re}\nu>0$	$\dfrac{\Gamma(\nu)}{\Gamma(\mu+\nu)} y^{\mu+\nu-1}\, {}_1F_1(\nu;\ \mu+\nu;\ \alpha y)$
(15)	$x^{\nu-1}\exp(\alpha x^k)$, $\quad \operatorname{Re}\mu>0,\ \operatorname{Re}\nu>0$ $\quad k=2,\ 3,\ 4,\ \ldots$	$\dfrac{\Gamma(\nu)}{\Gamma(\mu+\nu)} y^{\mu+\nu-1}\times$ $\times\, {}_kF_k\left(\dfrac{\nu}{k},\ \dfrac{\nu+1}{k},\ \ldots,\ \dfrac{\nu+k-1}{k};\ \dfrac{\mu+\nu}{k},\ \dfrac{\mu+\nu+1}{k},\ \ldots,\ \dfrac{\mu+\nu+k-1}{k};\ \alpha y^k\right)$

	$f(x)$	$[\Gamma(\mu)]^{-1}\int_0^y f(x)(y-x)^{\mu-1}\,dx$
(16)	$x^{-\mu-1}\exp(-\alpha/x)$, $\quad\operatorname{Re}\mu>0$	$\alpha^{-\mu}y^{\mu-1}\exp(-\alpha/y)$, $\quad\lvert\arg y\rvert<\pi$
(17)	$x^{-2\mu}\exp(-\alpha/x)$, $\quad\operatorname{Re}\mu>0$	$(\pi y)^{-1/2}\alpha^{1/2-\mu}\exp\left(-\frac{\alpha}{2y}\right)\times$ $\times K_{\mu-1/2}\left(\frac{\alpha}{2y}\right)$, $\quad\operatorname{Re}(\alpha/y)>0$
(18)	$x^{\nu-1}\exp(-\alpha/x)$, $\quad\operatorname{Re}\mu>0$	$\alpha^{\nu/2-1/2}y^{-\varkappa}\exp\left(-\frac{\alpha}{2y}\right)W_{\varkappa,\,\nu/2}\left(\frac{\alpha}{y}\right)$, $\varkappa=1/2-\mu-\nu/2$, $\quad\operatorname{Re}(\alpha/y)>0$
(19)	$\exp(\alpha x^{1/2})$, $\quad\operatorname{Re}\mu>0$	$\frac{y^{\mu}}{\Gamma(\mu+1)}+\pi^{1/2}(\alpha/2)^{1/2-\mu}y^{\mu/2+1/4}\times$ $\times\left[I_{\mu+1/2}(\alpha y^{1/2})+\mathbf{L}_{\mu+1/2}(\alpha y^{1/2})\right]$
(20)	$x^{-1/2}\exp(\alpha x^{1/2})$	$\pi^{1/2}(\alpha/2)^{1/2-\mu}y^{\mu/2-1/4}\times$ $\times\left[I_{\mu-1/2}(\alpha y^{1/2})+\mathbf{L}_{\mu-1/2}(\alpha y^{1/2})\right]$
(21)	$x^{\nu-1}\exp(\alpha x^{1/2})$, $\quad\operatorname{Re}\mu>0,\ \operatorname{Re}\nu>0$	$\frac{y^{\mu+\nu-1}\Gamma(\nu)}{\Gamma(\mu+\nu)}\times$ $\times{}_1F_2\left(\nu;\ \frac{1}{2},\ \mu+\nu;\ \frac{\alpha^2 y}{4}\right)+$ $+\frac{\alpha y^{\mu+\nu-1/2}\Gamma(\nu+1/2)}{2\Gamma(\mu+\nu+1/2)}\times$ $\times{}_1F_2\left(\nu+\frac{1}{2};\ \frac{3}{2},\ \mu+\nu+\frac{1}{2};\ \frac{\alpha^2 y}{4}\right)$
(22)	$x^{-3/2}\exp(-\alpha x^{-1/2})$, $\quad\mu=1$	$2\alpha^{-1}\exp(-\alpha y^{-1/2})$
(23)	$x^{-\mu-1/2}\exp(-\alpha x^{-1/2})$, $\quad\operatorname{Re}\mu>0$	$2^{\mu+1/2}\pi^{-1/2}\alpha^{1/2-\mu}y^{\mu/2-3/4}\times$ $\times K_{\mu-1/2}(\alpha y^{-1/2})$, $\quad\operatorname{Re}(\alpha y^{-1/2})>0$
(24)	$x^{\nu-1}\ln x$, $\quad\operatorname{Re}\mu>0,\ \operatorname{Re}\nu>0$	$\frac{y^{\mu+\nu-1}\Gamma(\nu)}{\Gamma(\mu+\nu)}\times$ $\times[\ln y+\psi(\nu)-\psi(\mu+\nu)]$
(25)	$x^{\mu-1}\sin(\alpha x)$, $\quad\operatorname{Re}\mu>0$	$\pi^{1/2}(y/\alpha)^{\mu-1/2}\sin(\alpha y/2)J_{\mu-1/2}(\alpha y/2)$
(26)	$x^{\nu-1}\sin(\alpha x)$, $\quad\operatorname{Re}\mu>0,\ \operatorname{Re}\nu>-1$	$\frac{y^{\mu+\nu-1}\Gamma(\nu)}{2i\,\Gamma(\mu+\nu)}[{}_1F_1(\nu;\mu+\nu;i\alpha y)-$ $-{}_1F_1(\nu;\mu+\nu;-i\alpha y)]$

	$f(x)$	$[\Gamma(\mu)]^{-1}\int\limits_0^y f(x)(y-x)^{\mu-1}\,dx$
(27)	$\sin(\alpha x^{1/2})$, $\quad\operatorname{Re}\mu>0$	$2^{\mu-1/2}\pi^{1/2}\alpha^{1/2-\mu}y^{\mu/2+1/4}J_{\mu+1/2}(\alpha y^{1/2})$
(28)	$x^{-1/2}\sin(\alpha x^{1/2})$, $\quad\operatorname{Re}\mu>0$	$\pi^{1/2}2^{\mu-1/2}\alpha^{1/2-\mu}y^{\mu/2-1/4}\mathbf{H}_{\mu-1/2}(\alpha y^{1/2})$
(29)	$x^{\nu-1}\sin(\alpha x^{1/2})$, $\operatorname{Re}\mu>0,\ \operatorname{Re}\nu>-1/2$	$\dfrac{\alpha y^{\mu+\nu-1/2}\,\Gamma(\nu+1/2)}{\Gamma(\mu+\nu+1/2)}\times$ $\times\,{}_1F_2\left(\nu+\frac{1}{2};\ \frac{3}{2},\ \mu+\nu+\frac{1}{2};\ -\frac{\alpha^2y}{4}\right)$
(30)	$x^{-\mu-1/2}\sin(ax^{-1/2})$, $0<\operatorname{Re}\mu<1,\ a>0$	$2^{\mu-1/2}\pi^{1/2}a^{1/2-\mu}y^{\mu/2-3/4}J_{1/2-\mu}(ay^{-1/2})$, $\lvert\arg y\rvert<\pi$
(31)	$x^{\mu-1}\cos(\alpha x)$, $\quad\operatorname{Re}\mu>0$	$\pi^{1/2}(y/\alpha)^{\mu-1/2}\cos(\alpha y/2)\,J_{\mu-1/2}(\alpha y/2)$
(32)	$x^{\nu-1}\cos(\alpha x)$, $\operatorname{Re}\mu>0,\ \operatorname{Re}\nu>0$	$\dfrac{y^{\mu+\nu-1}\,\Gamma(\nu)}{2\,\Gamma(\mu+\nu)}\,[{}_1F_1(\nu;\ \mu+\nu;\ i\alpha y)+$ $+\,{}_1F_1(\nu;\ \mu+\nu;\ -i\alpha y)]$
(33)	$\cos(\alpha x^{1/2})$, $\quad\operatorname{Re}\mu>0$	$\dfrac{y^{\mu}}{\Gamma(\mu+1)}-2^{\mu-1/2}\alpha^{1/2-\mu}\pi^{1/2}\times$ $\times\,y^{\mu/2+1/4}\,\mathbf{H}_{\mu+1/2}(\alpha y^{1/2})$
(34)	$x^{-1/2}\cos(\alpha y^{1/2})$, $\quad\operatorname{Re}\mu>0$	$2^{\mu-1/2}\pi^{1/2}\alpha^{1/2-\mu}y^{\mu/2-1/4}J_{\mu-1/2}(\alpha y^{1/2})$
(35)	$x^{\nu-1}\cos(\alpha x^{1/2})$, $\operatorname{Re}\mu>0,\ \operatorname{Re}\nu>0$	$\dfrac{y^{\mu+\nu-1}\,\Gamma(\nu)}{\Gamma(\mu+\nu)}\times$ $\times\,{}_1F_2\left(\nu;\ \frac{1}{2},\ \mu+\nu;\ -\frac{\alpha^2y}{4}\right)$
(36)	$x^{-\mu-1/2}\cos(ax^{-1/2})$, $0<\operatorname{Re}\mu<1,\ a>0$	$-2^{\mu-1/2}\pi^{1/2}a^{1/2-\mu}y^{\mu/2-3/4}\times$ $\times\,Y_{1/2-\mu}(ay^{-1/2})$, $\quad\lvert\arg y\rvert<\pi$
(37)	$P_n(1-\gamma x)$, $\quad\operatorname{Re}\mu>0$	$\dfrac{n!\,y^{\mu}}{\Gamma(\mu+n+1)}\,P_n^{(\mu,\,-\mu)}(1-\gamma y)$
(38)	$x^{\nu-1}P_n(1-\gamma x)$, $\operatorname{Re}\mu>0,\ \operatorname{Re}\nu>0$	$\dfrac{\Gamma(\nu)\,y^{\mu+\nu-1}}{\Gamma(\mu+\nu)}\times$ $\times\,{}_3F_2(-n,\ n+1,\ \nu;\ 1,\ \mu+\nu;\ 2^{-1}\gamma y)$

	$f(x)$	$[\Gamma(\mu)]^{-1}\int\limits_0^y f(x)(y-x)^{\mu-1}\,dx$
(39)	$x^{\lambda-1/2}C_n^\lambda(1-\gamma x)$, $\operatorname{Re}\lambda>-1,\ \lambda\neq 0,\ -1/2$ $\operatorname{Re}\mu>0$	$\frac{(2\lambda)_n\,\Gamma(\lambda+1/2)}{\Gamma(\lambda+\mu+n+1/2)}\times$ $\times\, y^{\lambda+\mu-1/2}P_n^{(\alpha,\beta)}(1-\gamma y)$, $\alpha=\lambda+\mu-1/2,\quad\beta=\lambda-\mu-1/2$
(40)	$x^{\nu-1}C_n^\lambda(1-\gamma x)$, $2\lambda\neq 0,\ -1,\ -2,\ \ldots$ $\operatorname{Re}\mu>0,\ \operatorname{Re}\nu>0$	$\frac{(2\lambda)_n\,\Gamma(\nu)}{n!\,\Gamma(\mu+\nu)}y^{\mu+\nu-1}\times$ $\times\,{}_3F_2(-n,\ n+2\lambda,\ \nu;\ \lambda+1/2;$ $\mu+\nu;\ 2^{-1}\gamma y)$
(41)	$x^{\nu-1}C_{2n}^\lambda(\gamma x^{1/2})$, $\operatorname{Re}\mu>0,\ \operatorname{Re}\nu>0$	$(-1)^n\frac{(\lambda)_n\,\Gamma(\nu)}{n!\,\Gamma(\mu+\nu)}y^{\mu+\nu-1}\times$ $\times\,{}_3F_2(-n,\ n+\lambda,\ \nu;\ 1/2,\ \mu+\nu;\ \gamma^2 y)$
(42)	$x^{\nu-1}C_{2n+1}^\lambda(\gamma x^{1/2})$, $\operatorname{Re}\mu>0,\ \operatorname{Re}\nu>-1/2$	$2(-1)^n\gamma y^{\mu+\nu-1/2}\times$ $\times\frac{(\lambda)_{n+1}\,\Gamma(\nu+1/2)}{n!\,\Gamma(\mu+\nu+1/2)}\times$ $\times\,{}_3F_2(-n,\ n+\lambda+1,\ \nu+1/2;$ $3/2,\ \mu+\nu+1/2;\ \gamma^2 y)$
(43)	$x^\alpha P_n^{(\alpha,\beta)}(1-\gamma x)$, $\operatorname{Re}\alpha>-1,\ \operatorname{Re}\mu>0$	$\frac{\Gamma(\alpha+n+1)}{\Gamma(\alpha+\mu+n+1)}y^{\alpha+\mu}\times$ $\times\,P_n^{(\alpha+\mu,\,\beta-\mu)}(1-\gamma y)$
(44)	$x^\beta P_n^{(\alpha,\beta)}(\gamma x-1)$, $\operatorname{Re}\beta>-1,\ \operatorname{Re}\mu>0$	$\frac{\Gamma(\beta+n+1)}{\Gamma(\beta+\mu+n+1)}y^{\beta+\mu}\times$ $\times\,P_n^{(\alpha-\mu,\,\beta+\mu)}(\gamma y-1)$
(45)	$x^{-\beta-\mu-n-1}(1-2^{-1}\gamma x)^\beta\times$ $\times\,P_n^{(\alpha,\beta)}(1-\gamma x)$, $0<\operatorname{Re}\mu<-\operatorname{Re}\beta-n$	$\frac{\Gamma(-\beta-\mu-n)}{\Gamma(-\beta-\mu)}y^{-\beta-n-1}\times$ $\times(1-2^{-1}\gamma y)^{\beta+\mu}P_n^{(\alpha,\,\beta+\mu)}(1-\gamma y)$
(46)	$x^{\lambda-1}P_n^{(\alpha,\beta)}(1-\gamma x)$, $\operatorname{Re}\lambda>0,\ \operatorname{Re}\mu>0$	$\frac{\Gamma(\alpha+n+1)\,\Gamma(\lambda)}{n!\,\Gamma(\alpha+1)\,\Gamma(\lambda+\mu)}y^{\lambda+\mu-1}\times$ $\times\,{}_3F_2(-n,\ n+\alpha+\beta+1,\ \lambda;$ $\alpha+1,\ \lambda+\mu;\ 2^{-1}\gamma y)$
(47)	$x^{\lambda-1}P_n^{(\alpha,\beta)}(\gamma x-1)$, $\operatorname{Re}\lambda>0,\ \operatorname{Re}\mu>0$	$(-1)^n\frac{\Gamma(\beta+n+1)\,\Gamma(\lambda)}{n!\,\Gamma(\beta+1)\,\Gamma(\lambda+\mu)}y^{\lambda+\mu-1}\times$ $\times\,{}_3F_2(-n,\ n+\alpha+\beta+1,\ \lambda;$ $\beta+1,\ \lambda+\mu;\ 2^{-1}\gamma y)$

	$f(x)$	$[\Gamma(\mu)]^{-1}\int\limits_0^y f(x)(y-x)^{\mu-1}dx$
(48)	$x^{\lambda-1}(1-2^{-1}\gamma x)^{\beta}P_n^{\alpha,\beta}(1-\gamma x)$, $\operatorname{Re}\lambda>0$, $\operatorname{Re}\mu>0$	$\frac{\Gamma(n+\alpha+1)\Gamma(\lambda)}{n!\Gamma(\alpha+1)\Gamma(\lambda+\mu)}y^{\lambda+\mu-1}\times$ $\times{}_3F_2(\alpha+n+1,-\beta-n,\lambda;$ $\alpha+1,\lambda+\mu;2^{-1}\gamma y)$
(49)	$x^{\alpha}L_n^{\alpha}(\beta x)$, $\operatorname{Re}\alpha>-1$, $\operatorname{Re}\mu>0$	$\frac{\Gamma(\alpha+n+1)}{\Gamma(\alpha+\mu+n+1)}y^{\alpha+\mu}L_n^{\alpha+\mu}(\beta y)$
(50)	$x^{\lambda-1}L_n^{\alpha}(\beta x)$, $\operatorname{Re}\lambda>0$, $\operatorname{Re}\mu>0$	$\frac{\Gamma(\alpha+n+1)\Gamma(\lambda)}{n!\Gamma(\alpha+1)\Gamma(\lambda+\mu)}y^{\lambda+\mu-1}\times$ $\times{}_2F_2(-n,\lambda;\alpha+1,\lambda+\mu;\beta y)$
(51)	$x^{\lambda-1}e^{-\beta x}L_n^{\alpha}(\beta x)$, $\operatorname{Re}\lambda>0$, $\operatorname{Re}\mu>0$	$\frac{\Gamma(\alpha+n+1)\Gamma(\lambda)}{n!\Gamma(\alpha+1)\Gamma(\lambda+\mu)}y^{\lambda+\mu-1}\times$ $\times{}_2F_2(\alpha+n+1,\lambda;$ $\alpha+1,\lambda+\mu;-\beta y)$
(52)	$[x(1+2^{-1}\gamma x)]^{-\lambda/2}P_\nu^{\lambda}(1+\gamma x)$, $\operatorname{Re}\lambda<1$, $\operatorname{Re}\mu>0$	$(2/\gamma)^{\mu/2}[y(1+2^{-1}\gamma y)]^{\mu/2-\lambda/2}\times$ $\times P_\nu^{\lambda-\mu}(1+\gamma y)$, $\|\arg\gamma y\|<\pi$
(53)	$x^{\varkappa+\lambda/2-1}(1+2^{-1}\gamma x)^{-\lambda/2}\times$ $\times P_\nu^{\lambda}(1+\gamma x)$, $\operatorname{Re}\varkappa>0$, $\operatorname{Re}\mu>0$	$\frac{(\gamma/2)^{-\lambda/2}\Gamma(\varkappa)}{\Gamma(1-\lambda)\Gamma(\varkappa+\mu)}y^{\varkappa+\mu-1}\times$ $\times{}_3F_2(-\nu,1+\nu,\varkappa;$ $1-\lambda,\varkappa+\mu;-2^{-1}\gamma y)$, $\|\gamma y\|<1$
(54)	$[x(1-x)]^{-\lambda/2}\mathrm{P}_\nu^{\lambda}(1-2x)$, $\operatorname{Re}\lambda<1$, $\operatorname{Re}\mu>0$	$[y(1-y)]^{\mu/2-\lambda/2}\mathrm{P}_\nu^{\lambda-\mu}(1-2y)$, $0<y<1$
(55)	$x^{\varkappa+\lambda/2-1}(1-x)^{-\lambda/2}\mathrm{P}_\nu^{\lambda}(1-2x)$, $\operatorname{Re}\varkappa>0$, $\operatorname{Re}\mu>0$	$\frac{\Gamma(\varkappa)y^{\varkappa+\mu-1}}{\Gamma(\varkappa+\mu)\Gamma(1-\lambda)}\times$ $\times{}_3F_2(-\nu,1+\nu,\varkappa;1-\lambda,\varkappa+\mu;y)$, $0<y<1$
(56)	$x^{\lambda-1}J_\nu(ax)$, $\operatorname{Re}\mu>0$, $\operatorname{Re}(\lambda+\nu)>0$	$\frac{\Gamma(\lambda+\nu)}{\Gamma(\nu+1)\Gamma(\lambda+\mu+\nu)}(a/2)^{\nu}\times$ $\times y^{\lambda+\mu+\nu-1}{}_2F_3\left(\frac{\lambda+\nu}{2},\frac{\lambda+\nu+1}{2};\right.$ $\left.\nu+1,\frac{\lambda+\mu+\nu}{2},\frac{\lambda+\mu+\nu+1}{2};-\frac{a^2y^2}{4}\right)$

	$f(x)$	$[\Gamma(\mu)]^{-1}\int\limits_0^y f(x)(y-x)^{\mu-1}\,dx$
(57)	$x^{\nu}e^{\pm i\alpha x}J_{\nu}(\alpha x)$, $\operatorname{Re}\mu>0$, $\operatorname{Re}\nu>-1/2$	$\dfrac{(2\alpha)^{\nu}y^{\mu+2\nu}\Gamma(\nu+1/2)}{\pi^{1/2}\Gamma(\mu+2\nu+1)}\times$ $\times{}_1F_1(\nu+1/2;\ \mu+2\nu+1;\ \pm 2i\alpha y)$
(58)	$x^{\lambda-\nu-1}e^{\pm i\alpha x}J_{\nu}(\alpha x)$, $\operatorname{Re}\lambda>0$, $\operatorname{Re}\mu>0$	$\dfrac{2^{-\nu}\alpha^{\nu}y^{\lambda+\mu-1}\Gamma(\lambda)}{\Gamma(\lambda+\mu)\Gamma(\nu+1)}\times$ $\times{}_2F_2(\lambda,\ \nu+1/2;\ \lambda+\mu,$ $2\nu+1;\ \pm 2i\alpha y)$
(59)	$x^{-1/2}J_{2\nu}(\alpha x^{1/2})$, $\mu=1/2$, $\operatorname{Re}\nu>-1/2$	$\pi^{1/2}[J_{\nu}(2^{-1}\alpha y^{1/2})]^2$
(60)	$x^{\nu/2-1/2}J_{\nu}(\alpha x^{1/2})$, $\mu=\nu+1/2$, $\operatorname{Re}\nu>-1/2$	$\pi^{1/2}\left(\dfrac{2y}{\alpha}\right)^{\nu}[J_{\nu}(2^{-1}\alpha y^{1/2})]^2$
(61)	$x^{\nu/2-1/2}J_{\nu}(\alpha x^{1/2})$, $\mu=\nu-1/2$, $\operatorname{Re}\nu>1/2$	$(\alpha/2)^{1-\nu}\pi^{1/2}y^{\nu-1/2}J_{\nu}(2^{-1}\alpha y^{1/2})\times$ $\times J_{\nu-1}(2^{-1}\alpha y^{1/2})$
(62)	$x^{-\nu/2-1/2}J_{\nu}(\alpha x^{1/2})$, $\mu=1/2-\nu$, $\operatorname{Re}\nu<1/2$	$\pi^{1/2}\left(\dfrac{\alpha}{2y}\right)^{\nu}J_{\nu}(2^{-1}\alpha y^{1/2})J_{-\nu}(2^{-1}\alpha y^{1/2})$
(63)	$x^{\nu/2}J_{\nu}(\alpha x^{1/2})$, $\operatorname{Re}\mu>0$, $\operatorname{Re}\nu>-1$	$2^{\mu}\alpha^{-\mu}y^{\mu/2+\nu/2}J_{\mu+\nu}(\alpha y^{1/2})$
(64)	$x^{-\nu/2}J_{\nu}(\alpha x^{1/2})$, $\operatorname{Re}\mu>0$	$\dfrac{2^{2-\nu}\alpha^{-\mu}y^{\mu/2-\nu/2}}{\Gamma(\mu)\Gamma(\nu)}\times$ $\times s_{\mu+\nu-1,\,\mu-\nu}(\alpha y^{1/2})$
(65)	$x^{\lambda-\nu/2-1}J_{\nu}(\alpha x^{1/2})$, $\operatorname{Re}\lambda>0$, $\operatorname{Re}\mu>0$	$\dfrac{\alpha^{\nu}y^{\lambda+\mu-1}\Gamma(\lambda)}{2^{\nu}\Gamma(\lambda+\mu)\Gamma(\nu+1)}\times$ $\times{}_1F_2(\lambda;\ \nu+1,\ \lambda+\mu;\ -2^{-2}\alpha^2 y)$
(66)	$x^{\lambda-\nu-1}[J_{\nu}(\alpha x^{1/2})]^2$, $\operatorname{Re}\lambda>0$, $\operatorname{Re}\mu>0$. Относительно многих частных случаев см. Bailey W. N., 1938; Quart. J. Math. Oxford Ser., 9, 141—147.	$\dfrac{(\alpha/2)^{2\nu}y^{\lambda+\mu-1}\Gamma(\lambda)}{[\Gamma(\nu+1)]^2\Gamma(\lambda+\mu)}\times$ $\times{}_2F_3(\lambda,\ \nu+1/2;\ \lambda+\mu,$ $\nu+1,\ 2\nu+1;\ -\alpha^2 y)$
(67)	$x^{\lambda-1}J_{\nu}(\alpha x^{1/2})J_{-\nu}(\alpha x^{1/2})$, $\operatorname{Re}\lambda>0$, $\operatorname{Re}\mu>0$	$\dfrac{\Gamma(\lambda)\sin(\nu\pi)}{\nu\pi\,\Gamma(\lambda+\mu)}y^{\lambda+\mu-1}\times$ $\times{}_2F_3(1/2,\ \lambda;\ 1+\nu,\ 1-\nu,$ $\lambda+\mu;\ -\alpha^2 y)$

	$f(x)$	$[\Gamma(\mu)]^{-1}\int\limits_0^y f(x)(y-x)^{\mu-1}\,dx$
(68)	$x^{-1/2}Y_\nu(\alpha x^{1/2})$, $\mu=1/2$, $-1<\operatorname{Re}\nu<1$	$\pi^{1/2}\operatorname{ctg}(\nu\pi)\left[J_{\nu/2}(2^{-1}\alpha y^{1/2})\right]^2-\frac{\pi^{1/2}[J_{-\nu/2}(2^{-1}\alpha y^{1/2})]^2}{\sin(\nu\pi)}$
(69)	$x^{\nu/2}Y_\nu(\alpha x^{1/2})$, $\mu=1/2$, $\operatorname{Re}\nu>-1$	$\frac{2^{1/2}y^{\nu/2+1/4}}{\alpha^{1/2}\sin(\nu\pi)}\times\left[\cos(\nu\pi)J_{\nu+1/2}(\alpha y^{1/2})-\mathbf{H}_{-\nu-1/2}(\alpha y^{1/2})\right]$
(70)	$x^{-\nu/2}Y_\nu(\alpha x^{1/2})$, $\mu=1/2$, $\operatorname{Re}\nu<1$	$\frac{2^{1/2}y^{1/4-\nu/2}}{\alpha^{1/2}\sin(\nu\pi)}\times\left[\cos(\nu\pi)\mathbf{H}_{\nu-1/2}(\alpha y^{1/2})-J_{1/2-\nu}(\alpha y^{1/2})\right]$
(71)	$x^{\nu/2-1/2}Y_\nu(\alpha x^{1/2})$, $\mu=\nu+1/2$, $\operatorname{Re}\nu>-1/2$	$\pi^{1/2}\left(\frac{2y}{\alpha}\right)^\nu J_\nu(2^{-1}\alpha y^{1/2})\,Y_\nu(2^{-1}\alpha y^{1/2})$
(72)	$x^{-\nu/2-1/2}Y_\nu(\alpha x^{1/2})$, $\mu=1/2-\nu$, $\operatorname{Re}\nu<1/2$	$\pi^{1/2}\left(\frac{\alpha}{2y}\right)^\nu J_{-\nu}(2^{-1}\alpha y^{1/2})\,Y_{-\nu}(2^{-1}\alpha y^{1/2})$
(73)	$x^{\nu/2}Y_\nu(\alpha x^{1/2})$, $\operatorname{Re}\mu>0$, $\operatorname{Re}\nu>-1$	$\left(\frac{2}{\alpha}\right)^\mu y^{\mu/2+\nu/2}\operatorname{ctg}(\nu\pi)J_{\mu+\nu}(\alpha y^{1/2})+\frac{2^{\nu+2}y^{\mu/2+\nu/2}\Gamma(\nu+1)}{\pi\alpha^\mu\Gamma(\mu)}\times s_{\mu-\nu-1,\,\mu+\nu}(\alpha y^{1/2})$
(74)	$x^{\nu/2}Y_\nu(\alpha x^{1/2})$, $\operatorname{Re}\mu>0$, $\operatorname{Re}\nu>-1$	$\left(\frac{2}{\alpha}\right)^\mu y^{\mu/2+\nu/2}Y_{\mu+\nu}(\alpha y^{1/2})+\frac{2^{\nu+2}y^{\mu/2+\nu/2}\Gamma(\nu+1)}{\pi\alpha^\mu\Gamma(\mu)}\times S_{\mu-\nu-1,\,\mu+\nu}(\alpha y^{1/2})$
(75)	$x^{-\nu/2}Y_\nu(\alpha x^{1/2})$, $\operatorname{Re}\mu>0$, $\operatorname{Re}\nu<1$	$\frac{y^{\mu/2-\nu/2}\operatorname{ctg}(\nu\pi)}{2^{\nu-2}\alpha^\mu\Gamma(\mu)\Gamma(\nu)}\times s_{\mu+\nu-1,\,\mu-\nu}(\alpha y^{1/2})-\frac{2^\mu y^{\mu/2-\nu/2}J_{\mu-\nu}(\alpha y^{1/2})}{\alpha^\mu\sin(\nu\pi)}$

	$f(x)$	$[\Gamma(\mu)]^{-1}\int_0^y f(x)(y-x)^{\mu-1}\,dx$
(76)	$x^{\lambda-1}Y_\nu(\alpha x^{1/2})$, $\operatorname{Re}\lambda>2^{-1}\lvert\operatorname{Re}\nu\rvert$, $\operatorname{Re}\mu>0$	$2^{-\nu}\alpha^\nu y^{\lambda+\mu+\nu/2-1}\operatorname{ctg}(\nu\pi)\times \times\frac{\Gamma(\lambda+\nu/2)}{\Gamma(1+\nu)\Gamma(\lambda+\mu+\nu/2)}\times \times{}_1F_2(\lambda+\nu/2;\ 1+\nu,\ \lambda+\mu+\nu/2;\ -2^{-2}\alpha^2y)-\frac{2^\nu y^{\lambda+\mu-\nu/2-1}}{\alpha^\nu\sin(\nu\pi)}\times \times\frac{\Gamma(\lambda-\nu/2)}{\Gamma(1-\nu)\Gamma(\lambda+\mu-\nu/2)}\times \times{}_1F_2(\lambda-\nu/2;\ 1-\nu,\ \lambda+\mu-\nu/2;\ -2^{-2}\alpha^2y)$
(77)	$x^\nu e^{\pm\alpha x}I_\nu(\alpha x)$, $\operatorname{Re}\mu>0$, $\operatorname{Re}\nu>-1/2$	$\frac{(2\alpha)^\nu y^{\mu+2\nu}\Gamma(\nu+1/2)}{\pi^{1/2}\Gamma(\mu+2\nu+1)}\times \times{}_1F_1(\nu+1/2;\ \mu+2\nu+1;\ \pm2\alpha y)$
(78)	$x^{\lambda-1}e^{\pm\alpha x}I_\nu(\alpha x)$, $\operatorname{Re}\mu>0$, $\operatorname{Re}(\lambda+\nu)>0$	$\frac{(\alpha/2)^\nu y^{\lambda+\mu+\nu-1}\Gamma(\lambda+\nu)}{\Gamma(\nu+1)\Gamma(\lambda+\mu+\nu)}\times \times{}_2F_2(\nu+1/2,\ \lambda+\nu;\ 2\nu+1,\ \mu+\lambda+\nu;\ \pm2\alpha y)$
(79)	$x^{-1/2}I_{2\nu}(\alpha x^{1/2})$, $\mu=1/2$, $\operatorname{Re}\nu>-1/2$	$\pi^{1/2}[I_\nu(2^{-1}\alpha y^{1/2})]^2$
(80)	$x^{\nu/2-1/2}I_\nu(\alpha x^{1/2})$, $\mu=\nu+1/2$, $\operatorname{Re}\nu>-1/2$	$\pi^{1/2}\left(\frac{2y}{\alpha}\right)^\nu[I_\nu(2^{-1}\alpha y^{1/2})]^2$
(81)	$x^{\nu/2-1/2}I_\nu(\alpha x^{1/2})$, $\mu=\nu-1/2$, $\operatorname{Re}\nu>1/2$	$(\alpha/2)^{1-\nu}\pi^{1/2}y^{\nu-1/2}I_\nu(2^{-1}\alpha y^{1/2})\times \times I_{\nu-1}(2^{-1}\alpha y^{1/2})$
(82)	$x^{-\nu/2-1/2}I_\nu(\alpha x^{1/2})$, $\mu=1/2-\nu$, $\operatorname{Re}\nu<1/2$	$\pi^{1/2}\left(\frac{\alpha}{2y}\right)^\nu I_\nu(2^{-1}\alpha y^{1/2})I_{-\nu}(2^{-1}\alpha y^{1/2})$
(83)	$x^{\nu/2}I_\nu(\alpha x^{1/2})$, $\operatorname{Re}\mu>0$, $\operatorname{Re}\nu>-1$	$2^\mu\alpha^{-\mu}y^{\mu/2+\nu/2}I_{\mu+\nu}(\alpha y^{1/2})$
(84)	$x^{\lambda-\nu/2-1}I_\nu(\alpha x^{1/2})$, $\operatorname{Re}\lambda>0$, $\operatorname{Re}\mu>0$	$\frac{\alpha^\nu y^{\lambda+\mu-1}\Gamma(\lambda)}{2^\nu\Gamma(\nu+1)\Gamma(\lambda+\mu)}\times \times{}_1F_2(\lambda;\ \nu+1,\ \lambda+\mu;\ 2^{-2}\alpha^2y)$

	$f(x)$	$[\Gamma(\mu)]^{-1}\int_0^y f(x)(y-x)^{\mu-1}\,dx$
(85)	$x^{-1/2}K_{2\nu}(\alpha x^{1/2})$, $\mu=1/2,\ -1/2<\operatorname{Re}\nu<1/2$	$\dfrac{I_\nu(2^{-1}\alpha y^{1/2})+I_{-\nu}(2^{-1}\alpha y^{1/2})}{2\cos(\nu\pi)}\times$ $\times K_\nu(2^{-1}\alpha y^{1/2})$
(86)	$x^{\nu/2-1/2}K_\nu(\alpha x^{1/2})$, $\mu=\nu+1/2,\ \operatorname{Re}\nu>-1/2$	$\pi^{1/2}(2y/\alpha)^\nu I_\nu(2^{-1}\alpha y^{1/2})K_\nu(2^{-1}\alpha y^{1/2})$
(87)	$x^{\lambda-1}K_\nu(\alpha x^{1/2})$, $\operatorname{Re}\lambda>2^{-1}\lvert\operatorname{Re}\nu\rvert,\ \operatorname{Re}\mu>0$	$2^{\nu-1}\alpha^{-\nu}y^{\lambda+\mu-\nu/2-1}\dfrac{\Gamma(\nu)\Gamma(\lambda-\nu/2)}{\Gamma(\lambda+\mu-\nu/2)}\times$ $\times{}_1F_2\left(\lambda-\frac{\nu}{2};\,1-\nu,\,\lambda+\mu-\frac{\nu}{2};\,\frac{\alpha^2y}{4}\right)+$ $+2^{1-\nu}\alpha^\nu y^{\lambda+\mu+\nu/2-1}\times$ $\times\dfrac{\Gamma(-\nu)\Gamma(\lambda+\nu/2)}{\Gamma(\lambda+\mu+\nu/2)}\times$ $\times{}_1F_2\left(\lambda+\frac{\nu}{2};\,1+\nu,\,\lambda+\mu+\frac{\nu}{2};\,\frac{\alpha^2y}{4}\right)$
(88)	$x^{\nu/2}\mathbf{H}_\nu(\gamma x^{1/2})$, $\operatorname{Re}\nu>-3/2,\ \operatorname{Re}\mu>0$	$(\gamma/2)^{-\mu}y^{\mu/2+\nu/2}\mathbf{H}_{\mu+\nu}(\gamma y^{1/2})$
(89)	$x^{\lambda-\nu/2-3/2}\mathbf{H}_\nu(\gamma x^{1/2})$, $\operatorname{Re}\lambda>0,\ \operatorname{Re}\mu>0$	$\dfrac{\Gamma(\lambda)\gamma^{\nu+1}y^{\lambda+\mu-1}}{2^\nu\pi^{1/2}\Gamma(\nu+3/2)\Gamma(\lambda+\mu)}\times$ $\times{}_2F_3\left(1,\,\lambda;\,\frac{3}{2},\,\nu+\frac{3}{2},\,\lambda+\mu;\,-\frac{\gamma^2y}{4}\right)$
(90)	$x^{\nu/2}\mathbf{L}_\nu(\gamma x^{1/2})$, $\operatorname{Re}\mu>0,\ \operatorname{Re}\nu>-3/2$	$(\gamma/2)^{-\mu}y^{\mu/2+\nu/2}\mathbf{L}_{\mu+\nu}(\gamma y^{1/2})$
(91)	$x^{\lambda-\nu/2-3/2}\mathbf{L}_\nu(\gamma x^{1/2})$, $\operatorname{Re}\lambda>0,\quad\operatorname{Re}\mu>0$	$\dfrac{\gamma^{\nu+1}y^{\lambda+\mu-1}\Gamma(\lambda)}{2^\nu\pi^{1/2}\Gamma(\nu+3/2)\Gamma(\lambda+\mu)}\times$ $\times{}_2F_3\left(1,\,\lambda;\,\frac{3}{2},\,\nu+\frac{3}{2},\,\lambda+\mu;\,\frac{\gamma^2y}{4}\right)$
(92)	$x^{\lambda-\varkappa/2-1/2}s_{\varkappa,\nu}(\alpha x^{1/2})$, $\operatorname{Re}\lambda>-1,\quad\operatorname{Re}\mu>0$	$\dfrac{\alpha^\varkappa y^{\lambda+\mu}\Gamma(\lambda+1)}{(\varkappa-\nu+1)(\varkappa+\nu+1)\Gamma(\lambda+\mu+1)}\times$ $\times{}_2F_3\left(1,\,\lambda+1;\,\frac{\varkappa-\nu+3}{2},\,\frac{\varkappa+\nu+3}{2},\right.$ $\left.\lambda+\mu+1;\,-\frac{\alpha^2y}{4}\right)$

	$f(x)$	$[\Gamma(\mu)]^{-1}\int\limits_0^y f(x)(y-x)^{\mu-1}\,dx$
(93)	$x^{\varkappa-\mu-1}e^{-\alpha x/2}W_{\varkappa,\lambda}(\alpha x)$, $0<\operatorname{Re}\mu<\operatorname{Re}\varkappa-\lvert\operatorname{Re}\lambda\rvert+1/2$	$\dfrac{y^{\varkappa-1}}{e^{\alpha y/2}\cos[(\varkappa-\mu-\lambda)\pi]}\times\left\{\sin(\mu\pi)\dfrac{\Gamma(\varkappa-\mu+\lambda+1/2)}{\Gamma(2\lambda+1)}\times M_{\varkappa-\mu,\lambda}(\alpha y)+\cos[(\varkappa-\lambda)\pi]W_{\varkappa-\mu,\lambda}(\alpha y)\right\}$
(94)	$x^{\nu-1}{}_pF_q(a_1,\ldots,a_p;\ \nu,b_2,\ldots,b_q;\ ax)$, $p\leqslant q+1$, $\operatorname{Re}\mu>0$, $\operatorname{Re}\nu>0$	$\dfrac{y^{\mu+\nu-1}\Gamma(\nu)}{\Gamma(\mu+\nu)}\times{}_pF_q(a_1,\ldots,a_p;\ \mu+\nu,b_2,\ldots,b_q;\ ay)$, $\lvert ay\rvert<1$, если $p=q+1$
(95)	$x^{\nu-1}{}_pF_q(a_1,\ldots,a_p;\ b_1,\ldots,b_q;\ ax)$, $p\leqslant q+1$, $\operatorname{Re}\mu>0$, $\operatorname{Re}\nu>0$	$\dfrac{y^{\mu+\nu-1}\Gamma(\nu)}{\Gamma(\mu+\nu)}\times{}_{p+1}F_{q+1}(\nu,a_1,\ldots,a_p;\ \mu+\nu,b_1,\ldots,b_q;\ ay)$, $\lvert ay\rvert<1$, если $p=q+1$
(96)	$G_{pq}^{mn}\left(ax\,\middle\vert\,\begin{matrix}a_1,\ldots,a_p\\ b_1,\ldots,b_q\end{matrix}\right)$, $p\leqslant q$, $\operatorname{Re}\mu>0$, $\operatorname{Re}b_j>-1$, $j=1,\ldots,m$	$y^{\mu}G_{p+1,q+1}^{m,n+1}\left(ay\,\middle\vert\,\begin{matrix}0,a_1,\ldots,a_p\\ b_1,\ldots,b_q,-\mu\end{matrix}\right)$, $\lvert ay\rvert<1$, если $p=q$
(97)	$G_{pq}^{mn}\left(\alpha x\,\middle\vert\,\begin{matrix}a_1,\ldots,a_p\\ b_1,\ldots,b_q\end{matrix}\right)$, $p+q<2(m+n)$, $\operatorname{Re}\mu>0$, $\operatorname{Re}b_j>-1$, $j=1,\ldots,m$	$y^{\mu}G_{p+1,q+1}^{m,n+1}\left(\alpha y\,\middle\vert\,\begin{matrix}0,a_1,\ldots,a_p\\ b_1,\ldots,b_q,-\mu\end{matrix}\right)$, $\lvert\arg\alpha y\rvert<(m+n-p/2-q/2)\pi$

13.2. Интегралы Вейля

	$f(x)$	$[\Gamma(\mu)]^{-1}\int\limits_y^\infty f(x)(x-y)^{\mu-1}\,dx=h(y;\mu)$
(1)	$f(\alpha x)$	$\alpha^{-\mu}h(\alpha y;\mu)$
(2)	$f(\alpha/x)$	$\alpha y^{\mu-1}\,\Re_{\mu}\{t^{-\mu-1}f(t);\ \alpha/y\}$ Относительно таблиц см. п. 13.1.
(3)	$f'(x)$	$-h(y;\mu-1)$

	$f(x)$	$[\Gamma(\mu)]^{-1}\int\limits_0^y f(x)(x-y)^{\mu-1}\,dx = h(y,\mu)$
(4)	$\int\limits_x^\infty f(t)\,dt$	$h(y;\ \mu+1)$
(5)	$h(x;\ \nu)$	$h(y;\ \mu+\nu)$
(6)	$x^{-\lambda}$, $\quad 0<\operatorname{Re}\mu<\operatorname{Re}\lambda$	$\frac{\Gamma(\lambda-\mu)}{\Gamma(\lambda)}y^{\mu-\lambda}$
(7)	$(x+\alpha)^{-\lambda}$, $\quad 0<\operatorname{Re}\mu<\operatorname{Re}\lambda$	$\frac{\Gamma(\lambda-\mu)}{\Gamma(\lambda)}(y+\alpha)^{\mu-\lambda}$, $\quad \|\arg(y/\alpha)\|<\pi$
(8)	$x^{-\lambda}(x+\alpha)^{\nu}$, $\quad 0<\operatorname{Re}\mu<\operatorname{Re}(\lambda-\nu)$	$y^{\mu+\nu-\lambda}\frac{\Gamma(\lambda-\mu-\nu)}{\Gamma(\lambda-\nu)}\times$ $\times{}_2F_1(-\nu,\lambda-\mu-\nu;\lambda-\nu;\ -\alpha/y)$, $\quad \|\arg(\alpha/y)\|<\pi$ или $\|\alpha/y\|<1$
(9)	$x^{-\lambda}(x^2+\alpha^2)^{\nu}$, $\quad 0<\operatorname{Re}\mu<\operatorname{Re}(\lambda-2\nu)$	$\frac{\Gamma(\lambda-\mu-2\nu)}{\Gamma(\lambda-\mu)}y^{\mu-\lambda+2\nu}\times$ $\times{}_3F_2\left(-\nu,\frac{\lambda-\mu}{2}-\nu,\frac{1+\lambda-\mu}{2}-\nu;\ \frac{\lambda}{2}-\nu,\frac{1+\lambda}{2}-\nu;\ -\frac{\alpha^2}{y^2}\right)$, $\quad \|y\|>\|\alpha\|$ или $\operatorname{Re}(\alpha/y)>0$
(10)	$(x^2-1)^{-1/2}[(x+1)^{1/2}-(x-1)^{1/2}]^{2\nu}$, $\quad 0<\operatorname{Re}\mu<1+\operatorname{Re}\nu$	$2^{\nu+1/2}\pi^{-1/2}e^{(\mu-1/2)\pi i}\times$ $\times(y^2-1)^{\mu/2-1/4}Q_{\nu-1/2}^{1/2-\mu}(y)$, $\quad \|\arg(y-1)\|<\pi$
(11)	$e^{-\alpha x}$, $\quad \operatorname{Re}\mu>0$	$\alpha^{-\mu}e^{-\alpha y}$, $\quad \operatorname{Re}(\alpha y)>0$
(12)	$x^{\mu-1}e^{-\alpha x}$, $\quad \operatorname{Re}\mu>0$	$\pi^{-1/2}(y/\alpha)^{\mu-1/2}\exp(-2^{-1}\alpha y)\times$ $\times K_{\mu-1/2}(2^{-1}\alpha y)$, $\quad \operatorname{Re}(\alpha y)>0$
(13)	$x^{-\lambda}e^{-\alpha x}$, $\quad \operatorname{Re}\mu>0$	$\alpha^{\nu-1/2}y^{-\nu-1/2}\exp(-2^{-1}\alpha y)\times$ $\times W_{\varkappa,\nu}(\alpha y)$, $\quad 2\varkappa=1-\lambda-\mu,\quad 2\nu=\lambda-\mu$, $\quad \operatorname{Re}(\alpha y)>0$
(14)	$x^{-2\mu}\exp(\alpha/x)$, $\quad \operatorname{Re}\mu>0$	$(\pi/y)^{1/2}\alpha^{1/2-\mu}\exp\left(\frac{\alpha}{2y}\right)I_{\mu-1/2}\left(\frac{\alpha}{2y}\right)$

	$f(x)$	$[\Gamma(\mu)]^{-1}\int_y^\infty f(x)(x-y)^{\mu-1}\,dx$
(15)	$x^{-\lambda}\exp(\alpha/x)$, $0<\operatorname{Re}\mu<\operatorname{Re}\lambda$	$\frac{\Gamma(\lambda-\mu)}{\Gamma(\lambda)}y^{\mu-\lambda}{}_1F_1(\lambda-\mu;\ \lambda;\ \alpha/y)$
(16)	$\exp(-\alpha x^{1/2})$, $\operatorname{Re}\mu>0$	$2^{\mu+1/2}\pi^{-1/2}\alpha^{1/2-\mu}\times$ $\times y^{\mu/2+1/4}K_{\mu+1/2}(\alpha y^{1/2})$, $\operatorname{Re}(\alpha y^{1/2})>0$
(17)	$x^{-1/2}\exp(-\alpha x^{1/2})$, $\operatorname{Re}\mu>0$	$2^{\mu+1/2}\pi^{-1/2}\alpha^{1/2-\mu}\times$ $\times y^{\mu/2-1/4}K_{\mu-1/2}(\alpha y^{1/2})$, $\operatorname{Re}(\alpha y^{1/2})>0$
(18)	$x^{-\lambda}\ln x$, $0<\operatorname{Re}\mu<\operatorname{Re}\lambda$	$\frac{\Gamma(\lambda-\mu)}{\Gamma(\lambda)}y^{\mu-\lambda}\times$ $\times[\ln y+\psi(\lambda)-\psi(\lambda-\mu)]$
(19)	$\sin(ax)$, $a>0$, $0<\operatorname{Re}\mu<1$	$a^{-\mu}\sin(ay+2^{-1}\mu\pi)$
(20)	$x^{\mu-1}\sin(ax)$, $a>0$, $0<\operatorname{Re}\mu<1/2$	$2^{-1}\pi^{1/2}(y/a)^{\mu-1/2}\times$ $\times\left[\cos(2^{-1}ay)J_{1/2-\mu}(2^{-1}ay)-\right.$ $\left.-\sin(2^{-1}ay)Y_{1/2-\mu}(2^{-1}ay)\right]$
(21)	$x^{-2\mu}\sin(\alpha/x)$, $\operatorname{Re}\mu>0$	$\left(\frac{\pi}{y}\right)^{1/2}\alpha^{1/2-\mu}\sin\left(\frac{\alpha}{2y}\right)J_{\mu-1/2}\left(\frac{\alpha}{2y}\right)$
(22)	$\sin(ax^{1/2})$, $a>0$, $0<\operatorname{Re}\mu<1/2$	$2^{\mu-1/2}\pi^{1/2}a^{1/2-\mu}y^{\mu/2+1/4}Y_{-1/2-\mu}(ay^{1/2})$
(23)	$x^{-1/2}\sin(ax^{1/2})$, $a>0$, $0<\operatorname{Re}\mu<1$	$2^{\mu-1/2}\pi^{1/2}a^{1/2-\mu}y^{\mu/2-1/4}J_{1/2-\mu}(ay^{1/2})$
(24)	$\cos(ax)$, $a>0$, $0<\operatorname{Re}\mu<1$	$a^{-\mu}\cos(ay+2^{-1}\mu\pi)$
(25)	$x^{\mu-1}\cos(ax)$, $a>0$, $0<\operatorname{Re}\mu<1/2$	$-2^{-1}\pi^{1/2}(y/a)^{\mu-1/2}\times$ $\times\left[\sin(2^{-1}ay)J_{1/2-\mu}(2^{-1}ay)+\right.$ $\left.+\cos(2^{-1}ay)Y_{1/2-\mu}(2^{-1}ay)\right]$
(26)	$x^{-2\mu}\cos(\alpha/x)$, $\operatorname{Re}\mu>0$	$\left(\frac{\pi}{y}\right)^{1/2}\alpha^{1/2-\mu}\cos\left(\frac{\alpha}{2y}\right)J_{\mu-1/2}\left(\frac{\alpha}{2y}\right)$

	$f(x)$	$[\Gamma(\mu)]^{-1}\int_y^\infty f(x)\,(x-y)^{\mu-1}\,dx$
(27)	$\cos(ax^{1/2})$, $a>0$, $0<\operatorname{Re}\mu<1/2$	$2^{\mu-1/2}\pi^{1/2}a^{1/2-\mu}\,y^{\mu/2+1/4}J_{-1/2-\mu}(ay^{1/2})$
(28)	$x^{-1/2}\cos(ax^{1/2})$, $a>0$, $0<\operatorname{Re}\mu<1$	$-2^{\mu-1/2}\pi^{1/2}a^{1/2-\mu}\,y^{\mu/2-1/4}Y_{1/2-\mu}(ay^{1/2})$
(29)	$Q_\nu(x)$, $0<\operatorname{Re}\mu<1+\operatorname{Re}\nu$	$e^{\mu\pi i}\,(y^2-1)^{\mu/2}\,Q_\nu^{-\mu}(y)$, $\|\arg(y-1)\|<\pi$
(30)	$(x^2-1)^{\lambda/2}\,Q_\nu^{-\lambda}(x)$, $0<\operatorname{Re}\mu<1+\operatorname{Re}(\nu-\lambda)$	$e^{\mu\pi i}\,(y^2-1)^{\lambda/2+\mu/2}\,Q_\nu^{-\lambda-\mu}(y)$, $\|\arg(y-1)\|<\pi$
(31)	$x^{-\nu}\,e^{iax}\,J_\nu(ax)$, $a>0$, $0<\operatorname{Re}\mu<1/2+\operatorname{Re}\nu$	$\frac{\exp(2^{-1}i\mu\pi)\,(2a)^{\nu-\mu}\,\Gamma(1/2-\mu+\nu)}{\pi^{1/2}\,\Gamma(1-\mu+2\nu)}\times$ $\times\,{}_1F_1(1/2-\mu+\nu;\ 1-\mu+2\nu;\ 2aiy)$, $y>0$
(32)	$x^{\nu/2-1/2}\,J_\nu(ax^{1/2})$, $a>0$, $\mu=\nu+1/2$, $-1/2<\operatorname{Re}\nu<1/2$	$-\frac{\pi^{1/2}}{2}\left(\frac{2y}{a}\right)^\nu\left[J_\nu\left(\frac{ay^{1/2}}{2}\right)Y_{-\nu}\left(\frac{ay^{1/2}}{2}\right)+\right.$ $\left.+J_{-\nu}\left(\frac{ay^{1/2}}{2}\right)Y_\nu\left(\frac{ay^{1/2}}{2}\right)\right]$, $y>0$
(33)	$x^{\nu/2-1/2}\,J_{-\nu}(ax^{1/2})$, $a>0$, $\mu=\nu+1/2$, $-1/2<\operatorname{Re}\nu<1/2$	$-\pi^{1/2}\left(\frac{2y}{a}\right)^\nu J_{-\nu}\left(\frac{ay^{1/2}}{2}\right)Y_{-\nu}\left(\frac{ay^{1/2}}{2}\right)$, $y>0$
(34)	$x^{-\nu/2}\,J_\nu(ax^{1/2})$, $a>0$, $0<\operatorname{Re}\mu<2^{-1}\operatorname{Re}\nu+3/4$	$2^\mu a^{-\mu}y^{\mu/2-\nu/2}\,J_{\nu-\mu}(ay^{1/2})$, $y>0$
(35)	$x^{-\nu/2}\,J_{-\nu}(ax^{1/2})$, $a>0$, $0<\operatorname{Re}\mu<2^{-1}\operatorname{Re}\nu+3/4$	$2^\mu a^{-\mu}y^{\mu/2-\nu/2}[\cos(\nu\pi)\,J_{\nu-\mu}(ay^{1/2})-$ $-\sin(\nu\pi)\,Y_{\nu-\mu}(ay^{1/2})]$, $y>0$
(36)	$x^\lambda\,J_\nu(ax^{1/2})$, $a>0$, $0<\operatorname{Re}\mu<1/4-\operatorname{Re}\lambda$	$2^{2\lambda}a^{-2\lambda}y^\mu\times$ $\times\,G_{13}^{20}\left(\frac{a^2y}{4}\,\middle\|\,\begin{matrix}0\\ -\mu,\ \lambda+\nu/2,\ \lambda-\nu/2\end{matrix}\right)$, $y>0$
(37)	$x^{-\nu}\,[J_\nu(ax^{1/2})]^2$, $a>0$, $\mu=\nu-1/2$, $\operatorname{Re}\nu>1/2$	$\pi^{-1/2}a^{-\nu}y^{-\nu/2-1/2}\,\mathbf{H}_\nu(2ay^{1/2})$, $y>0$

	$f(x)$	$[\Gamma(\mu)]^{-1}\int\limits_y^\infty f(x)(x-y)^{\mu-1}\,dx$
(38)	$x^{\nu/2-1/2}Y_\nu(ax^{1/2})$, $a>0$ $\mu=\nu+1/2$, $-1/2<\operatorname{Re}\nu<1/2$	$\frac{\pi^{1/2}}{2}\left(\frac{2y}{a}\right)^\nu\left[J_\nu\left(\frac{ay^{1/2}}{2}\right)J_{-\nu}\left(\frac{ay^{1/2}}{2}\right)-\right.$ $\left.-Y_\nu\left(\frac{ay^{1/2}}{2}\right)Y_{-\nu}\left(\frac{ay^{1/2}}{2}\right)\right]$, $y>0$
(39)	$x^{\nu/2}Y_\nu(ax^{1/2})$, $a>0$ $0<\operatorname{Re}\mu<3/4-2^{-1}\operatorname{Re}\nu$	$2^\mu a^{-\mu}y^{\mu/2+\nu/2}\times$ $\times[\cos(\nu\pi)Y_{-\mu-\nu}(ay^{1/2})-$ $-\sin(\nu\pi)J_{-\mu-\nu}(ay^{1/2})]$, $y>0$
(40)	$x^{-\nu/2}Y_\nu(ax^{1/2})$, $a>0$ $0<\operatorname{Re}\mu<2^{-1}\operatorname{Re}\nu+3/4$	$2^\mu a^{-\mu}y^{\mu/2-\nu/2}Y_{\nu-\mu}(ay^{1/2})$, $y>0$
(41)	$x^{-\nu}J_\nu(ax^{1/2})Y_\nu(ax^{1/2})$, $a>0$ $\mu=\nu-1/2$, $\operatorname{Re}\nu>1/2$	$-\pi^{-1/2}a^{-\nu}y^{-\nu/2-1/2}J_\nu(2ay^{1/2})$
(42)	$x^{-\nu}[Y_\nu(ax^{1/2})]^2$, $a>0$ $\mu=\nu-1/2$, $\operatorname{Re}\nu>1/2$	$\pi^{-1/2}a^{-\nu}y^{-\nu/2-1/2}[\mathbf{H}_\nu(2ay^{1/2})-$ $-2Y_\nu(2ay^{1/2})]$, $y>0$
(43)	$x^{\nu/2-1/2}H_\nu^{(1)}(\alpha x^{1/2})$, $\mu=\nu+1/2$, $\operatorname{Re}\nu>-1/2$	$\frac{\pi^{1/2}i}{2}\left(\frac{2y}{\alpha}\right)^\nu H_\nu^{(1)}\left(\frac{\alpha y^{1/2}}{2}\right)\times$ $\times H_{-\nu}^{(1)}\left(\frac{\alpha y^{1/2}}{2}\right)$, $\operatorname{Im}(\alpha y^{1/2})>0$
(44)	$x^{\nu/2-1/2}H_{-\nu}^{(1)}(\alpha x^{1/2})$, $\mu=\nu+1/2$, $\operatorname{Re}\nu>-1/2$	$\frac{\pi^{1/2}i}{2}\left(\frac{2y}{\alpha}\right)^\nu\left[H_{-\nu}^{(1)}\left(\frac{\alpha y^{1/2}}{2}\right)\right]^2$, $\operatorname{Im}(\alpha y^{1/2})>0$
(45)	$x^{-\nu/2}H_\nu^{(1)}(\alpha x^{1/2})$, $\operatorname{Re}\mu>0$	$2^\mu\alpha^{-\mu}y^{\mu/2-\nu/2}H_{\nu-\mu}^{(1)}(\alpha y^{1/2})$, $\operatorname{Im}(\alpha y^{1/2})>0$
(46)	$x^{\nu/2-1/2}H_\nu^{(2)}(\alpha x^{1/2})$, $\mu=\nu+1/2$, $\operatorname{Re}\nu>-1/2$	$\frac{\pi^{1/2}i}{2}\left(\frac{2y}{\alpha}\right)^\nu H_\nu^{(2)}\left(\frac{\alpha y^{1/2}}{2}\right)\times$ $\times H_{-\nu}^{(2)}\left(\frac{\alpha y^{1/2}}{2}\right)$, $\operatorname{Im}(\alpha y^{1/2})<0$
(47)	$x^{\nu/2-1/2}H_{-\nu}^{(2)}(\alpha x^{1/2})$, $\mu=\nu+1/2$, $\operatorname{Re}\nu>-1/2$	$\frac{\pi^{1/2}i}{2}\left(\frac{2y}{\alpha}\right)^\nu\left[H_{-\nu}^{(2)}\left(\frac{\alpha y^{1/2}}{2}\right)\right]^2$, $\operatorname{Im}(\alpha y^{1/2})<0$

	$f(x)$	$[\Gamma(\mu)]^{-1}\int\limits_y^\infty f(x)\,(x-y)^{\mu-1}\,dx$
(48)	$x^{-\nu/2}H_\nu^{(2)}(\alpha x^{1/2})$, $\operatorname{Re}\mu>0$	$2^\mu\alpha^{-\mu}y^{\mu/2-\nu/2}H_{\nu-\mu}^{(2)}(\alpha y^{1/2})$, $\operatorname{Im}(\alpha y^{1/2})<0$
(49)	$x^{-\nu}e^{-\alpha x}I_\nu(\alpha x)$, $0<\operatorname{Re}\mu<1/2+\operatorname{Re}\nu$	$\dfrac{(2\alpha)^{\nu-\mu}\,\Gamma(1/2-\mu+\nu)}{\pi^{1/2}\,\Gamma(1-\mu+2\nu)}\times$ $\times{}_1F_1(1/2-\mu+\nu;\,1-\mu+2\nu;\,-2\alpha y)$, $\operatorname{Re}(\alpha y)>0$
(50)	$x^{-\lambda}e^{-\alpha x}I_\nu(\alpha x)$, $0<\operatorname{Re}\mu<1/2+\operatorname{Re}\lambda$	$\pi^{-1/2}(2\alpha)^\lambda y^\mu\times$ $\times G_{23}^{21}\left(2\alpha y\,\middle\vert\,{1/2-\lambda,\ 0 \atop -\mu,\ \nu-\lambda,\ -\nu-\lambda}\right)$, $\operatorname{Re}(\alpha y)>0$
(51)	$x^{-\mu-1/2}e^{-\alpha x}K_\nu(\alpha x)$, $\operatorname{Re}\mu>0$	$\pi^{1/2}(2\alpha)^{-1/2}y^{-1}e^{-\alpha y}W_{-\mu,\nu}(2\alpha y)$, $\operatorname{Re}(\alpha y)>0$
(52)	$x^{-\nu}e^{\alpha x}K_\nu(\alpha x)$, $0<\operatorname{Re}\mu<1/2+\operatorname{Re}\nu$	$\dfrac{\pi^{1/2}y^{\mu/2-\nu-1/2}\,\Gamma(1/2-\mu+\nu)}{(2\alpha)^{\mu/2+1/2}\,\Gamma(1/2+\nu)}\times$ $\times e^{\alpha y}W_{\mu/2,\,\nu-\mu/2}(2\alpha y)$, $\vert\arg(\alpha y)\vert<3\pi/2$
(53)	$x^{-\nu}e^{-\alpha x}K_\nu(\alpha x)$, $\operatorname{Re}\mu>0$	$\pi^{1/2}(2\alpha)^{-\mu/2-1/2}y^{\mu/2-\nu-1/2}\times$ $\times e^{-\alpha y}W_{-\mu/2,\,\nu-\mu/2}(2\alpha y)$, $\operatorname{Re}(\alpha y)>0$
(54)	$x^{-\lambda}e^{\alpha x}K_\nu(\alpha x)$, $0<\operatorname{Re}\mu<1/2+\operatorname{Re}\lambda$	$\pi^{-1/2}(2\alpha)^\lambda y^\mu\cos(\nu\pi)\times$ $\times G_{23}^{31}\left(2\alpha y\,\middle\vert\,{1/2-\lambda,\ 0 \atop -\mu,\ \nu-\lambda,\ -\nu-\lambda}\right)$, $\vert\arg(\alpha y)\vert<3\pi/2$
(55)	$x^{-\lambda}e^{-\alpha x}K_\nu(\alpha x)$, $\operatorname{Re}\mu>0$	$\pi^{1/2}(2\alpha)^\lambda y^\mu\times$ $\times G_{23}^{30}\left(2\alpha y\,\middle\vert\,{0,\ 1/2-\lambda \atop -\mu,\ \nu-\lambda,\ -\nu-\lambda}\right)$, $\operatorname{Re}(\alpha y)>0$
(56)	$x^{-1/2}K_{2\nu}(\alpha x^{1/2})$, $\mu=1/2$	$\pi^{-1/2}[K_\nu(2^{-1}\alpha y^{1/2})]^2$, $\operatorname{Re}(\alpha y^{1/2})>0$
(57)	$x^{\nu/2-1/2}K_\nu(\alpha x^{1/2})$, $\mu=\nu+1/2$, $\operatorname{Re}\nu>-1/2$	$\pi^{-1/2}\left(\dfrac{2y}{\alpha}\right)^\nu\left[K_\nu\left(\dfrac{\alpha y^{1/2}}{2}\right)\right]^2$, $\operatorname{Re}(\alpha y^{1/2})>0$

	$f(x)$	$[\Gamma(\mu)]^{-1}\int_y^\infty f(x)(x-y)^{\mu-1}\,dx$
(58)	$x^{\nu/2-1/2}K_\nu(\alpha x^{1/2})$, $\mu=\nu-1/2$, $\operatorname{Re}\nu>1/2$	$\pi^{-1/2}(2/\alpha)^{\nu-1}y^{\nu-1/2}\times$ $\times K_\nu(2^{-1}\alpha y^{1/2})K_{\nu-1}(2^{-1}\alpha y^{1/2})$, $\operatorname{Re}(\alpha y^{1/2})>0$
(59)	$x^{-\nu/2}K_\nu(\alpha x^{1/2})$ $\operatorname{Re}\mu>0$	$2^{\mu}\alpha^{-\mu}y^{\mu/2-\nu/2}K_{\nu-\mu}(\alpha y^{1/2})$, $\operatorname{Re}(\alpha y^{1/2})>0$
(60)	$x^{-\lambda}K_\nu(\alpha x^{1/2})$, $\operatorname{Re}\mu>0$	$2^{-2\lambda-1}\alpha^{2\lambda}y^{\mu}\times$ $\times G_{13}^{30}\left(\frac{\alpha^2 y}{4}\middle\vert\begin{matrix}0\\ -\mu,\ \nu/2-\lambda,\ -\nu/2-\lambda\end{matrix}\right)$, $\operatorname{Re}(\alpha y^{1/2})>0$
(61)	$x^{-\nu}I_\nu(\alpha x^{1/2})K_\nu(\alpha x^{1/2})$, $\mu=\nu-1/2$, $\operatorname{Re}\nu>1/2$	$2^{-1}\pi^{1/2}\alpha^{-\nu}y^{-\nu/2-1/2}\times$ $\times[I_\nu(2\alpha y^{1/2})-\mathbf{L}_\nu(2\alpha y^{1/2})]$, $\operatorname{Re}(\alpha y^{1/2})>0$
(62)	$x^{-\nu}[K_\nu(\alpha x^{1/2})]^2$, $\mu=\nu-1/2$, $\operatorname{Re}\nu>1/2$	$\pi^{1/2}\alpha^{-\nu}y^{-\nu/2-1/2}K_\nu(2\alpha y^{1/2})$, $\operatorname{Re}(\alpha y^{1/2})>0$
(63)	$x^{\mu/2}\mathbf{H}_{-\mu}(ax^{1/2})$, $a>0$, $0<\operatorname{Re}\mu<1/2$	$\left(\frac{2y}{a}\right)^{\mu}\left[Y_{-2\mu}(ay^{1/2})+\right.$ $\left.+\frac{2}{\pi}S_{0,2\mu}(ay^{1/2})\right]$, $y>0$
(64)	$x^{\nu/2-1/2}\mathbf{H}_{-\nu}(ax^{1/2})$, $a>0$, $\mu=\nu+1/2$, $-1/2<\operatorname{Re}\nu<1/2$	$\pi^{1/2}\left(\frac{2y}{a}\right)^{\nu}\left[J_{-\nu}\left(\frac{ay^{1/2}}{2}\right)\right]^2$, $y>0$
(65)	$x^{\nu/2}\mathbf{H}_\nu(ax^{1/2})$, $a>0$, $\operatorname{Re}\mu>0$, $\operatorname{Re}(\mu+\nu)<1/2$, $\operatorname{Re}(\mu+\nu/2)<3/4$	$\frac{(2/a)^{\mu}y^{\nu/2+\mu/2}}{\cos[(\mu+\nu)\pi]}[\cos(\nu\pi)\mathbf{H}_{\mu+\nu}(ay^{1/2})+$ $+\sin(\mu\pi)J_{-\mu-\nu}(ay^{1/2})]$, $y>0$
(66)	$x^{\mu/2}[\mathbf{H}_{-\mu}(\alpha x^{1/2})-Y_{-\mu}(\alpha x^{1/2})]$, $0<\operatorname{Re}\mu<1/2$	$\frac{2}{\pi}\left(\frac{2y}{\alpha}\right)^{\mu}S_{0,2\mu}(\alpha y^{1/2})$, $\vert\arg(\alpha y^{1/2})\vert<\pi$
(67)	$x^{\nu/2-1/2}[\mathbf{H}_{-\nu}(\alpha x^{1/2})-Y_{-\nu}(\alpha x^{1/2})]$, $\mu=\nu+1/2$, $-1/2<\operatorname{Re}\nu<1/2$	$\frac{\pi^{1/2}}{2}\left(\frac{2y}{\alpha}\right)^{\nu}\left\{\left[J_\nu\left(\frac{\alpha y^{1/2}}{2}\right)\right]^2+\right.$ $\left.+\left[Y_\nu\left(\frac{\alpha y^{1/2}}{2}\right)\right]^2\right\}$, $\vert\arg(\alpha y^{1/2})\vert<\pi$

	$f(x)$	$[\Gamma(\mu)]^{-1}\int_y^\infty f(x)(x-y)^{\mu-1}\,dx$
(68)	$x^{\nu/2}[\mathbf{H}_\nu(\alpha x^{1/2})-Y_\nu(\alpha x^{1/2})]$, $0<\operatorname{Re}\mu<1/2-\operatorname{Re}\nu$	$\frac{(2/\alpha)^\mu\cos(\nu\pi)}{\cos[(\mu+\nu)\pi]}y^{\nu/2+\mu/2}\times$ $\times[\mathbf{H}_{\mu+\nu}(\alpha y^{1/2})-Y_{\mu+\nu}(\alpha y^{1/2})]$, $\lvert\arg(\alpha y^{1/2})\rvert<\pi$
(69)	$x^{\nu/2-1/2}[I_{-\nu}(\alpha x^{1/2})-\mathbf{L}_{-\nu}(\alpha x^{1/2})]$, $\mu=\nu+1/2,\quad -1/2<\operatorname{Re}\nu<1/2$	$\frac{2}{\pi^{1/2}}\left(\frac{2y}{\alpha}\right)^\nu I_{-\nu}\left(\frac{\alpha y^{1/2}}{2}\right)K_\nu\left(\frac{\alpha y^{1/2}}{2}\right)$, $\operatorname{Re}(\alpha y^{1/2})>0$
(70)	$x^{\nu/2}[I_{-\nu}(\alpha x^{1/2})-\mathbf{L}_\nu(\alpha x^{1/2})]$, $0<\operatorname{Re}\mu<1/2-\operatorname{Re}\nu$	$\frac{\cos(\nu\pi)}{\cos[(\mu+\nu)\pi]}\left(\frac{2}{\alpha}\right)^\mu y^{\mu/2+\nu/2}\times$ $\times[I_{-\mu-\nu}(\alpha y^{1/2})-\mathbf{L}_{\mu+\nu}(\alpha y^{1/2})]$, $\operatorname{Re}(\alpha y^{1/2})>0$
(71)	$x^{\nu/2}S_{\lambda,\nu}(\alpha x^{1/2})$, $0<2\operatorname{Re}\mu<1-\operatorname{Re}(\lambda+\nu)$	$\frac{\Gamma(1/2-\lambda/2-\mu-\nu/2)}{\Gamma(1/2-\lambda/2-\nu/2)}\alpha^{-\mu}y^{\mu/2+\nu/2}\times$ $\times S_{\lambda+\mu,\,\mu+\nu}(\alpha y^{1/2})$, $\lvert\arg(\alpha y^{1/2})\rvert<\pi$
(72)	$x^{\lambda-1/2}\exp(2^{-1}\alpha x)W_{\varkappa,\lambda}(\alpha x)$, $0<\operatorname{Re}\mu<1/2-\operatorname{Re}(\varkappa+\lambda)$	$\frac{\Gamma(1/2-\varkappa-\lambda-\mu)}{\Gamma(1/2-\varkappa-\lambda)}\alpha^{-\mu/2}y^{\mu/2+\lambda-1/2}\times$ $\times\exp(2^{-1}\alpha y)W_{\varkappa+\mu/2,\,\lambda+\mu/2}(\alpha y)$, $\lvert\arg(\alpha y)\rvert<3\pi/2$
(73)	$x^{\varkappa-\mu-1}\exp(-2^{-1}\alpha x)W_{\varkappa,\lambda}(\alpha x)$, $\operatorname{Re}\mu>0$	$y^{\varkappa-1}\exp(-2^{-1}\alpha y)W_{\varkappa-\mu,\,\lambda}(\alpha y)$, $\operatorname{Re}(\alpha y)>0$
(74)	$x^{\lambda-1/2}\exp(-2^{-1}\alpha x)W_{\varkappa,\lambda}(\alpha x)$, $\operatorname{Re}\mu>0$	$\alpha^{-\mu/2}y^{\mu/2+\lambda-1/2}\exp(-2^{-1}\alpha y)\times$ $\times W_{\varkappa-\mu/2,\,\lambda-\mu/2}(\alpha y)$, $\operatorname{Re}(\alpha y)>0$
(75)	$x^{-\rho}\exp(2^{-1}\alpha x)W_{\varkappa,\lambda}(\alpha x)$, $0<\operatorname{Re}\mu<\operatorname{Re}(\rho-\varkappa)$	$\frac{y^{\mu-\rho}}{\Gamma(1/2+\lambda-\varkappa)\Gamma(1/2-\lambda-\varkappa)}\times$ $\times G_{23}^{31}\left(\alpha y\,\middle\vert\,\begin{matrix}1+\varkappa,\ \rho\\ \rho-\mu,\ 1/2+\lambda,\ 1/2-\lambda\end{matrix}\right)$, $\lvert\arg(\alpha y)\rvert<3\pi/2$
(76)	$x^{-\rho}\exp(-2^{-1}\alpha x)W_{\varkappa,\lambda}(\alpha x)$, $\operatorname{Re}\mu>0$	$y^{\mu-\rho}G_{23}^{30}\left(\alpha y\,\middle\vert\,\begin{matrix}\rho,\ 1-\varkappa\\ \rho-\mu,\ 1/2+\lambda,\ 1/2-\lambda\end{matrix}\right)$, $\operatorname{Re}(\alpha y)>0$

	$f(x)$	$[\Gamma(\mu)]^{-1}\int\limits_y^\infty f(x)(x-y)^{\mu-1}\,dx$
(77)	$x^{-\lambda}\,{}_pF_q(a_1, \ldots, a_p; b_1, \ldots, b_q; -\alpha/x)$, $0<\operatorname{Re}\mu<\operatorname{Re}\lambda$, $p\leqslant q+1$	$\frac{\Gamma(\lambda-\mu)}{\Gamma(\lambda)}y^{\mu-\lambda}\times$ $\times\,{}_{p+1}F_{q+1}(\lambda-\mu, a_1, \ldots, a_p; \lambda, b_1, \ldots, b_q; -\alpha/y)$, $\lvert y\rvert>\lvert\alpha\rvert$ или $\lvert\arg(\alpha/y)\rvert<\pi$, если $p=q+1$
(78)	$G_{pq}^{mn}\left(\alpha x\,\middle\vert\,\begin{matrix}a_1, \ldots, a_p\\ b_1, \ldots, b_q\end{matrix}\right)$, $p\geqslant q$ $0<\operatorname{Re}\mu<1-\operatorname{Re}a_j$ $j=1, \ldots, n$	$y^\mu\, G_{p+1,\,q+1}^{m+1,\,n}\left(\alpha y\,\middle\vert\,\begin{matrix}a_1, \ldots, a_p, 0\\ -\mu, b_1, \ldots, b_q\end{matrix}\right)$, $\lvert\alpha y\rvert>1$, если $p=q$
(79)	$G_{pq}^{mn}\left(\alpha x\,\middle\vert\,\begin{matrix}a_1, \ldots, a_p\\ b_1, \ldots, b_q\end{matrix}\right)$, $p+q<2(m+n)$ $0<\operatorname{Re}\mu<1-\operatorname{Re}a_j$ $j=1, \ldots, n$	$y^\mu\, G_{p+1,\,q+1}^{m+1,\,n}\left(\alpha y\,\middle\vert\,\begin{matrix}a_1, \ldots, a_p, 0\\ -\mu, b_1, \ldots, b_q\end{matrix}\right)$, $\lvert\arg(\alpha y)\rvert<(m+n-p/2-q/2)\pi$

ГЛАВА XIV

ПРЕОБРАЗОВАНИЯ СТИЛТЬЕСА

Мы называем функцию

$$g(y) = \mathfrak{S}\{f(x);\ y\} = \int_0^\infty f(x)(x+y)^{-1}\,dx$$

преобразованием Стилтьеса функции $f(x)$. Здесь интегрирование ведется по положительной вещественной полуоси, а y является комплексным переменным, изменяющимся в комплексной плоскости, разрезанной вдоль отрицательной вещественной полуоси.

Преобразования Стилтьеса являются итерированными преобразованиями Лапласа

$$\mathfrak{S}\{f(x);\ y\} - \mathfrak{L}\{\mathfrak{L}\{f(x);\ t\};\ y\}.$$

Поэтому информацию относительно преобразований Стилтьеса можно найти в трудах по преобразованиям Лапласа, в частности в книгах Уиддера (Widder, 1941, гл. VIII) и Титчмарша (1948, пп. 11.8 и 11.9). Преобразования Стилтьеса связаны также с проблемой моментов для полубесконечного промежутка (Shohat and Tamarkin, 1943) и, следовательно, с некоторыми цепными дробями.

Мы даем также краткий список *обобщенных преобразований Стилтьеса порядка* ρ

$$g(y;\ \rho) = \mathfrak{S}_\rho\{f(x);\ y\} = \int_0^\infty f(x)(x+y)^{-\rho}\,dx,$$

где x и y имеют тот же смысл, что и выше, а ρ — комплексный параметр. Обобщенные преобразования Стилтьеса различных порядков связаны друг с другом, а также с преобразованием Стилтьеса с помощью интегрирования дробного порядка

$$\Gamma(\rho)\,\mathfrak{W}_{\rho-1}\mathfrak{S}_\rho = \mathfrak{S},$$

$$\Gamma(\rho)\,\mathfrak{W}_\mu\mathfrak{S}_\rho = \Gamma(\rho-\mu)\,\mathfrak{S}_{\rho-\mu}.$$

Из приведенных в нижеследующих таблицах пар преобразований могут быть получены дальнейшие интегралы с помощью методов, указанных во введении к первому тому, общих формул, указанных в таблицах, а также путем применения приведенных выше формул и таблиц преобразований Лапласа и интегралов дробного порядка.

14.1. Общие формулы

	$f(x)$	$\int\limits_0^\infty f(x)(x+y)^{-1}dx = g(y)$, $\lvert \arg y \rvert < \pi$
(1)	$f(x)$	$g(y)$
(2)	$x f(x)$	$\int\limits_0^\infty f(x)\,dx - y\,g(y)$
(3)	$(x+\alpha)^{-1} f(x)$, $\lvert \arg \alpha \rvert < \pi$	$(y-\alpha)^{-1}[g(\alpha) - g(y)]$
(4)	$\frac{f(x)-f(a)}{x-a}$, $a > 0$	$(y+a)^{-1}[2^{-1} g(a e^{i\pi}) + 2^{-1} g(a e^{-i\pi}) - f(a) \ln(y/a) - g(y)]$
(5)	$g(x e^{i\pi}) - g(x e^{-i\pi})$	$2\pi i\, g(y)$
(6)	$f(ax)$, $a > 0$	$g(ay)$
(7)	$x^{-1} f(a/x)$, $a > 0$	$y^{-1} g(a/y)$
(8)	$f(x^{1/2})$	$g(iy^{1/2}) + g(-iy^{1/2})$
(9)	$f'(x)$	$-y^{-1} f(0) - g'(y)$

14.2. Элементарные функции

(1)	-1, $2n < x < 2n+1$ 1, $2n+1 < x < 2n+2$ $n = 0, 1, 2, \ldots$	$\ln\left\{\frac{y}{2}\left[\frac{\Gamma(y/2)}{\Gamma(y/2+1/2)}\right]^2\right\}$
(2)	$(\alpha+x)^{-1}$, $\lvert \arg \alpha \rvert < \pi$	$(\alpha - y)^{-1} \ln(\alpha/y)$
(3)	$\frac{1}{\alpha^2+x^2}$, $\operatorname{Re}\alpha > 0$	$\frac{1}{\alpha^2+y^2}\left[\frac{\pi y}{2\alpha} - \ln\left(\frac{y}{\alpha}\right)\right]$
(4)	$\frac{x}{\alpha^2+x^2}$, $\operatorname{Re}\alpha > 0$	$\frac{1}{\alpha^2+y^2}\left[\frac{\pi\alpha}{2} + y \ln\left(\frac{y}{\alpha}\right)\right]$
(5)	x^ν, $-1 < \operatorname{Re}\nu < 0$	$-\frac{\pi y^\nu}{\sin(\pi\nu)}$

	$f(x)$	$\int\limits_0^\infty f(x)(x+y)^{-1}\,dx,\ \lvert\arg y\rvert<\pi$
(6)	$\frac{x^\nu}{\alpha+x}$, $\lvert\arg\alpha\rvert<\pi,\ -1<\operatorname{Re}\nu<1$	$\frac{\pi(\alpha^\nu-y^\nu)}{(\alpha-y)\sin(\nu\pi)}$
(7)	$\frac{x^\nu}{\alpha^2+x^2}$, $\operatorname{Re}\alpha>0,\ -1<\operatorname{Re}\nu<2$	$\frac{\pi}{\alpha^2+y^2}\left[\frac{\alpha^{\nu-1}y}{2\cos(\nu\pi/2)}+\frac{\alpha^\nu}{2\sin(\nu\pi/2)}-\frac{y^\nu}{\sin(\nu\pi)}\right]$
(8)	$\frac{x^\nu-\alpha^\nu}{x-\alpha}$, $-1<\operatorname{Re}\nu<1$	$\frac{\pi}{\alpha+y}\left[\frac{y^\nu}{\sin(\nu\pi)}-\alpha^\nu\operatorname{ctg}(\nu\pi)+\frac{\alpha^\nu}{\pi}\ln\left(\frac{\alpha}{y}\right)\right]$
(9)	$x^{\nu-1}(\alpha+x)^{1-\mu}$, $\lvert\arg\alpha\rvert<\pi,\ 0<\operatorname{Re}\nu<\operatorname{Re}\mu$	$\frac{\Gamma(\nu)\Gamma(\mu-\nu)\,y^{\nu-1}}{\Gamma(\mu)\,\alpha^{\mu-1}}\times{}_2F_1(\mu-1,\ \nu;\ \mu;\ 1-y/\alpha)$
(10)	$x^{-\rho}(\alpha+x)^{-\sigma}$, $\lvert\arg\alpha\rvert<\pi$, $-\operatorname{Re}\sigma<\operatorname{Re}\rho<1$	$\frac{\pi(\alpha-y)^{-\sigma}I_{1-y/\alpha}(\sigma,\ \rho)}{y^\rho\sin(\rho\pi)}$
(11)	$e^{-\alpha x}$, $\operatorname{Re}\alpha>0$	$-e^{\alpha y}\operatorname{Ei}(-\alpha y)$
(12)	$e^{-\alpha x}$, $0<x<b$; 0, $b<x<\infty$	$e^{\alpha y}[\operatorname{Ei}(-\alpha b-\alpha y)-\operatorname{Ei}(-\alpha y)]$
(13)	0, $0<x<b$; $e^{-\alpha x}$, $b<x<\infty$; $\operatorname{Re}\alpha>0$	$-e^{\alpha y}\operatorname{Ei}(-\alpha b-\alpha y)$
(14)	$x^n e^{-\alpha x}$, $\operatorname{Re}\alpha>0$	$(-1)^{n+1}y^n e^{\alpha y}\operatorname{Ei}(-\alpha y)+\sum\limits_{r=1}^{n}(-1)^{n-r}(r-1)!\,\alpha^{-r}y^{n-r}$
(15)	$x^{-1/2}e^{-\alpha x}$, $\operatorname{Re}\alpha>0$	$\pi y^{-1/2}e^{\alpha y}\operatorname{Erfc}(\alpha^{1/2}y^{1/2})$
(16)	$x^{1/2}e^{-\alpha x}$, $\operatorname{Re}\alpha>0$	$\pi^{1/2}\alpha^{-1/2}-\pi y^{1/2}e^{\alpha y}\operatorname{Erfc}(\alpha^{1/2}y^{1/2})$
(17)	$x^{-\nu}e^{-\alpha x}$, $\operatorname{Re}\alpha>0,\ \operatorname{Re}\nu<1$	$\Gamma(1-\nu)\,y^{-\nu}e^{\alpha y}\,\Gamma(\nu,\ \alpha y)$

	$f(x)$	$\int\limits_0^\infty f(x)(x+y)^{-1}\,dx, \ \lvert\arg y\rvert<\pi$
(18)	$x^{-1}(1-e^{-\alpha x}),\quad \mathrm{Re}\,\alpha>0$	$y^{-1}[\ln(\alpha\gamma y)-e^{\alpha y}\,\mathrm{Ei}(-\alpha y)]$
(19)	$x^{\nu-1}e^{-\alpha/x},\quad \mathrm{Re}\,\alpha>0,\ \mathrm{Re}\,\nu<1$	$\Gamma(1-\nu)\,y^{\nu-1}e^{\alpha/y}\,\Gamma(\nu;\ \alpha/y)$
(20)	$\exp(-\alpha x^{1/2}),\quad \mathrm{Re}\,\alpha>0$	$2\cos(\alpha y^{1/2})\,\mathrm{ci}(\alpha y^{1/2})-$ $-2\sin(\alpha y^{1/2})\,\mathrm{si}(\alpha y^{1/2})$
(21)	$x^{-1/2}\exp(-\alpha x^{1/2}),\quad \mathrm{Re}\,\alpha>0$	$-2y^{-1/2}[\sin(\alpha y^{1/2})\,\mathrm{ci}(\alpha y^{1/2})+$ $+\cos(\alpha y^{1/2})\,\mathrm{si}(\alpha y^{1/2})]$
(22)	$x^{\lambda}\exp(-\alpha x^{1/2}),$ $\mathrm{Re}\,\alpha>0,\ \mathrm{Re}\,\lambda>-1$	$\Gamma(2\lambda+1)\,y^{\lambda}[\exp(i\alpha y^{1/2}+\lambda\pi i)\times$ $\times\Gamma(-2\lambda,\ i\alpha y^{1/2})+$ $+\exp(-i\alpha y^{1/2}-\lambda\pi i)\times$ $\times\Gamma(-2\lambda,\ -i\alpha y^{1/2})]$
(23)	$[\exp(\alpha x^{1/2})-1]^{-1},\quad \mathrm{Re}\,\alpha>0$	$\ln(\alpha y^{1/2})-(2\alpha y^{1/2})^{-1}-\psi(\alpha y^{1/2})$
(24)	$(\alpha+x)^{-1}\ln x,\quad \lvert\arg\alpha\rvert<\pi$	$2^{-1}(y-\alpha)^{-1}[(\ln y)^2-(\ln\alpha)^2]$
(25)	$(\alpha+x)^{-1}\ln(x/\alpha),\quad \lvert\arg\alpha\rvert<\pi$	$\frac{1}{2(y-\alpha)}\left[\ln\left(\frac{y}{\alpha}\right)\right]^2$
(26)	$(x-a)^{-1}\ln(x/a),\quad a>0$	$2^{-1}(y+a)^{-1}\{\pi^2+[\ln(y/a)]^2\}$
(27)	$x^{-1/2}\ln(\alpha x+\beta),$ $\mathrm{Re}\,\alpha>0,\ \mathrm{Re}\,\beta>0$	$2\pi y^{-1/2}\ln(\alpha^{1/2}y^{1/2}+\beta^{1/2})$
(28)	$x^{\nu}\ln x,\quad -1<\mathrm{Re}\,\nu<0$	$-\frac{\pi y^{\nu}[\ln y-\pi\,\mathrm{ctg}(\nu\pi)]}{\sin(\nu\pi)}$
(29)	$x^{\nu}(\alpha+x)^{-1}\ln x,$ $\lvert\arg\alpha\rvert<\pi,\ -1<\mathrm{Re}\,\nu<1$	$-\frac{\pi}{(y-\alpha)\sin(\nu\pi)}[\alpha^{\nu}\ln\alpha-y^{\nu}\ln y-$ $-\pi\,\mathrm{ctg}(\nu\pi)(\alpha^{\nu}-y^{\nu})]$
(30)	$x^{\nu}(\alpha+x)^{-1}\ln(x/\alpha),$ $\lvert\arg\alpha\rvert<\pi,\ -1<\mathrm{Re}\,\nu<1$	$\frac{\pi}{(y-\alpha)\sin(\nu\pi)}[y^{\nu}\ln(y/\alpha)+$ $+\pi\,\mathrm{ctg}(\nu\pi)(\alpha^{\nu}-y^{\nu})]$
(31)	$\sin(ax),\quad a>0$	$-\sin(ay)\,\mathrm{ci}(ay)-\cos(ay)\,\mathrm{si}(ay)$

	$f(x)$	$\int_0^\infty f(x)(x+y)^{-1}\,dx,\ \lvert\arg y\rvert<\pi$
(32)	$x^{1/2}\sin(ax),\qquad a>0$	$\pi y^{1/2}[\sin(ay)-2^{1/2}\sin(ay+\pi/4)\,C(ay)+2^{1/2}\cos(ay+\pi/4)\,S(ay)]-2^{-1/2}\pi^{1/2}a^{-1/2}$
(33)	$x^{-1/2}\sin(ax),\qquad a>0$	$\pi y^{-1/2}[2^{1/2}\sin(ay+\pi/4)\,C(ay)-2^{1/2}\cos(ay+\pi/4)\,S(ay)-\sin(ay)]$
(34)	$x^{-\nu}\sin(ax),$ $a>0,\ -1<\operatorname{Re}\nu<2$	$2^{-1}i\,\Gamma(1-\nu)\,y^{-\nu}\times[e^{-iay}\,\Gamma(\nu,\ -iay)-e^{iay}\,\Gamma(\nu,\ iay)]$
(35)	$0,\qquad 0<x<a$ $(x^2-a^2)^{-1/2}\sin(bx),\qquad a<x<\infty$	См. Erdélyi Arthur, 1939; Proc. Edinburgh Math. Soc. (2), 6, 94—104.
(36)	$\sin(ax^{1/2}),\qquad a>0$	$\pi\exp(-ay^{1/2})$
(37)	$x^{-1}\sin(ax^{1/2}),\qquad a>0$	$\pi y^{-1}[1-\exp(-ay^{1/2})]$
(38)	$x^{-1/2}\sin(ax^{1/2}),\qquad a>0$	$y^{-1/2}[\exp(-ay^{1/2})\,\overline{\mathrm{Ei}}(ay^{1/2})-\exp(ay^{1/2})\,\mathrm{Ei}(-ay^{1/2})]$
(39)	$x^{\lambda}\sin(ax^{1/2}),$ $a>0,\ -3/2<\operatorname{Re}\lambda<1/2$	$-\dfrac{\pi y^{\lambda}\,\mathrm{sh}(ay^{1/2})}{\cos(\lambda\pi)}-a^{-2\lambda}\,\Gamma(2\lambda)\sin(\lambda\pi)\times[{}_1F_1(1;\ 1-2\lambda;\ ay^{1/2})+{}_1F_1(1;\ 1-2\lambda;\ -ay^{1/2})]$
(40)	$(x+\beta)^{-1}\sin(ax^{1/2}),$ $a>0,\ \lvert\arg\beta\rvert<\pi$	$\pi(y-\beta)^{-1}[\exp(-a\beta^{1/2})-\exp(-ay^{1/2})]$
(41)	$x^{-\beta}\sin(ax^{1/2}+\beta\pi),$ $a>0,\ -1/2<\operatorname{Re}\beta<1$	$\pi y^{-\beta}\exp(-ay^{1/2})$
(42)	$\sin(ax^{1/2}-bx^{-1/2}),\qquad a,\ b>0$	$\pi\exp(-ay^{1/2}-by^{-1/2})$
(43)	$x^{-1/2}[\sin(ax^{1/2})]^2,\qquad a>0$	$2^{-1}\pi y^{-1/2}[1-\exp(-2ay^{1/2})]$

	$f(x)$	$\int_0^\infty f(x)(x+y)^{-1}\,dx, \ \lvert\arg y\rvert<\pi$
(44)	$x^{-1/2}\sin(ax^{1/2})\sin(bx^{1/2}),$ $a>0,\ b>0$	$2^{-1}\pi y^{-1/2}\{\exp[-\lvert a-b\rvert y^{1/2}]-$ $-\exp[-(a+b)y^{1/2}]\}$
(45)	$x^{-1/2}\sin(ax^{1/2})\sin(bx^{1/2}),$ $a\geqslant b>0$	$\pi y^{-1/2}\exp(-ay^{1/2})\operatorname{sh}(by^{1/2})$
(46)	$\ln(\beta x)\sin(ax^{1/2}),$ $a>0,\ \lvert\arg\beta\rvert<\pi$	$\pi[\ln(\beta y)\exp(-ay^{1/2})-$ $-\exp(ay^{1/2})\operatorname{Ei}(-ay^{1/2})-$ $-\exp(-ay^{1/2})\operatorname{Ei}(ay^{1/2})]$
(47)	$x^{-1/2}\ln\lvert\sin(ax^{1/2})\rvert,$ $a>0$	$\pi y^{-1/2}\ln[1/2-2^{-1}\exp(-2ay^{1/2})]$
(48)	$\cos(ax),$ $a>0$	$\cos(ay)\operatorname{ci}(ay)-\sin(ay)\operatorname{si}(ay)$
(49)	$x^{-1}[\cos(bx)-\cos(ax)],$ $a,\ b>0$	$y^{-1}[-\operatorname{ci}(by)\cos(by)+$ $+\operatorname{si}(by)\sin(by)+\operatorname{ci}(ay)\cos(ay)-$ $-\operatorname{si}(ay)\sin(ay)+\ln(ab^{-1})]$
(50)	$x^{1/2}\cos(ax),$ $a>0$	$2^{-1/2}\pi^{1/2}a^{-1/2}-\pi y^{1/2}[\cos(ay)-$ $-2^{1/2}\cos(ay+\pi/4)\,C(ay)-$ $-2^{1/2}\sin(ay+\pi/4)\,S(ay)]$
(51)	$x^{-1/2}\cos(ax),$ $a>0$	$\pi y^{-1/2}[\cos(ay)-$ $-2^{1/2}\cos(ay+\pi/4)\,C(ay)-$ $-2^{1/2}\sin(ay+\pi/4)\,S(ay)]$
(52)	$x^{-\nu}\cos(ax),$ $a>0,\ -1<\operatorname{Re}\nu<1$	$2^{-1}\Gamma(1-\nu)\,y^{-\nu}[e^{iay}\Gamma(\nu,\ iay)+$ $+e^{-iay}\Gamma(\nu,\ -iay)]$
(53)	$0,$ $0<x<a$; $(x^2-a^2)^{-1/2}\cos(bx),$ $a<x<\infty$	См. Erdélyi Arthur, 1939: Proc. Edinburgh Math. Soc. (2), 6, 94—104.
(54)	$\cos(ax^{1/2}),$ $a>0$	$-\exp(-ay^{1/2})\,\overline{\operatorname{Ei}}(ay^{1/2})-$ $-\exp(ay^{1/2})\operatorname{Ei}(-ay^{1/2})$
(55)	$x^{-1/2}\cos(ax^{1/2}),$ $a>0$	$\pi y^{-1/2}\exp(-ay^{1/2})$

	$f(x)$	$\int_0^\infty f(x)(x+y)^{-1}\,dx, \quad \lvert\arg y\rvert<\pi$
(56)	$x^\lambda \cos(ax^{1/2})$, $a>0,\ -1<\operatorname{Re}\lambda<1/2$	$-\frac{\pi y^\lambda \operatorname{ch}(ay^{1/2})}{\sin(\lambda\pi)}-$ $-a^{-2\lambda}\cos(\lambda\pi)\,\Gamma(2\lambda)\times$ $\times[{}_1F_1(1;\,1-2\lambda;\,ay^{1/2})+$ $+{}_1F_1(1;\,1-2\lambda;\,-ay^{1/2})]$
(57)	$x^{-1/2}(x+\beta)^{-1}\cos(ax^{1/2})$, $a>0,\ \lvert\arg\beta\rvert<\pi$	$\pi(y-\beta)^{-1}[\beta^{-1/2}\exp(-a\beta^{1/2})-$ $-y^{-1/2}\exp(-ay^{1/2})]$
(58)	$x^{-1/2}\cos(ax^{1/2}-bx^{-1/2})$, $a,\ b>0$	$\pi y^{-1/2}\exp(-ay^{1/2}-by^{-1/2})$
(59)	$x^{-3/2}[\cos(ax^{1/2})-\cos(bx^{1/2})]$, $a>0,\ b>0$	$\pi y^{-3/2}[(b-a)\,y^{1/2}+$ $+\exp(-by^{1/2})-\exp(-ay^{1/2})]$
(60)	$x^{-1/2}[\cos(ax^{1/2})]^2$, $a>0$	$2^{-1}\pi y^{-1/2}[1-\exp(-2ay^{1/2})]$
(61)	$x^{-1/2}\cos(ax^{1/2})\cos(bx^{1/2})$, $a>0,\ b>0$	$2^{-1}\pi y^{-1/2}\{\exp(-\lvert a-b\rvert\,y^{1/2})+$ $+\exp[-(a+b)\,y^{1/2}]\}$
(62)	$x^{-1/2}\cos(ax^{1/2})\cos(bx^{1/2})$, $a\geqslant b>0$	$\pi y^{-1/2}\exp(-ay^{1/2})\operatorname{ch}(by^{1/2})$
(63)	$\frac{x^{-1/2}}{[\beta\sin(ax^{1/2})]^2+[\gamma\cos(ax^{1/2})]^2}$, $\lvert\arg(\beta/\gamma)\rvert<\pi$	$\frac{\pi y^{-1/2}[2^{-1}(\beta/\gamma-\gamma/\beta)\operatorname{sh}(2ay^{1/2})-1]}{[\beta\operatorname{sh}(ay^{1/2})]^2-[\gamma\operatorname{ch}(ay^{1/2})]^2}$
(64)	$\frac{\sin(2ax^{1/2})}{[\beta\sin(ax^{1/2})]^2+[\gamma\cos(ax^{1/2})]^2}$, $\lvert\arg(\beta/\gamma)\rvert<\pi$	$\frac{\pi[(\beta-\gamma)/(\beta+\gamma)-\exp(-2ay^{1/2})]}{[\beta\operatorname{sh}(ay^{1/2})]^2-[\gamma\operatorname{ch}(ay^{1/2})]^2}$
(65)	$x^{-1/2}\ln(\beta x)\cos(ax^{1/2})$, $a>0,\ \lvert\arg\beta\rvert<\pi$	$\pi y^{-1/2}[\ln(\beta y)\exp(-ay^{1/2})+$ $+\exp(ay^{1/2})\operatorname{Ei}(-ay^{1/2})-$ $-\exp(-ay^{1/2})\overline{\operatorname{Ei}}(ay^{1/2})]$
(66)	$x^{-1/2}\ln\lvert\cos(ax^{1/2})\rvert$, $a>0$	$\pi y^{-1/2}\ln[1/2+2^{-1}\exp(-2ay^{1/2})]$
(67)	$x^{-1/2}\ln[1+2\beta\cos(ax^{1/2})+\beta^2]$, $a>0,\ \lvert\beta\rvert<1$	$2\pi y^{-1/2}\ln[1+\beta\exp(-ay^{1/2})]$

	$f(x)$	$\int_0^\infty f(x)(x+y)^{-1}\,dx, \quad \lvert\arg y\rvert<\pi$
(68)	$x^{-1/2}\ln\{[b\sin(ax^{1/2})]^2+$ $+[c\cos(ax^{1/2})]^2\}$, $a,\ b,\ c>0$	$2\pi y^{-1/2}\ln[b\operatorname{sh}(ay^{1/2})+$ $+c\operatorname{ch}(ay^{1/2})]-2\pi a$
(69)	$[\cos(ax^{1/2})]^n\sin(nax^{1/2})$, $a>0,\ n=1,\ 2,\ \ldots$	$2^{-n}\pi\{[1+\exp(-2ay^{1/2})]^n-1\}$
(70)	$x^{-1/2}\ln[\operatorname{tg}(ax^{1/2})]$, $a>0$	$\pi y^{-1/2}\ln[\operatorname{th}(ay^{1/2})]$
(71)	$x^{-1/2}\ln\{1+[b\operatorname{tg}(ax^{1/2})]^2\}$, $a,\ b>0$	$2\pi y^{-1/2}\ln[1+b\operatorname{th}(ay^{1/2})]$
(72)	$x^{-1/2}\ln\{1+[b\operatorname{ctg}(ax^{1/2})]^2\}$, $a,\ b>0$	$2\pi y^{-1/2}\ln[1+b\operatorname{cth}(ay^{1/2})]$
(73)	$\dfrac{1}{\operatorname{sh}(\pi x^{1/2})}$	$-y^{-1/2}+\psi(2^{-1}y^{1/2}+1/2)-$ $-\psi(2^{-1}y^{1/2})$
(74)	$\dfrac{1}{x^{1/2}\operatorname{ch}(\pi x^{1/2})}$	$y^{-1/2}[\psi(2^{-1}y^{1/2}+3/4)-$ $-\psi(2^{-1}y^{1/2}+1/4)]$
(75)	$\dfrac{x^{-1/2}\sin(ax^{1/2})}{\operatorname{sh}(bx^{1/2})}$, $\dfrac{x^{-1/2}\cos(ax^{1/2})}{\operatorname{ch}(bx^{1/2})}$, $\dfrac{x^{-1/2}\cos(ax^{1/2})}{c+\operatorname{ch}(bx^{1/2})}$	См. Ramanujan Srinivasa, 1914; Messenger of Math. 44, 75—85.

14.3. Высшие трансцендентные функции

(1)	$\operatorname{ci}(ax)$, $a>0$	$\dfrac{[\operatorname{ci}(ay)]^2+[\operatorname{si}(ay)]^2}{2}$
(2)	$J_\nu(ax)$, $a>0$, $\operatorname{Re}\nu>-1$	$\dfrac{\pi[\mathbf{J}_\nu(ay)-J_\nu(ay)]}{\sin(\nu\pi)}$
(3)	$x^\nu J_\nu(ax)$, $a>0,\ -1/2<\operatorname{Re}\nu<3/2$	$\dfrac{\pi y^\nu\ [\mathbf{H}_{-\nu}(ay)-Y_{-\nu}(ay)]}{2\cos(\nu\pi)}$

	$f(x)$	$\int\limits_0^\infty f(x)(x+y)^{-1}\,dx, \quad \lvert\arg y\rvert<\pi$
(4)	$x^{\nu+1}J_\nu(ax)$, $a>0,\ -1<\operatorname{Re}\nu<{}^1/_2$	$2^\nu\pi^{-1/2}a^{-\nu-1}\Gamma(\nu+{}^1/_2)+ \dfrac{\pi y^{\nu+1}[Y_{-\nu}(ay)-\mathbf{H}_{-\nu}(ay)]}{2\cos(\nu\pi)}$
(5)	$x^{-\nu}J_\nu(ax)$, $a>0,\ \operatorname{Re}\nu>-{}^3/_2$	$2^{-1}\pi y^{-\nu}[\mathbf{H}_\nu(ay)-Y_\nu(ay)]-\dfrac{2^{1-\nu}y^{-\nu}}{\Gamma(\nu)}S_{\nu-1,\,\nu}(ay)$
(6)	$x^\lambda J_\nu(ax),\ a>0,\ \operatorname{Re}\lambda<{}^3/_2$, $\operatorname{Re}(\lambda+\nu)>-1$	$-\dfrac{\pi y^\lambda J_\nu(ay)}{\sin[(\lambda+\nu)\pi]}+\dfrac{2^{\lambda-1}a^{-\lambda}\Gamma(\lambda/2+\nu/2)}{\Gamma(1-\lambda/2+\nu/2)}\times {}_1F_2\left(1;\,1-\dfrac{\lambda+\nu}{2},\,1-\dfrac{\lambda-\nu}{2};\,-\dfrac{a^2y^2}{4}\right)-\dfrac{2^{\lambda-2}a^{1-\lambda}y\,\Gamma(\lambda/2+\nu/2-{}^1/_2)}{\Gamma({}^3/_2-\lambda/2+\nu/2)}\times {}_1F_2\left(1;\,\dfrac{3-\lambda-\nu}{2},\,\dfrac{3-\lambda+\nu}{2};\,-\dfrac{a^2y^2}{4}\right)$
(7)	$x^\nu\sin(ax)J_\nu(ax)$, $a>0,\ -1<\operatorname{Re}\nu<{}^1/_2$	$\dfrac{\pi y^\nu}{2\cos(\nu\pi)}[\cos(ay-\nu\pi)J_\nu(ay)+\sin(ay-\nu\pi)Y_\nu(ay)]$
(8)	$x^\nu\cos(ax)J_\nu(ax)$, $a>0,\ -{}^1/_2<\operatorname{Re}\nu<{}^1/_2$	$\dfrac{\pi y^\nu}{2\cos(\nu\pi)}[\sin(ay-\nu\pi)J_\nu(ay)-\cos(ay-\nu\pi)Y_\nu(ay)]$
(9)	$x^\nu\cos(ax+\beta)J_\nu(ax)$, $a>0,\ -{}^1/_2<\operatorname{Re}\nu<{}^1/_2$	$\dfrac{\pi y^\nu}{2\cos(\nu\pi)}[\sin(ay-\nu\pi-\beta)J_\nu(ay)-\cos(ay-\nu\pi-\beta)Y_\nu(ay)]$
(10)	$x^{\nu/2+k}J_\nu(ax^{1/2})$, $a>0,\ k=0,\ 1,\ 2$, $-k-1<\operatorname{Re}\nu<-2k+{}^3/_2$	$2(-1)^k y^{\nu/2+k}K_\nu(ay^{1/2})$
(11)	$x^{\nu/2+k-1/2}J_\nu(ax^{1/2})$, $a>0,\ k=0,\ 1,\ 2$, $-k-{}^1/_2<\operatorname{Re}\nu<-2k+{}^5/_2$	$\dfrac{(-1)^k\pi y^{\nu/2+k-1/2}}{\cos(\nu\pi)}\times[I_\nu(ay^{1/2})-\mathbf{L}_{-\nu}(ay^{1/2})]$
(12)	$x^{k-\nu/2-1/2}J_\nu(ax^{1/2})$, $a>0,\ k=0,\ 1,\ 2,\ \ldots$, $\operatorname{Re}\nu>2k-{}^5/_2$	$\pi y^{k-\nu/2-1/2}[I_\nu(ay^{1/2})-\mathbf{L}_\nu(ay^{1/2})]$

	$f(x)$	$\int_0^\infty f(x)(x+y)^{-1}\,dx, \quad \lvert\arg y\rvert<\pi$
(13)	$x^\lambda J_\nu(ax^{1/2})$, $a>0$, $\operatorname{Re}(\lambda+\nu/2)>-1$ $\operatorname{Re}\lambda<3/4$	$\left(\frac{2}{a}\right)^{2\lambda}\frac{\Gamma(\lambda+\nu/2)}{\Gamma(1-\lambda+\nu/2)}\times$ $\times\,{}_1F_2\left(1;1-\lambda-\frac{\nu}{2},1-\lambda+\frac{\nu}{2};\frac{a^2y}{4}\right)-$ $-\frac{\pi y^\lambda I_\nu(ay^{1/2})}{\sin[(\lambda+\nu/2)\pi]}$
(14)	$\sin(ax^{1/2})J_0(bx^{1/2})$, $0<b<a$	$\pi\exp(-ay^{1/2})I_0(by^{1/2})$
(15)	$x^{-1/2}\sin(ax^{1/2})J_0(bx^{1/2})$, $0<a<b$	$2y^{-1/2}\operatorname{sh}(ay^{1/2})K_0(by^{1/2})$
(16)	$\cos(ax^{1/2})J_0(bx^{1/2})$, $0<a<b$	$2\operatorname{ch}(ay^{1/2})K_0(by^{1/2})$
(17)	$x^{-1/2}\cos(ax^{1/2})J_0(bx^{1/2})$, $0<b<a$	$\pi y^{-1/2}\exp(-ay^{1/2})I_0(by^{1/2})$
(18)	$x^{\nu/2-1/2}\sin(ax^{1/2})J_\nu(bx^{1/2})$, $0<a<b$, $-1<\operatorname{Re}\nu<3/2$	$2y^{\nu/2-1/2}\operatorname{sh}(ay^{1/2})K_\nu(by^{1/2})$
(19)	$x^{-\nu/2}\sin(ax^{1/2})J_\nu(bx^{1/2})$, $0<b<a$, $\operatorname{Re}\nu>-1/2$	$\pi y^{-\nu/2}\exp(-ay^{1/2})I_\nu(by^{1/2})$
(20)	$x^{\nu/2}\cos(ax^{1/2})J_\nu(bx^{1/2})$, $0<a<b$, $-1<\operatorname{Re}\nu<1/2$	$2y^{\nu/2}\operatorname{ch}(ay^{1/2})K_\nu(by^{1/2})$
(21)	$x^{-\nu/2-1/2}\cos(ax^{1/2})J_\nu(bx^{1/2})$, $0<b<a$, $\operatorname{Re}\nu>-3/2$	$\pi y^{-\nu/2-1/2}\exp(-ay^{1/2})I_\nu(by^{1/2})$
(22)	$[J_\nu(ax)]^2$, $a>0$	$2I_\nu(ay^{1/2})K_\nu(ay^{1/2})$
(23)	$J_\nu(ax^{1/2})J_\nu(bx^{1/2})$, $a, b>0$, $\operatorname{Re}\nu>-1$	$2I_\nu(ay^{1/2})K_\nu(by^{1/2})$, $b>a$ $2I_\nu(by^{1/2})K_\nu(ay^{1/2})$, $b<a$
(24)	$x^{\nu/2-\mu/2}J_\nu(bx^{1/2})J_\mu(ax^{1/2})$, $0<a<b$ $2+\operatorname{Re}\mu>\operatorname{Re}\nu>-1$	$2y^{\nu/2-\mu/2}I_\mu(ay^{1/2})K_\nu(by^{1/2})$

	$f(x)$	$\int_0^\infty f(x)(x+y)^{-1}\,dx,\ \lvert\arg y\rvert<\pi$
(25)	$x^\lambda J_\mu(ax^{1/2}) J_\nu(ax^{1/2})$, $a>0$, $\operatorname{Re}\lambda<1$, $\operatorname{Re}(2\lambda+\mu+\nu)>-2$	$a^{-2\lambda}\pi^{-1/2} G_{35}^{23}\left(a^2 y \,\middle\vert\, {0,\ \lambda,\ \lambda+1/2 \atop 0,\ p,\ q,\ r,\ s}\right)$, $p=\lambda+\mu/2+\nu/2$, $q=\lambda+\mu/2-\nu/2$, $r=\lambda-\mu/2+\nu/2$, $s=\lambda-\mu/2-\nu/2$
(26)	$x^{\mu/2+n}(x+\gamma)^{-\nu/2} J_\mu(ax^{1/2}) \times$ $\times J_\nu[b(x+\gamma)^{1/2}]$, $a>b>0$, $n=0,\ 1,\ 2,\ \ldots$ $-1-n<\operatorname{Re}\mu<2-2n+\operatorname{Re}\nu$	$2(-1)^n y^{\mu/2+n}(y-\gamma)^{-\nu/2} \times$ $\times K_\mu(ay^{1/2}) I_\mu[b(y-\gamma)^{1/2}]$
(27)	$x^{-1/2}[\sin(ax) J_\nu(ax) +$ $+\cos(ax) Y_\nu(ax)]$, $a>0$, $-1/2<\operatorname{Re}\nu<1/2$	$\dfrac{\pi}{y^{1/2}\cos(\nu\pi)}[-\sin(ay) J_\nu(ay) +$ $+\cos(ay) Y_\nu(ay)]$
(28)	$x^{-1/2}[\cos(ax) J_\nu(ax) -$ $-\sin(ax) Y_\nu(ax)]$, $a>0$, $-1/2<\operatorname{Re}\nu>1/2$	$\dfrac{\pi}{y^{1/2}\cos(\nu\pi)}[\cos(ay) J_\nu(ay) +$ $+\sin(ay) Y_\nu(ay)]$
(29)	$x^\lambda Y_\nu(ax)$, $a>0$ $-1+\lvert\operatorname{Re}\nu\rvert<\operatorname{Re}\lambda<3/2$	$-\dfrac{\pi\operatorname{ctg}(\nu\pi)\,y^\lambda}{\sin[(\nu+\lambda)\pi]} J_\nu\left(\dfrac{ay}{2}\right)-$ $-\dfrac{\pi y^\lambda}{\sin(\nu\pi)\sin[(\nu-\lambda)\pi]} J_{-\nu}\left(\dfrac{ay}{2}\right)-$ $-2^{\lambda-1}\pi^{-1}a^{-\lambda}\cos[2^{-1}(\lambda-\nu)\pi]\times$ $\times\Gamma\left(\dfrac{\lambda-\nu}{2}\right)\Gamma\left(\dfrac{\lambda+\nu}{2}\right)\times$ $\times {}_1F_2\left(1;1-\dfrac{\lambda-\nu}{2},1-\dfrac{\lambda+\nu}{2};-\dfrac{a^2y^2}{4}\right)+$ $+2^{\lambda-2}\pi^{-1}a^{1-\lambda}y\sin\left(\dfrac{\lambda-\nu}{2}\pi\right)\times$ $\times\Gamma\left(\dfrac{\lambda-\nu-1}{2}\right)\Gamma\left(\dfrac{\lambda+\nu-1}{2}\right)\times$ $\times {}_1F_2\left(1;\dfrac{3-\lambda+\nu}{2},\dfrac{3-\lambda-\nu}{2};-\dfrac{a^2y^2}{4}\right)$
(30)	$x^{\nu/2-1/2} Y_\nu(ax^{1/2})$, $a>0$, $-1/2<\operatorname{Re}\nu<5/2$	$-2y^{\nu/2-1/2} K_\nu(ay^{1/2})$
(31)	$x^{\nu/2+1/2} Y_\nu(ax^{1/2})$, $a>0$, $-3/2<\operatorname{Re}\nu<1/2$	$2y^{\nu/2+1/2} K_\nu(ay^{1/2})$
(32)	$x^\lambda Y_\nu(ax^{1/2})$, $a>0$ $-1+2^{-1}\lvert\operatorname{Re}\nu\rvert<\operatorname{Re}\lambda<3/4$	$y^\lambda G_{24}^{31}\left(\dfrac{a^2y}{4}\,\middle\vert\, {-\lambda,\ -\frac{\nu+1}{2} \atop -\lambda,\ -\frac{\nu}{2},\ \frac{\nu}{2},\ -\frac{\nu+1}{2}}\right)$

	$f(x)$	$\int_0^\infty f(x)(x+y)^{-1}\,dx, \quad \lvert\arg y\rvert<\pi$
(33)	$x^{\lambda-1}\{\cos[(\lambda-\nu/2)\pi]\,J_\nu(ax^{1/2})+$ $+\sin[(\lambda-\nu/2)\pi]\,Y_\nu(ax^{1/2})\}$, $a>0,\ \lvert\operatorname{Re}\nu\rvert<2\operatorname{Re}\lambda<7/2$	$-2y^{\lambda-1}K_\nu(ay^{1/2})$
(34)	$x^{\mu/2+n-1/2}(x+\gamma)^{-\nu/2}Y_\mu(ax^{1/2})\times$ $\times J_\nu[b(x+\gamma)^{1/2}]$, $a>b>0,\ n=0,1,2,\ldots$ $-1/2-n<\operatorname{Re}\mu<3-2n+\operatorname{Re}\nu$	$2(-1)^{n+1}y^{\mu/2+n-1/2}(y-\gamma)^{-\nu/2}\times$ $\times K_\mu(ay^{1/2})\,I_\nu[b(y-\gamma)^{1/2}]$
(35)	$x^{\lambda-1}(x+\gamma)^{-\mu/2}J_\mu[b(x+\gamma)^{1/2}]\times$ $\times\{\cos[(\lambda-\nu/2)\pi]\,J_\nu(ax^{1/2})+$ $+\sin[(\lambda-\nu/2)\pi]\,Y_\nu(ax^{1/2})\}$, $a>b>0$ $\lvert\operatorname{Re}\nu\rvert<2\operatorname{Re}\lambda<4+\operatorname{Re}\mu$	$-2y^{\lambda-1}(y-\gamma)^{-\mu/2}\times$ $\times I_\mu[b(y-\gamma)^{1/2}]\,K_\nu(ay^{1/2})$
(36)	$x^\nu e^{-\alpha x}I_\nu(\alpha x)$, $\operatorname{Re}\alpha>0,\ -1/2<\operatorname{Re}\nu<1/2$	$\dfrac{y^\nu e^{\alpha y}K_\nu(\alpha y)}{\cos(\nu\pi)}$
(37)	$x^\lambda e^{-\alpha x}I_\nu(\alpha x)$, $\operatorname{Re}\alpha>0,\ \operatorname{Re}\lambda<1/2$ $\operatorname{Re}(\lambda+\nu)>-1$	$\pi^{-1/2}y^\lambda G_{23}^{22}\left(2\alpha y\,\middle\vert\,{-\lambda,\ 1/2 \atop -\lambda,\ \nu,\ -\nu}\right)$
(38)	$x^\lambda e^{\alpha x}K_\nu(\alpha x)$, $\lvert\arg\alpha\rvert<3\pi/2$ $\operatorname{Re}\lambda-\lvert\operatorname{Re}\nu\rvert>-1$	$\pi^{-1/2}\cos(\nu\pi)\,y^\lambda\times$ $\times G_{23}^{32}\left(2\alpha y\,\middle\vert\,{-\lambda,\ 1/2 \atop -\lambda,\ \nu,\ -\nu}\right)$
(39)	$x^{-1/2}e^{-\alpha x}K_\nu(\alpha x)$, $\operatorname{Re}\alpha>0,\ -1/2<\operatorname{Re}\nu<1/2$	$\dfrac{\pi e^{\alpha y}K_\nu(\alpha y)}{y^{1/2}\cos(\nu\pi)}$
(40)	$x^\lambda e^{-\alpha x}K_\nu(\alpha x)$, $\operatorname{Re}\alpha>0,\ \operatorname{Re}\lambda-\lvert\operatorname{Re}\nu\rvert>-1$	$\pi^{1/2}y^\lambda G_{23}^{31}\left(2\alpha y\,\middle\vert\,{-\lambda,\ 1/2 \atop -\lambda,\ \nu,\ -\nu}\right)$
(41)	$x^{-\nu/2-1/2}K_\nu(\alpha x^{1/2})$, $\operatorname{Re}\alpha>0,\ \operatorname{Re}\nu<1/2$	$\dfrac{\pi^2}{2y^{\nu/2+1/2}\cos(\nu\pi)}\times$ $\times[\mathbf{H}_\nu(\alpha y^{1/2})-Y_\nu(\alpha y^{1/2})]$
(42)	$x^\lambda K_\nu(\alpha x^{1/2})$, $\operatorname{Re}\alpha>0,\ \operatorname{Re}\lambda>2^{-1}\lvert\operatorname{Re}\nu\rvert-1$	$2^{2\lambda+1}y^\lambda\,\Gamma(1+\lambda+\nu/2)\times$ $\times\Gamma(1+\lambda-\nu/2)\,S_{-2\lambda-1,\,\nu}(\alpha y^{1/2})$
(43)	$x^{-1/2}[2\pi^{-1}K_0(\alpha x^{1/4})-Y_0(\alpha x^{1/4})]$, $\lvert\arg\alpha\rvert<\pi/4$	$4y^{-1/4}\operatorname{ker}(\alpha y^{1/4})$

	$f(x)$	$\int_0^\infty f(x)(x+y)^{-1}\,dx, \quad \mid\arg y\mid<\pi$
(44)	$x^{\nu/2}\,\mathbf{H}_\nu(ax^{1/2})$, $a>0,\ -1/2<\operatorname{Re}\nu<1/2$	$\dfrac{\pi y^{\nu/2}\left[I_{-\nu}(ay^{1/2})-\mathbf{L}_\nu(ay^{1/2})\right]}{\cos(\nu\pi)}$
(45)	$x^{-\nu/2}\,\mathbf{H}_\nu(ax^{1/2})$, $a>0,\ \operatorname{Re}\nu>-3/2$	$\pi y^{-\nu/2}\left[I_\nu(ay^{1/2})-\mathbf{L}_\nu(ay^{1/2})\right]$
(46)	$x^\lambda\,\mathbf{H}_\nu(ax^{1/2})$, $a>0,\ \operatorname{Re}\lambda<3/4$ $-1/2<\operatorname{Re}(\lambda+\nu/2)<1/2$	$\dfrac{\pi}{\cos[(\lambda+\nu/2)\pi]}\times$ $\times\left[\dfrac{(2/a)^{2\lambda}}{\Gamma(1-\lambda+\nu/2)\,\Gamma(1-\lambda-\nu/2)}\times\right.$ $\times\,{}_1F_2(1;\ 1-\lambda+\nu/2,\ 1-\lambda-\nu/2;$ $\left.2^{-2}a^2y)-y^\lambda\mathbf{L}_\nu(ay^{1/2})\right]$
(47)	$x^{-1/2}\left[\cos(2^{-1}\nu\pi)\,J_\nu(ax^{1/2})+\right.$ $\left.+\sin(2^{-1}\nu\pi)\,\mathbf{H}_\nu(ax^{1/2})\right]$, $a>0,\ -1/2<\operatorname{Re}\nu<2$	$\pi y^{-1/2}\left[I_\nu(ay^{1/2})-\mathbf{L}_\nu(ay^{1/2})\right]$
(48)	$x^{-1/2}\left[I_\nu(\alpha x^{1/2})-\mathbf{L}_\nu(\alpha x^{1/2})\right]$, $\operatorname{Re}\alpha>0,\ -1<\operatorname{Re}\nu<2$	$\dfrac{\pi\left[\mathbf{H}_\nu(\alpha y^{1/2})+\mathbf{E}_\nu(\alpha y^{1/2})\right]}{\alpha\sin(\nu\pi/2)}$
(49)	$x^\lambda\left[I_\nu(\alpha x^{1/2})-\mathbf{L}_\nu(\alpha x^{1/2})\right]$, $\operatorname{Re}\alpha>0$ $-2<\operatorname{Re}(2\lambda+\nu)<1$	$\dfrac{y^\lambda}{\pi}\,G_{24}^{32}\left(\dfrac{\alpha^2y}{4}\,\middle\vert\,\begin{matrix}-\lambda,\ \nu/2+1/2\\ -\lambda,\ \nu/2,\ \nu/2+1/2,\ -\nu/2\end{matrix}\right)$

Относительно других интегралов, содержащих функции Бесселя, см. Г. Н. Ватсон (1949), в частности пп. 13.5—13.55.

(50)	$x^{\mu-1/2}\exp(-\alpha x/2)\,M_{\varkappa,\mu}(\alpha x)$, $\operatorname{Re}\alpha>0$ $-1/2<\operatorname{Re}\mu<\operatorname{Re}\varkappa+1/2$	$\Gamma(2\mu+1)\,\Gamma(\varkappa-\mu+1/2)\,y^{\mu-1/2}\times$ $\times\exp(\alpha x/2)\,W_{-\varkappa,\mu}(\alpha x)$
(51)	$x^\lambda\exp(-\alpha x/2)\,M_{\varkappa,\mu}(\alpha x)$, $\operatorname{Re}\alpha>0$ $-3/2-\operatorname{Re}\mu<\operatorname{Re}\lambda<\operatorname{Re}\varkappa$	$\dfrac{\Gamma(2\mu+1)\,y^\lambda}{\Gamma(\varkappa+\mu+1/2)}\times$ $\times\,G_{23}^{22}\left(\alpha x\,\middle\vert\,\begin{matrix}-\lambda,\ 1-\varkappa\\ -\lambda,\ \mu+1/2,\ 1/2-\mu\end{matrix}\right)$
(52)	$x^\lambda\exp(\alpha x/2)\,W_{\varkappa,\mu}(\alpha x)$, $\mid\arg\alpha\mid<3\pi/2$ $\operatorname{Re}(\varkappa+\lambda)<0$ $\operatorname{Re}\lambda>\mid\operatorname{Re}\mu\mid-3/2$	$y^\lambda\left[\Gamma(1/2+\mu-\varkappa)\,\Gamma(1/2-\mu-\varkappa)\right]^{-1}\times$ $\times\,G_{23}^{32}\left(\alpha y\,\middle\vert\,\begin{matrix}-\lambda,\ \varkappa+1\\ -\lambda,\ 1/2+\mu,\ 1/2-\mu\end{matrix}\right)$

	$f(x)$	$\int_0^\infty f(x)(x+y)^{-1}\,dx,\quad \lvert\arg y\rvert<\pi$
(53)	$x^{\varkappa-1}\exp(-\alpha x/2)\,W_{\varkappa,\mu}(\alpha x)$, $\operatorname{Re}\alpha>0$, $\operatorname{Re}\varkappa>\lvert\operatorname{Re}\mu\rvert-1/2$	$\Gamma(\varkappa+\mu+1/2)\,\Gamma(\varkappa-\mu+1/2)\,y^{\varkappa-1}\times$ $\times\exp(\alpha y/2)\,W_{-\varkappa,\mu}(\alpha y)$
(54)	$x^{\lambda}\exp(-\alpha x/2)\,W_{\varkappa,\mu}(\alpha x)$, $\operatorname{Re}\alpha>0$, $\operatorname{Re}\lambda>\lvert\operatorname{Re}\mu\rvert-3/2$	$y^{\lambda}\,G_{23}^{31}\left(\alpha y\,\middle\vert\,\begin{matrix}-\lambda,\ 1-\varkappa\\ -\lambda,\ 1/2+\mu,\ 1/2-\mu\end{matrix}\right)$
(55)	$G_{pq}^{mn}\left(\alpha x\,\middle\vert\,\begin{matrix}a_1,\dots,a_p\\ b_1,\dots,b_q\end{matrix}\right)$, $p+q<2(m+n)$, $\lvert\arg\alpha\rvert<(m+n-p/2-q/2)\pi$, $\operatorname{Re}a_j<1$, $j=1,\dots,n$, $\operatorname{Re}b_j>-1$, $j=1,\dots,m$	$G_{p+1,\,q+1}^{m+1,\,n+1}\left(\alpha y\,\middle\vert\,\begin{matrix}0,\ a_1,\dots,a_p\\ 0,\ b_1,\dots,b_q\end{matrix}\right)$
(56)	$G_{pq}^{mn}\left(\alpha x^2\,\middle\vert\,\begin{matrix}a_1,\dots,a_p\\ b_1,\dots,b_q\end{matrix}\right)$, $p+q<2(m+n)$, $\lvert\arg\alpha\rvert<(m+n-p/2-q/2)\pi$, $\operatorname{Re}a_j<1$, $j=1,\dots,n$, $\operatorname{Re}b_j>-1/2$, $j=1,\dots,m$	$\frac{1}{2\pi}\,G_{p+2,\,q+2}^{m+2,\,n+2}\left(\alpha y^2\,\middle\vert\,\begin{matrix}0,\ 1/2,\ a_1,\dots,a_p\\ 0,\ 1/2,\ b_1,\dots,b_q\end{matrix}\right)$

14.4. Обобщенные преобразования Стилтьеса

	$f(x)$	$\int_0^\infty f(x)(x+y)^{-\rho}\,dx=g(y;\rho)$, $\lvert\arg y\rvert<\pi$
(1)	$f(x)$	$g(y;\rho)$
(2)	$x\,f(x)$	$g(y;\rho-1)-y\,g(y;\rho)$
(3)	$f(ax)$, $a>0$	$a^{\rho-1}\,g(ay;\rho)$
(4)	$x^{\rho-2}\,f(a/x)$, $a>0$	$a^{\rho-1}y^{-\rho}\,g(a/y)$
(5)	$f'(x)$	$\rho\,g(y;\rho+1)-y^{-\rho}f(0)$
(6)	$\int_0^x f(t)\,dt$	$(\rho-1)^{-1}\,g(y;\rho-1)$, $\operatorname{Re}\rho>1$

	$f(x)$	$\int\limits_0^\infty f(x)(x+y)^{-\rho}\,dx, \quad \lvert\arg y\rvert<\pi$
(7)	$[\Gamma(\mu)]^{-1}\int\limits_0^x f(t)(x-t)^{\mu-1}\,dt,$ $0<\operatorname{Re}\mu<\operatorname{Re}\rho$	$\frac{\Gamma(\rho-\mu)}{\Gamma(\rho)}\, g(y;\ \rho-\mu)$
(8)	$x^{\nu-1}, \quad \operatorname{Re}\nu>0$	$\frac{\Gamma(\nu)\,\Gamma(\rho-\nu)}{\Gamma(\rho)}\, y^{\nu-\rho}, \quad \operatorname{Re}\rho>\operatorname{Re}\nu$
(9)	$x^{\nu-1}(\alpha+x)^{-\mu},$ $\lvert\arg\alpha\rvert<\pi,\ \operatorname{Re}\nu>0$	$\frac{\Gamma(\nu)\,\Gamma(\mu-\nu+\rho)\, y^{\nu-\rho}}{\Gamma(\mu+\rho)\,\alpha^{\mu}}\times$ $\times\, {}_2F_1(\mu,\ \nu;\ \mu+\rho;\ 1-y/\alpha),$ $\operatorname{Re}\rho>\operatorname{Re}(\nu-\mu)$
(10)	$e^{-\alpha x}, \quad \operatorname{Re}\alpha>0$	$\alpha^{\rho-1}e^{\alpha y}\,\Gamma(1-\rho,\ \alpha y)$
(11)	$x^{-\rho}e^{-\alpha x}, \quad \operatorname{Re}\alpha>0,\ \operatorname{Re}\rho<1$	$\pi^{-1/2}\,\Gamma(1-\rho)\,(\alpha/y)^{\rho-1/2}\times$ $\times\exp(\alpha y/2)\, K_{\rho-1/2}(\alpha y/2)$
(12)	$x^{\lambda}e^{-\alpha x}, \quad \operatorname{Re}\alpha>0, \quad \operatorname{Re}\lambda>-1$	$\Gamma(\lambda+1)\,\alpha^{\rho/2-\lambda/2-1}y^{\lambda/2-\rho/2}\times$ $\times\exp(\alpha y/2)\, W_{k,m}(\alpha y),$ $2k=-\lambda-\rho,\ 2m=\lambda-\rho+1$
(13)	$x^{\lambda}\exp(-\alpha/x), \quad \operatorname{Re}\alpha>0$	$\Gamma(\rho-\lambda-1)\,\alpha^{\lambda/2}y^{-\lambda/2-1}\times$ $\times\exp\left(\frac{\alpha}{2y}\right) W_{k,m}\left(\frac{\alpha}{y}\right),$ $k=\lambda/2-\rho+1,\ m=\lambda/2+1/2$ $\operatorname{Re}\rho>\operatorname{Re}\lambda+1$
(14)	$x^{-1/2}\exp(-\alpha x^{1/2}), \quad \operatorname{Re}\alpha>0$	$\pi^{1/2}(2\alpha^{-1}y^{1/2})^{1/2-\rho}\,\Gamma(1-\rho)\times$ $\times\left[\mathbf{H}_{1/2-\rho}(\alpha y^{1/2})-Y_{1/2-\rho}(\alpha y^{1/2})\right]$
(15)	$x^{\lambda}\exp(-\alpha x^{1/2}),$ $\operatorname{Re}\alpha>0,\ \operatorname{Re}\lambda>-1$	$\frac{y^{\lambda-\rho+1}}{\pi^{1/2}\,\Gamma(\rho)}\, G_{13}^{31}\left(\frac{\alpha^2 y}{4}\middle\vert \begin{matrix}-\lambda\\ \rho-\lambda-1,\ 0,\ 1/2\end{matrix}\right)$
(16)	$\sin(ax^{1/2}), \quad a>0$	$\frac{2\pi^{1/2}y^{1/2}}{\Gamma(\rho)}\left(\frac{2y^{1/2}}{a}\right)^{1/2-\rho} K_{\rho-3/2}(ay^{1/2}),$ $\operatorname{Re}\rho>1/2$
(17)	$x^{\lambda}\sin(ax^{1/2}),$ $a>0,\ \operatorname{Re}\lambda>-1/2$	$\frac{\pi^{1/2}y^{\lambda-\rho+1}}{\Gamma(\rho)}\, G_{13}^{21}\left(\frac{a^2 y}{4}\middle\vert \begin{matrix}-\lambda\\ \rho-\lambda-1,\ 1/2,\ 0\end{matrix}\right),$ $\operatorname{Re}\rho>\operatorname{Re}\lambda+1/2$

	$f(x)$	$\int_0^\infty f(x)(x+y)^{-\rho}\,dx, \quad \lvert\arg y\rvert<\pi$
(18)	$x^{-1/2}\cos(ax^{1/2}), \quad a>0$	$\frac{2\pi^{1/2}}{\Gamma(\rho)}\left(\frac{2y^{1/2}}{a}\right)^{1/2-\rho} K_{\rho-1/2}(ay^{1/2})$, $\operatorname{Re}\rho>0$
(19)	$x^{\lambda}\cos(ax^{1/2})$, $a>0, \operatorname{Re}\lambda>-1$	$\frac{\pi^{1/2}y^{\lambda-\rho+1}}{\Gamma(\rho)} G_{13}^{21}\left(\frac{a^2y}{4}\middle\vert \begin{matrix}-\lambda\\ \rho-\lambda-1,\ 0,\ 1/2\end{matrix}\right)$, $\operatorname{Re}\rho>\operatorname{Re}\lambda+1/2$
(20)	$x^{\nu/2}J_{\nu}(ax^{1/2})$, $a>0, \operatorname{Re}\nu>-1$	$\frac{a^{\rho-1}}{2^{\rho}\Gamma(\rho)} y^{\nu/2-\rho/2+1/2} K_{\nu-\rho+1}(ay^{1/2})$, $\operatorname{Re}\rho>2^{-1}\operatorname{Re}\nu+1/4$
(21)	$x^{\lambda}J_{\nu}(ax^{1/2})$, $a>0, \operatorname{Re}(\lambda+\nu/2)>-1$	$\frac{2^{2\lambda}y^{1-\rho}}{a^{2\lambda}\Gamma(\rho)}\times$ $\times G_{13}^{21}\left(\frac{a^2y}{4}\middle\vert \begin{matrix}0\\ \rho-1,\ \lambda+\nu/2,\ \lambda-\nu/2\end{matrix}\right)$, $\operatorname{Re}\rho>\operatorname{Re}\lambda+1/4$
(22)	$x^{\nu}e^{-\alpha x}I_{\nu}(\alpha x)$, $\operatorname{Re}\alpha>0, \operatorname{Re}\nu>-1/2$	$\frac{\Gamma(\nu+1/2)\Gamma(\rho-\nu-1/2)}{\pi^{1/2}\Gamma(\rho)}(2\alpha)^{\rho/2-1}\times$ $\times y^{\nu-\rho/2}e^{\alpha y}W_{k,m}(2\alpha y)$, $k=1/2-\rho/2,\ m=1/2-\rho/2+\nu$ $\operatorname{Re}\rho>\operatorname{Re}\nu+1/2$
(23)	$x^{\lambda}e^{-\alpha x}I_{\nu}(\alpha x)$, $\operatorname{Re}\alpha>0, \operatorname{Re}(\lambda+\nu)>-1$	$\frac{y^{\lambda+1-\rho}}{\pi^{1/2}\Gamma(\rho)} G_{23}^{22}\left(2\alpha y\middle\vert \begin{matrix}-\lambda,\ 1/2\\ \rho-\lambda-1,\ \nu,\ -\nu\end{matrix}\right)$, $\operatorname{Re}\rho>\operatorname{Re}\lambda+1/2$
(24)	$x^{\lambda}e^{\alpha x}K_{\nu}(\alpha x) \quad \lvert\arg\alpha\rvert<3\pi/2$ $\operatorname{Re}\lambda>\lvert\operatorname{Re}\nu\rvert-1$	$\frac{\cos(\nu\pi)}{\pi^{1/2}\Gamma(\rho)} y^{\lambda+1-\rho}\times$ $\times G_{23}^{32}\left(2\alpha y\middle\vert \begin{matrix}-\lambda,\ 1/2\\ \rho-\lambda-1,\ \nu,\ -\nu\end{matrix}\right)$, $\operatorname{Re}\rho>\operatorname{Re}\lambda+1/2$
(25)	$x^{\rho-3/2}e^{-\alpha x}K_{\nu}(\alpha x)$, $\operatorname{Re}\alpha>0, \operatorname{Re}\rho>\lvert\operatorname{Re}\nu\rvert+1/2$	$\frac{\pi^{1/2}\Gamma(\rho+\nu-1/2)\Gamma(\rho-\nu-1/2)}{(2\alpha)^{1/2}y\,\Gamma(\rho)}\times$ $\times e^{\alpha y}W_{1-\rho,\nu}(2\alpha y)$
(26)	$x^{\lambda}e^{-\alpha x}K_{\nu}(\alpha x)$, $\operatorname{Re}\alpha>0, \operatorname{Re}\lambda>\lvert\operatorname{Re}\nu\rvert-1$	$\frac{\pi^{1/2}}{\Gamma(\rho)} y^{\lambda+1-\rho}\times$ $\times G_{23}^{31}\left(2\alpha y\middle\vert \begin{matrix}-\lambda,\ 1/2\\ \rho-\lambda-1,\ \nu,\ -\nu\end{matrix}\right)$

	$f(x)$	$\int\limits_0^\infty f(x)(x+y)^{-\rho}\,dx, \quad \lvert\arg y\rvert<\pi$
(27)	$x^{\lambda}K_{\nu}(\alpha x^{1/2}), \quad \operatorname{Re}\alpha>0$ $\operatorname{Re}\lambda>2^{-1}\lvert\operatorname{Re}\nu\rvert-1$	$\dfrac{y^{\lambda-\rho+1}}{2\Gamma(\rho)}\times$ $\times G_{13}^{31}\left(\dfrac{\alpha^2 y}{4}\,\middle\vert\,\begin{matrix}-\lambda\\ \rho-\lambda-1,\ \nu/2,\ -\nu/2\end{matrix}\right)$
(28)	$x^{\mu-1/2}\exp(-\alpha x/2)M_{\varkappa,\mu}(\alpha x),$ $\operatorname{Re}\alpha>0,\ \operatorname{Re}\mu>-1/2$	$\Gamma(2\mu+1)\Gamma(\varkappa+\rho-\mu-1/2)\times$ $\times[\Gamma(\rho)]^{-1}\alpha^{\rho/2-1/2}y^{\lambda+1/2-\rho/2}\times$ $\times\exp(\alpha y/2)\,W_{k,m}(\alpha y),$ $k=1/2-\rho/2-\varkappa,\ m=1/2-\rho/2+\mu$ $\operatorname{Re}\rho>\operatorname{Re}(\mu-\varkappa)+1/2$
(29)	$x^{\lambda}\exp(-\alpha x/2)M_{\varkappa,\mu}(\alpha x),$ $\operatorname{Re}\alpha>0,\ \operatorname{Re}(\lambda+\mu)>-3/2$	$\dfrac{\Gamma(2\mu+1)\,y^{\lambda-\rho+1}}{\Gamma(\rho)\Gamma(\varkappa+\mu+1/2)}\times$ $\times G_{23}^{22}\left(\alpha y\,\middle\vert\,\begin{matrix}-\lambda,\ 1-\varkappa\\ \rho-\lambda-1,\ 1/2+\mu,\ 1/2-\mu\end{matrix}\right),$ $\operatorname{Re}\rho>\operatorname{Re}(\lambda-\varkappa)+1$
(30)	$x^{\lambda}\exp(\alpha x/2)W_{\varkappa,\mu}(\alpha x),$ $\lvert\arg\alpha\rvert<3\pi/2$ $\operatorname{Re}\lambda>\lvert\operatorname{Re}\mu\rvert-3/2$	$\dfrac{y^{\lambda-\rho+1}}{\Gamma(\rho)\Gamma(1/2-\varkappa+\mu)\Gamma(1/2-\varkappa-\mu)}\times$ $\times G_{23}^{32}\left(\alpha y\,\middle\vert\,\begin{matrix}-\lambda,\ 1+\varkappa\\ \rho-\lambda-1,\ 1/2+\mu,\ 1/2-\mu\end{matrix}\right),$ $\operatorname{Re}\rho>\operatorname{Re}(\lambda+\varkappa)+1$
(31)	$x^{\varkappa+\rho-2}\exp(-\alpha x/2)W_{\varkappa,\mu}(\alpha x),$ $\operatorname{Re}\alpha>0$ $\operatorname{Re}\rho>\lvert\operatorname{Re}\mu\rvert-\operatorname{Re}\varkappa+1/2$	$\Gamma(\varkappa+\rho+\mu-1/2)\times$ $\times\Gamma(\varkappa+\rho-\mu-1/2)[\Gamma(\rho)]^{-1}\times$ $\times y^{\varkappa-1}\exp(\alpha y/2)\,W_{1-\varkappa-\rho,\mu}(\alpha y)$
(32)	$x^{\lambda}\exp(-\alpha x/2)W_{\varkappa,\mu}(\alpha x),$ $\operatorname{Re}\alpha>0$ $\operatorname{Re}\lambda>\lvert\operatorname{Re}\mu\rvert-3/2$	$\dfrac{y^{\lambda+1-\rho}}{\Gamma(\rho)}\times$ $\times G_{23}^{31}\left(\alpha y\,\middle\vert\,\begin{matrix}-\lambda,\ 1-\varkappa\\ \rho-\lambda-1,\ 1/2+\mu,\ 1/2-\mu\end{matrix}\right)$
(33)	$G_{pq}^{mn}\left(\alpha x\,\middle\vert\,\begin{matrix}a_1,\ldots,a_p\\ b_1,\ldots,b_q\end{matrix}\right),$ $p+q<2(m+n)$ $\lvert\arg\alpha\rvert<(m+n-p/2-q/2)\pi$ $\operatorname{Re}b_j>-1,\quad j=1,\ldots,m$	$\dfrac{y^{1-\rho}}{\Gamma(\rho)}\times$ $\times G_{p+1,q+1}^{m+1,n+1}\left(\alpha y\,\middle\vert\,\begin{matrix}0,\ a_1,\ldots,a_p\\ \rho-1,\ b_1,\ldots,b_q\end{matrix}\right),$ $\operatorname{Re}\rho>\operatorname{Re}a_j,\quad j=1,\ldots,n$

ГЛАВА XV

ПРЕОБРАЗОВАНИЯ ГИЛЬБЕРТА

Мы называем функцию

$$g(y)=\pi^{-1}\,\fint_{-\infty}^{\infty} f(x)\,(x-y)^{-1}\,dx$$

преобразованием Гильберта функции $f(x)$. Здесь x и y - вещественные переменные и

$$\fint_{-\infty}^{\infty}=\lim_{\varepsilon\to+0}\left(\int_{-\infty}^{y-\varepsilon}+\int_{y+\varepsilon}^{\infty}\right)$$

— главное значение интеграла $\int_{-\infty}^{\infty}$ в смысле Коши.

Относительно теории преобразований Гильберта см. главу V книги Титчмарша (1948) и указанные там ссылки. На стр. 321 приводятся ссылки на литературу, вышедшую после опубликования книги Титчмарша. Конечное преобразование Гильберта

$$g(y)=\pi^{-1}\,\fint_{a}^{b} f(x)\,(x-y)^{-1}\,dx$$

и его приложения к теории крыла были рассмотрены Трикоми (Tricomi 1951 a, b) и Никкелем (Nickel, 1951, 1953). Второй из авторов дает ссылки на более ранние работы по этому вопросу.

В указанных выше соотношениях $g(x)$ называют функцией, *сопряженной* к $f(x)$. Это отношение является *косовзаимным*, т. е. сопряженным к $g(x)$ является $-f(x)$. Относительно связи между преобразованиями Гильберта и интегралами Фурье см. Титчмарш (1948) и Кобер (Kober, 1942, 1943 a, b). Связь с преобразованиями Лапласа устанавливается, если заметить, что, формально, мнимая часть преобразования Лапласа, вычисленного вдоль прямой, параллельной мнимой оси, сопряжена с вещественной частью этого же преобразования, вычисленного вдоль той же прямой. Формулы

$$g(y)=\pi^{-1}\mathfrak{S}\{f(x);\ -y\}-(2\pi)^{-1}\mathfrak{S}\{f(-x);\ |y|\,e^{i\pi}\}-$$
$$-(2\pi)^{-1}\mathfrak{S}\{f(-x);\ |y|\,e^{-i\pi}\},\qquad -\infty<y<0,$$

$$g(y)=(2\pi)^{-1}\mathfrak{S}\{f(x);\ ye^{i\pi}\}+(2\pi)^{-1}\mathfrak{S}\{f(x);\ ye^{-i\pi}\}-$$
$$-\pi^{-1}\mathfrak{S}\{f(-x);\ y\},\qquad 0<y<\infty,$$

позволяют свести вычисление преобразования Гильберта к использованию таблиц преобразования Стилтьеса (см. гл. XIV).

С преобразованиями Гильберта связаны преобразования

$$\int_{-\pi}^{\pi} f(x) \operatorname{ctg}\left[2^{-1}(x-y)\right] dx,$$

$$\int_{0}^{\pi} f(x)(\cos x-\cos y)^{-1} dx.$$

Они сводятся к преобразованиям Гильберта путем замены переменной.

Из приведенных здесь пар преобразований могут быть получены дальнейшие пары преобразований с помощью методов, указанных во введении к первому тому, общих формул, указанных в п. 15.1, и использования связей с другими интегральными преобразованиями.

15.1. Общие формулы

	$f(x)$	$\pi^{-1} \int_{-\infty}^{\infty} f(x)(x-y)^{-1} dx=g(y)$ *)
(1)	$f(x)$	$g(y)$
(2)	$g(x)$	$-f(y)$
(3)	$f(a+x)$, a — вещественное	$g(a+y)$
(4)	$f(ax)$, $a>0$	$g(ay)$
(5)	$f(-ax)$, $a>0$	$-g(-ay)$
(6)	$x f(x)$	$y g(y)+\pi^{-1} \int_{-\infty}^{\infty} f(x) dx$
(7)	$(x+a) f(x)$	$(y+a) g(y)+\pi^{-1} \int_{-\infty}^{\infty} f(x) dx$
(8)	$f'(x)$	$g'(y)$

*) y вещественно, и интеграл понимается в смысле главного значения по Коши.

15.2. Элементарные функции

	$f(x)$		$\pi^{-1}\int_{-\infty}^{\infty} f(x)\,(x-y)^{-1}\,dx$
(1)	1		0
(2)	$0,$ $1,$ $0,$	$-\infty<x<a$ $a<x<b$ $b<x<\infty$	$\frac{1}{\pi}\ln\left\lvert\frac{b-y}{a-y}\right\rvert$
(3)	$0,$ $x^{-1},$	$-\infty<x<a$ $a<x<\infty$ $a>0$	$\frac{1}{\pi y}\ln\left\lvert\frac{a}{a-y}\right\rvert,\quad y\neq 0,\ y\neq a$
(4)	$x^{-1},$ $0,$ $x^{-1},$	$-\infty<x<a$ $a<x<b$ $b<x<\infty$ $a<0<b$	$\frac{1}{\pi y}\ln\left\lvert\frac{(y-a)\,b}{a\,(b-y)}\right\rvert,$ $y\neq 0,\ a,\ b$
(5)	$0,$ $x^{-2},$	$-\infty<x<a$ $a<x<\infty$ $a>0$	$\frac{1}{\pi y^2}\ln\left\lvert\frac{a}{a-y}\right\rvert-\frac{1}{\pi a y},$ $y\neq 0,\ y\neq a$
(6)	$(x+\alpha)^{-1},$	$\operatorname{Im}\alpha>0$	$i\,(y+\alpha)^{-1}$
(7)	$(x+\alpha)^{-1},$	$\operatorname{Im}\alpha<0$	$-i\,(y+\alpha)^{-1}$
(8)	$0,$ $(ax+b)^{-1},$	$-\infty<x<0$ $0<x<\infty$ $a,\ b>0$	$\frac{1}{\pi\,(ay+b)}\ln\left\lvert\frac{b}{ay}\right\rvert,$ $y\neq -b/a,\ y\neq 0$
(9)	$0,$ $(ax+b)^{-2},$	$-\infty<x<0$ $0<x<\infty$ $a,\ b>0$	$\frac{1}{\pi\,(ay+b)^2}\ln\left\lvert\frac{b}{ay}\right\rvert-\frac{1}{\pi b\,(ay+b)},$ $y\neq 0,\ y\neq -b/a$
(10)	$(x^2+\alpha^2)^{-1},$	$\operatorname{Re}\alpha>0$	$-\frac{y}{\alpha\,(y^2+\alpha^2)}$
(11)	$\frac{x}{x^2+\alpha^2},$	$\operatorname{Re}\alpha>0$	$\frac{\alpha}{y^2+\alpha^2}$
(12)	$\frac{\lambda x+\mu\alpha}{x^2+\alpha^2},$	$\operatorname{Re}\alpha>0$	$\frac{\lambda\alpha-\mu y}{y^2+\alpha^2}$

y вещественно, и интеграл понимается в смысле главного значения по Коши.

	$f(x)$	$\pi^{-1}\int_{-\infty}^{\infty} f(x)(x-y)^{-1}\,dx$
(13)	$0, \quad -\infty<x<0$ $\dfrac{cx+d}{(ax+b)^2}, \quad 0<x<\infty$ $a,\ b>0$	$\dfrac{cy+d}{\pi(ay+b)^2}\ln\left\|\dfrac{b}{ay}\right\|-\dfrac{ad-bc}{\pi ab(ay+b)},$ $y\neq 0,\ y\neq -b/a$
(14)	$(a-x)^{1/2}-(b-x)^{1/2},$ $-\infty<x<a$ $-(b-x)^{1/2}, \quad a<x<b$ $0, \quad b<x<\infty$	$0, \quad -\infty<y<a$ $(y-a)^{1/2}, \quad a<y<b$ $(y-a)^{1/2}-(y-b)^{1/2}, \quad b<y<\infty$
(15)	$0, \quad -\infty<x<a$ $(x-a)^{1/2}, \quad a<x<b$ $(x-a)^{1/2}-(x-b)^{1/2}, \quad b<x<\infty$	$(b-y)^{1/2}-(a-y)^{1/2}, \quad -\infty<y<a$ $(b-y)^{1/2}, \quad a<y<b$ $0, \quad b<y<\infty$
(16)	$\|a-x\|^{1/2}-\|b-x\|^{1/2},$ $a>0,\ b>0$	$(b-y)^{1/2}-(a-y)^{1/2}, \quad -\infty<y<a$ $(b-y)^{1/2}-(y-a)^{1/2}, \quad a<y<b$ $(y-a)^{1/2}-(y-b)^{1/2}, \quad b<y<\infty$
(17)	$0, \quad -\infty<x<0$ $(ax+b)^{-1/2}, \quad 0<x<\infty$ $a,\ b>0$	$2\pi^{-1}(-ay-b)^{-1/2}\times$ $\times\operatorname{arctg}\{[-(ay+b)/b]^{1/2}\},$ $-\infty<y<-b/a$ $2\pi^{-1}b^{-1}, \quad y=-b/a$ $\dfrac{1}{(ay+b)^{1/2}}\ln\left\|\dfrac{b^{1/2}+(ay+b)^{1/2}}{b^{1/2}-(ay+b)^{1/2}}\right\|,$ $-b/a<y<\infty$
(18)	$0, \quad -\infty<x<0$ $(a^2-x^2)^{1/2}, \quad 0<x<a$ $0, \quad a<x<\infty$	$-\pi^{-1}a-y/2-\pi^{-1}(y^2-a^2)^{1/2}\times$ $\times\arccos(-a/y), \quad -\infty<y<-a$ $-\pi^{-1}a-y/2+\pi^{-1}(a^2-y^2)^{1/2}\times$ $\times\ln\left\|\dfrac{a+(a^2-y^2)^{1/2}}{-y}\right\|, \quad -a<y<a$ $-\pi^{-1}a-y/2+\pi^{-1}(y^2-a^2)^{1/2}\times$ $\times\arccos(-a/y), \quad a<y<\infty$ $0<\arccos(-a/y)<\pi$

y вещественно, и интеграл понимается в смысле главного значения по Коши.

	$f(x)$	$\pi^{-1}\int\limits_{-\infty}^{\infty} f(x)\,(x-y)^{-1}\,dx$
(19)	$0, \quad -\infty<x<-a$ $(a^2-x^2)^{1/2}, \quad -a<x<a$ $0, \quad a<x<\infty$	$-y-(y^2-a^2)^{1/2}, \quad -\infty<y<-a$ $-y, \quad -a<y<a$ $-y+(y^2-a^2)^{1/2} \quad a<y<\infty$
(20)	$0, \quad -\infty<x<0$ $(a^2-x^2)^{-1/2}, \quad 0<x<a$ $0, \quad a<x<\infty$	$\frac{\arccos(-a/y)}{\pi(y^2-a^2)^{1/2}}, \quad -\infty<y<-a$ $\frac{1}{\pi(a^2-y^2)^{1/2}}\ln\left\lvert\frac{a+(a^2-y^2)^{1/2}}{-y}\right\rvert,$ $-a<y<a$ $-\frac{\arccos(-a/y)}{\pi(y^2-a^2)^{1/2}}, \quad a<y<\infty$ $0<\arccos(-a/y)<\pi$
(21)	$0, \quad -\infty<x<-a$ $(a^2-x^2)^{-1/2}, \quad -a<x<a$ $0, \quad a<x<\infty$	$(y^2-a^2)^{-1/2}, \quad -\infty<y<-a$ $0, \quad -a<y<a$ $-(y^2-a^2)^{-1/2}, \quad a<y<\infty$
(22)	$0, \quad -\infty<x<a$ $(x^2-a^2)^{-1/2}, \quad a<x<\infty$ $a>0$	$\frac{1}{\pi(y^2-a^2)^{1/2}}\ln\left\lvert\frac{-y+(y^2-a^2)^{1/2}}{a}\right\rvert,$ $-\infty<y<-a$ $\frac{1}{\pi(a^2-y^2)^{1/2}}\arccos(-y/a),$ $-a<y<a$ $\frac{1}{\pi(y^2-a^2)^{1/2}}\ln\left\lvert\frac{-y+(y^2-a^2)^{1/2}}{a}\right\rvert,$ $a<y<\infty$ $0<\arccos(-y/a)<\pi$
(23)	$-(x^2-a^2)^{-1/2}, \quad -\infty<x<-a$ $0, \quad -a<x<a$ $(x^2-a^2)^{-1/2}, \quad a<x<\infty$	$0, \quad -\infty<y<-a$ $(a^2-y^2)^{-1/2}, \quad -a<y<a$ $0, \quad a<y<\infty$
(24)	$0, \quad -\infty<x<0$ $(a-x)^{1/2}(a+x)^{-1/2}, \quad 0<x<a$ $0, \quad a<x<\infty$	$-\frac{1}{2}+\frac{1}{\pi}\left\lvert\frac{a-y}{a+y}\right\rvert^{1/2}\arccos\left(-\frac{a}{y}\right),$ $-\infty<y<-a$ $-\frac{1}{2}+\frac{1}{\pi}\left(\frac{a-y}{a+y}\right)^{1/2}\ln\left\lvert\frac{a+(a^2-y^2)^{1/2}}{-y}\right\rvert,$ $-a<y<a$ $-\frac{1}{2}+\frac{1}{\pi}\left(\frac{y-a}{y+a}\right)^{1/2}\arccos\left(-\frac{a}{y}\right),$ $a<y<\infty$ $0<\arccos(-a/y)<\pi$

y вещественно, и интеграл понимается в смысле главного значения по Коши.

	$f(x)$	$\pi^{-1} \int_{-\infty}^{\infty} f(x)(x-y)^{-1}\,dx$
(25)	$0, \quad -\infty < x < -a$ $(a-x)^{1/2}(a+x)^{-1/2}, \quad -a < x < a$ $0, \quad a < x < \infty$	$-1+(a-y)^{1/2}\,\lvert y+a\rvert^{-1/2}, \quad -\infty < y < -a$ $-1, \quad -a < y < a$ $-1+(y-a)^{1/2}(y+a)^{-1/2}, \quad a < y < \infty$
(26)	$0, \quad -\infty < x < 0$ $(a+x)^{1/2}(a-x)^{-1/2}, \quad 0 < x < a$ $0, \quad a < x < \infty$	$\frac{1}{2}-\frac{1}{\pi}\left\lvert\frac{a+y}{a-y}\right\rvert^{1/2}\arccos\left(-\frac{a}{y}\right), \quad -\infty < y < -a$ $\frac{1}{2}+\frac{1}{\pi}\left(\frac{a+y}{a-y}\right)^{1/2}\ln\left\lvert\frac{a+(a^2-y^2)^{1/2}}{-y}\right\rvert, \quad -a < y < a$ $\frac{1}{2}-\frac{1}{\pi}\left(\frac{y+a}{y-a}\right)^{1/2}\arccos\left(-\frac{a}{y}\right), \quad a < y < \infty$ $0 < \arccos(-a/y) < \pi$
(27)	$0, \quad -\infty < x < -a$ $x(a-x)^{1/2}(a+x)^{-1/2}, \quad -a < x < a$ $0, \quad a < x < \infty$	$a-y+y\left\lvert\frac{a-y}{a+y}\right\rvert^{1/2}, \quad -\infty < y < -a$ $a-y, \quad -a < y < a$ $a-y+y\left(\frac{y-a}{y+a}\right)^{1/2}, \quad a < y < \infty$
(28)	$0, \quad -\infty < x < 0$ $x^{\nu-1}, \quad 0 < x < \infty$ $0 < \operatorname{Re}\nu < 1$	$\frac{(-y)^{\nu-1}}{\sin(\nu\pi)}, \quad -\infty < y < 0$ $-\operatorname{ctg}(\nu\pi)\,y^{\nu-1}, \quad 0 < y < \infty$
(29)	$\lvert x\rvert^{\nu-1}, \quad 0 < \operatorname{Re}\nu < 1$	$-\operatorname{ctg}(\nu\pi/2)\operatorname{sgn} y\,\lvert y\rvert^{\nu-1}$
(30)	$\operatorname{sgn} x\,\lvert x\rvert^{\nu-1}, \quad 0 < \operatorname{Re}\nu < 1$	$\operatorname{tg}(\nu\pi/2)\,\lvert y\rvert^{\nu-1}$
(31)	$0, \quad -\infty < x < a$ $(x-a)^{\nu}(b-x)^{-\nu}, \quad a < x < b$ $0, \quad b < x < \infty$ $\lvert\operatorname{Re}\nu\rvert < 1$	$\frac{1}{\sin(\nu\pi)}\left[1-\left(\frac{a-y}{b-y}\right)^{\nu}\right], \quad -\infty < y < a$ $\frac{1}{\sin(\nu\pi)}\left[1-\cos(\nu\pi)\left(\frac{y-a}{b-y}\right)^{\nu}\right], \quad a < y < b$ $\frac{1}{\sin(\nu\pi)}\left[1-\left(\frac{y-a}{y-b}\right)^{\nu}\right], \quad b < y < \infty$

y вещественно, и интеграл понимается в смысле главного значения по Коши.

	$f(x)$	$\pi^{-1}\int_{-\infty}^{\infty} f(x)(x-y)^{-1}\,dx$
(32)	$0, \quad -\infty<x<a$ $(x-a)^{\nu-1}(b-x)^{-\nu}, \quad a<x<b$ $0, \quad b<x<\infty$ $0<\operatorname{Re}\nu<1$	$\frac{1}{(b-y)\sin(\nu\pi)}\left\|\frac{a-y}{b-y}\right\|^{\nu-1}$, $-\infty<y<a$ или $b<y<\infty$ $-(y-a)^{\nu-1}(b-y)^{-\nu}\operatorname{ctg}(\nu\pi)$, $a<y<b$
(33)	$0, \quad -\infty<x<a$ $(x-a)^{\rho-1}(b-x)^{\sigma-1}$, $a<x<b$ $0, \quad b<x<\infty$ $\operatorname{Re}\rho>0,\ \operatorname{Re}\sigma>0$	$\frac{\Gamma(\rho)\Gamma(\sigma)(b-a)^{\rho+\sigma-1}}{(b-y)\pi\Gamma(\rho+\sigma)}\times$ $\times\,{}_2F_1\left(1,\ \sigma;\ \rho+\sigma;\ \frac{b-a}{b-y}\right)$, $-\infty<y<a$ или $b<y<\infty$ $(y-a)^{\rho-1}(b-y)^{\sigma-1}\operatorname{ctg}(\sigma\pi)-$ $-\frac{\Gamma(\rho)\Gamma(\sigma-1)}{\pi\Gamma(\rho+\sigma-1)}(b-a)^{\rho+\sigma-2}\times$ $\times\,{}_2F_1\left(2-\rho-\sigma,\ 1;\ 2-\sigma;\ \frac{b-y}{b-a}\right)$, $a<y<b$
(34)	$0, \quad -\infty<x<0$ $x^{\nu-1}(x+a)^{1-\mu}, \quad 0<x<\infty$ $a>0,\ 0<\operatorname{Re}\nu<\operatorname{Re}\mu$	$\frac{\Gamma(\mu-\nu)\Gamma(\nu)(-y)^{\nu-1}}{\pi\Gamma(\mu)a^{\mu-1}}\times$ $\times\,{}_2F_1(\mu-1,\ \nu;\ \mu;\ 1+y/a)$, $-\infty<y<0$ $y^{\nu-1}(y+a)^{1-\mu}\operatorname{ctg}[(\mu-\nu)\pi]-$ $-\frac{\Gamma(\mu-\nu-1)\Gamma(\nu)a^{1-\mu+\nu}}{(y+a)\pi\Gamma(\mu-1)}\times$ $\times\,{}_2F_1\left(2-\mu,\ 1;\ 2-\mu+\nu;\ \frac{a}{y+a}\right)$, $0<y<\infty$
(35)	$\exp(-a\|x\|), \quad a>0$	$\pi^{-1}\operatorname{sgn}y\,[\exp(a\|y\|)\operatorname{Ei}(-a\|y\|)-$ $-\exp(-a\|y\|)\overline{\operatorname{Ei}}(a\|y\|)]$
(36)	$\operatorname{sgn}x\exp(-a\|x\|), \quad a>0$	$-\pi^{-1}[\exp(a\|y\|)\operatorname{Ei}(-a\|y\|)+$ $+\exp(-a\|y\|)\overline{\operatorname{Ei}}(a\|y\|)]$
(37)	$0, \quad -\infty<x<a$ $e^{-bx}, \quad a<x<\infty$ $b>0$	$-\pi^{-1}e^{-by}\operatorname{Ei}(by-ab)$, $-\infty<y<a$ $-\pi^{-1}e^{-by}\overline{\operatorname{Ei}}(by-ab)$, $a<y<\infty$

y вещественно, и интеграл понимается в смысле главного значения по Коши.

	$f(x)$	$\pi^{-1}\int\limits_{-\infty}^{\infty} f(x)(x-y)^{-1}\,dx$
(38)	$e^{iax}, \quad a>0$	$i\,e^{iay}$
(39)	$0, \quad -\infty<x<0$ $\exp(-ax^{1/2}), \quad 0<x<\infty$ $a>0$	$2\pi^{-1}\cos(a\lvert y\rvert^{1/2})\,\mathrm{ci}(a\lvert y\rvert^{1/2})-$ $-2\pi^{-1}\sin(a\lvert y\rvert^{1/2})\,\mathrm{si}(a\lvert y\rvert^{1/2}),$ $-\infty<y<0$ $-\pi^{-1}\exp(ay^{1/2})\,\mathrm{Ei}(-ay^{1/2})-$ $-\pi^{-1}\exp(-ay^{1/2})\,\overline{\mathrm{Ei}}(ay^{1/2}),$ $0<y<\infty$
(40)	$\ln\left\lvert\frac{b-x}{x-a}\right\rvert, \quad a<b$	$0, \quad -\infty<y<a$ $-\pi, \quad a<y<b$ $0, \quad b<y<\infty$
(41)	$\frac{1}{x}\ln\left\lvert\frac{1+ax}{1-bx}\right\rvert, \quad a>0,\ b>0$	$-\pi y^{-1}, \quad -\infty<y<-a^{-1}$ $0, \quad -a^{-1}<y<b^{-1}$ $-\pi y^{-1}, \quad -b^{-1}<y<\infty$
(42)	$\ln\left\lvert\frac{x^2-a^2}{x^2-b^2}\right\rvert, \quad 0<a<b$	$-\pi, \quad -b<y<-a$ $\pi, \quad a<y<b$ 0 в остальных случаях
(43)	$\sin(ax), \quad a>0$	$\cos(ay)$
(44)	$\frac{\sin(ax)}{x}, \quad a>0$	$\frac{\cos(ay)-1}{y}$
(45)	$0, \quad -\infty<x<0$ $\sin(ax^{1/2}), \quad 0<x<\infty$ $a>0$	$\exp(-a\lvert y\rvert^{1/2}), \quad -\infty<y<0$ $\cos(ay^{1/2}), \quad 0<y<\infty$
(46)	$\operatorname{sgn} x\sin(a\lvert x\rvert^{1/2}), \quad a>0$	$\cos(a\lvert y\rvert^{1/2})+\exp(-a\lvert y\rvert^{1/2})$
(47)	$\cos(ax), \quad a>0$	$-\sin(ay)$
(48)	$\frac{1-\cos(ax)}{x}, \quad a>0$	$\frac{\sin(ay)}{y}$

y вещественно, и интеграл понимается в смысле главного значения по Коши.

15.3. Высшие трансцендентные функции

	$f(x)$	$\pi^{-1}\int_{-\infty}^{\infty} f(x)(x-y)^{-1}\,dx$
(1)	$e^{-ax}\,\mathrm{Ei}(ax),\quad -\infty<x<0$ $e^{-ax}\,\overline{\mathrm{Ei}}(ax),\quad 0<x<\infty$	$0,\quad -\infty<y<0$ $\pi e^{-ay},\quad 0<y<\infty$
(2)	$\mathrm{ci}(a\lvert x\rvert),\quad a>0$	$\mathrm{sgn}\,y\,\mathrm{si}(a\lvert y\rvert)$
(3)	$\mathrm{sgn}\,x\,\mathrm{si}(a\lvert x\rvert),\quad a>0$	$\mathrm{Ci}(a\lvert y\rvert)$
(4)	$\cos(ax)\,\mathrm{ci}(a\lvert x\rvert)-$ $-\sin(a\lvert x\rvert)\,\mathrm{si}(a\lvert x\rvert),$ $a>0$	$\mathrm{sgn}\,y\cos(ay)\,\mathrm{si}(a\lvert y\rvert)+$ $+\sin(ay)\,\mathrm{ci}(a\lvert y\rvert)$
(5)	$\sin(ax)\,\mathrm{ci}(a\lvert x\rvert)+$ $+\mathrm{sgn}\,x\cos(ax)\,\mathrm{si}(a\lvert x\rvert),$ $a>0$	$\sin(a\lvert y\rvert)\,\mathrm{si}(a\lvert y\rvert)-$ $-\cos(ay)\,\mathrm{ci}(a\lvert y\rvert)$
(6)	$0,\quad -\infty<x<1,\ 1<x<\infty$ $\mathrm{P}_n(x),\quad -1<x<1$ $n=0,1,2,\ldots$	$-2\pi^{-1}Q_n(y),$ $-\infty<y<-1,\ 1<y<\infty$ $-2\pi^{-1}\mathrm{Q}_n(y),\quad -1<y<1$
(7)	$0,\quad -\infty<x<-1,\ 1<x<\infty$ $(1-x^2)^{-1/2}T_n(x),\quad -1<x<1$ $n=1,2,\ldots$	$U_{n-1}(y),\quad -1<y<1$
(8)	$0,\quad -\infty<x<-1,\ 1<x<\infty$ $(1-x^2)^{1/2}U_n(x),\quad -1<x<1$ $n=0,1,2,\ldots$	$-T_{n+1}(y),\quad -1<y<1$
(9)	$0,\quad -\infty<x<-1,\ 1<x<\infty$ $(1-x)^{\alpha}(1+x)^{\beta}P_n^{(\alpha,\beta)}(x),$ $-1<x<1$ $\mathrm{Re}\,\alpha>-1,\ \mathrm{Re}\,\beta>-1$	$-2\pi^{-1}(y-1)^{\alpha}(y+1)^{\beta}Q_n^{(\alpha,\beta)}(y),$ $-\infty<y<-1,\ 1<y<\infty$ $-2\pi^{-1}(1-y)^{\alpha}(y+1)^{\beta}\mathrm{Q}_n^{(\alpha,\beta)}(y),$ $-1<y<1$
(10)	$0,\quad -\infty<x<0$ $J_\nu(ax),\quad 0<x<\infty$ $a>0,\ \mathrm{Re}\,\nu>-1$	$\dfrac{\mathrm{J}_\nu(-ay)-J_\nu(-ay)}{\sin(\nu\pi)},\quad -\infty<y<0$ $\dfrac{\mathrm{J}_\nu(-ay)-\cos(\nu\pi)J_\nu(ay)}{\sin(\nu\tau)},$ $0<y<\infty$

y вещественно, и интеграл понимается в смысле главного значения по Коши.

	$f(x)$	$\pi^{-1}\int_{-\infty}^{\infty} f(x)\,(x-y)^{-1}\,dx$
(11)	$-J_{-\nu}(-ax), \quad -\infty<x<0$ $J_\nu(ax), \quad 0<x<\infty$ $a>0, \ -1<\operatorname{Re}\nu<1$	$-Y_{-\nu}(-ay), \quad -\infty<y<0$ $-Y_\nu(ay), \quad 0<y<\infty$
(12)	$0, \quad -\infty<x<0$ $x^\nu J_\nu(ax), \quad 0<x<\infty$ $a>0, \ -1/2<\operatorname{Re}\nu<3/2$	$2^{-1}\lvert y\rvert^\nu\Big[\operatorname{tg}(\nu\pi)\operatorname{sgn} y\, J_\nu(a\lvert y\rvert)-$ $-Y_\nu(a\lvert y\rvert)-\dfrac{\operatorname{sgn} y\,\mathbf{H}_{-\nu}(a\lvert y\rvert)}{\cos(\nu\pi)}\Big]$
(13)	$\lvert x\rvert^\nu J_\nu(a\lvert x\rvert),$ $a>0, \ -1/2<\operatorname{Re}\nu<3/2$	$\operatorname{sgn} y\lvert y\rvert^\nu\Big[\operatorname{tg}(\nu\pi)J_\nu(a\lvert y\rvert)-$ $-\dfrac{\mathbf{H}_{-\nu}(a\lvert y\rvert)}{\cos(\nu\pi)}\Big]$
(14)	$\operatorname{sgn} x\lvert x\rvert^\nu J_\nu(a\lvert x\rvert),$ $a>0, \ -1/2<\operatorname{Re}\nu<3/2$	$-\lvert y\rvert^\nu Y_\nu(a\lvert y\rvert)$
(15)	$\lvert x\rvert^{-\nu} J_\nu(a\lvert x\rvert),$ $a>0, \ \operatorname{Re}\nu>-3/2$	$-\operatorname{sgn} y\lvert y\rvert^{-\nu}\mathbf{H}_\nu(a\lvert y\rvert)$
(16)	$0, \quad -\infty<x<0$ $x^\lambda J_\nu(ax), \quad 0<x<\infty$ $a>0$ $-1-\operatorname{Re}\nu<\operatorname{Re}\lambda<3/2$	$\dfrac{2^{\lambda-1}\Gamma(\lambda/2+\nu/2)}{\pi a^\lambda\,\Gamma(1-\lambda/2+\nu/2)}\times$ $\times{}_1F_2\Big(1;\,1-\dfrac{\lambda+\nu}{2},\,1-\dfrac{\lambda-\nu}{2};\,-\dfrac{a^2y^2}{4}\Big)+$ $+\dfrac{2^{\lambda-2}y\,\Gamma(\lambda/2+\nu/2-1/2)}{\pi a^{\lambda-1}\,\Gamma(3/2-\lambda/2+\nu/2)}\times$ $+{}_1F_2\Big(1;\,\dfrac{3-\lambda-\nu}{2},\,\dfrac{3-\lambda+\nu}{2};\,-\dfrac{a^2y^2}{4}\Big)-$ $-h(y)\lvert y\rvert^\lambda J_\nu(a\lvert y\rvert),$ $h(y)=\begin{cases}\dfrac{1}{\sin[(\lambda+\nu)\pi]}, & -\infty<y<0\\ \operatorname{ctg}[(\lambda+\nu)\pi], & 0<y<\infty\end{cases}$
(17)	$\sin(ax)\,J_1(ax), \quad a>0$	$\cos(ay)\,J_1(ay)$
(18)	$\sin(ax)\,J_n(bx),$ $0<b<a, \ n=0,1,2,\ldots$	$\cos(ay)\,J_n(by)$
(19)	$\cos(ax)\,J_1(ax), \quad a>0$	$-\sin(ay)\,J_1(ay)$
(20)	$\cos(ax)\,J_n(bx),$ $0<b<a, \ n=0,1,2,\ldots$	$-\sin(ay)\,J_n(by)$

y вещественно, и интеграл понимается в смысле главного значения по Коши.

	$f(x)$	$\pi^{-1}\int\limits_{-\infty}^{\infty} f(x)(x-y)^{-1}\,dx$
(21)	$\operatorname{sgn} x\,\lvert x\rvert^{\nu}\sin(a\lvert x\rvert-\pi\nu)\times$ $\times J_\nu(a\lvert x\rvert)$, $a>0$, $-1/2<\operatorname{Re}\nu<1/2$	$\lvert y\rvert^{\nu}\cos(a\lvert y\rvert-\pi\nu)\,J_\nu(a\lvert y\rvert)$
(22)	$\lvert x\rvert^{\nu}\cos(a\lvert x\rvert-\pi\nu)\,J_\nu(a\lvert x\rvert)$, $a>0$, $-1/2<\operatorname{Re}\nu<1/2$	$-\operatorname{sgn} y\,\lvert y\rvert^{\nu}\sin(a\lvert y\rvert-\pi\nu)\,J_\nu(a\lvert y\rvert)$
(23)	$\lvert x\rvert^{-\nu}\sin(ax)\,J_\nu(a\lvert x\rvert)$, $a>0$, $\operatorname{Re}\nu>-1/2$	$\lvert y\rvert^{-\nu}\cos(ay)\,J_\nu(a\lvert y\rvert)$
(24)	$\lvert x\rvert^{-\nu}\cos(ax)\,J_\nu(a\lvert x\rvert)$, $a>0$, $\operatorname{Re}\nu>1/2$	$-\lvert y\rvert^{-\nu}\sin(ay)\,J_\nu(a\lvert y\rvert)$
(25)	$\lvert x\rvert^{1/2}J_{\nu-1/4}(a\lvert x\rvert)\,J_{-\nu-1/4}(a\lvert x\rvert)$, $a>0$	$-\operatorname{sgn} y\,\lvert y\rvert^{1/2}J_{1/4+\nu}(a\lvert y\rvert)\times$ $\times J_{1/4-\nu}(a\lvert y\rvert)$
(26)	$\operatorname{sgn} x\,\lvert x\rvert^{1/2}J_{1/4+\nu}(a\lvert x\rvert)\times$ $\times J_{1/4-\nu}(a\lvert x\rvert)$, $a>0$	$\lvert y\rvert^{1/2}J_{\nu-1/4}(a\lvert y\rvert)\,J_{-\nu-1/4}(a\lvert y\rvert)$
(27)	0, $-\infty<x<0$ $x^{\nu/2}J_\nu(ax^{1/2})$, $0<x<\infty$ $a>0$, $-1<\operatorname{Re}\nu<3/2$	$2\pi^{-1}(-y)^{\nu/2}K_\nu[a(-y)^{1/2}]$, $-\infty<y<0$ $-y^{\nu/2}Y_\nu(ay^{1/2})$, $0<y<\infty$
(28)	$\lvert x\rvert^{\nu/2}J_\nu(a\lvert x\rvert^{1/2})$, $a>0$, $-1<\operatorname{Re}\nu<3/2$	$-\operatorname{sgn} y\,\lvert y\rvert^{\nu/2}[2\pi^{-1}K_\nu(a\lvert y\rvert^{1/2})+$ $+Y_\nu(a\lvert y\rvert^{1/2})]$
(29)	$\operatorname{sgn} x\,\lvert x\rvert^{\nu/2}J_\nu(a\lvert x\rvert^{1/2})$, $a>0$, $-1<\operatorname{Re}\nu<3/2$	$2\pi^{-1}\lvert y\rvert^{\nu/2}K_\nu(a\lvert y\rvert^{1/2})-$ $-\lvert y\rvert^{\nu/2}Y_\nu(a\lvert y\rvert^{1/2})$
(30)	0, $-\infty<x<0$ $x^{\nu/2-1/2}J_\nu(ax^{1/2})$, $0<x<\infty$ $a>0$, $-1/2<\operatorname{Re}\nu<5/2$	$\dfrac{\lvert y\rvert^{\nu/2-1/2}}{\cos(\nu\pi)}[I_\nu(a\lvert y\rvert^{1/2})-$ $-\mathbf{L}_{-\nu}(a\lvert y\rvert^{1/2})]$, $-\infty<y<0$ $y^{\nu/2-1/2}\left[\operatorname{tg}(\nu\pi)\,J_\nu(ay^{1/2})-\right.$ $\left.-\dfrac{\mathbf{H}_{-\nu}(ay^{1/2})}{\cos(\nu\pi)}\right]$, $0<y<\infty$

y вещественно, и интеграл понимается в смысле главного значения по Коши.

	$f(x)$	$\pi^{-1}\int_{-\infty}^{\infty} f(x)(x-y)^{-1}\,dx$
(31)	$\lvert x\rvert^{\nu/2-1/2} J_\nu(a\lvert x\rvert^{1/2})$, $a>0$, $-1/2<\operatorname{Re}\nu<5/2$	$\operatorname{sgn} y\lvert y\rvert^{\nu/2-1/2}\times$ $\times\{\operatorname{tg}(\nu\pi) J_\nu(a\lvert y\rvert^{1/2})+$ $+\frac{1}{\cos(\nu\pi)}[\mathbf{L}_{-\nu}(a\lvert y\rvert^{1/2})-$ $-\mathbf{H}_{-\nu}(a\lvert y\rvert^{1/2})-I_\nu(a\lvert y\rvert^{1/2})]\}$
(32)	$\operatorname{sgn} x\lvert x\rvert^{\nu/2-1/2} J_\nu(a\lvert x\rvert^{1/2})$, $a>0$, $-1/2<\operatorname{Re}\nu<5/2$	$\lvert y\rvert^{\nu/2-1/2}\{\operatorname{tg}(\nu\pi) J_\nu(a\lvert y\rvert^{1/2})+$ $+\frac{1}{\cos(\nu\pi)}[I_\nu(a\lvert y\rvert^{1/2})-$ $-\mathbf{L}_{-\nu}(a\lvert y\rvert^{1/2})-\mathbf{H}_{-\nu}(a\lvert y\rvert^{1/2})]\}$
(33)	0, $-\infty<x<0$ $x^{-\nu/2-1/2} J_\nu(ax^{1/2})$, $0<x<\infty$ $a>0$, $\operatorname{Re}\nu>-5/2$	$\lvert y\rvert^{-\nu/2-1/2}\times$ $\times[I_\nu(a\lvert y\rvert^{1/2})-\mathbf{L}_\nu(a\lvert y\rvert^{1/2})]$, $-\infty<y<0$ $-y^{-\nu/2-1/2}\mathbf{H}_\nu(ay^{1/2})$, $0<y<\infty$
(34)	$\lvert x\rvert^{-\nu/2-1/2} J_\nu(a\lvert x\rvert^{1/2})$, $a>0$, $\operatorname{Re}\nu>-5/2$	$\operatorname{sgn} y\lvert y\rvert^{-\nu/2-1/2}[\mathbf{L}_\nu(a\lvert y\rvert^{1/2})-$ $-\mathbf{H}_\nu(a\lvert y\rvert^{1/2})-I_\nu(a\lvert y\rvert^{1/2})]$
(35)	$\operatorname{sgn} x\lvert x\rvert^{-\nu/2-1/2} J_\nu(a\lvert x\rvert^{1/2})$, $a>0$, $\operatorname{Re}\nu>-5/2$	$\lvert y\rvert^{-\nu/2-1/2}[I_\nu(a\lvert y\rvert^{1/2})-$ $-\mathbf{L}_\nu(a\lvert y\rvert^{1/2})-\mathbf{H}_\nu(a\lvert y\rvert^{1/2})]$
(36)	0, $-\infty<x<0$ $x^\lambda J_\nu(ax^{1/2})$, $0<x<\infty$ $a>0$ $-1-2^{-1}\operatorname{Re}\nu<\operatorname{Re}\lambda<3/4$	$\frac{2^{2\lambda}\Gamma(\lambda+\nu/2)}{\pi a^{2\lambda}\Gamma(1-\lambda+\nu/2)}\times$ $\times{}_1F_2\left(1;\ 1-\lambda-\frac{\nu}{2},\ 1-\lambda+\frac{\nu}{2};\ -\frac{a^2y}{4}\right)-h(y)$, $h(y)=\frac{\lvert y\rvert^\lambda I_\nu(a\lvert y\rvert^{1/2})}{\sin[(\lambda+\nu/2)\pi]}$, $-\infty<y<0$ $h(y)=y^\lambda\operatorname{ctg}[(\lambda+\nu/2)\pi]\times$ $\times J_\nu(a\lvert y\rvert^{1/2})$, $0<y<\infty$

y вещественно, и интеграл понимается в смысле главного значения по Коши.

	$f(x)$	$\pi^{-1} \int_{-\infty}^{\infty} f(x)(x-y)^{-1}\,dx$
(37)	$0, \quad -a<x<a$ $\operatorname{sgn} x\,(x^2-a^2)^{\nu/2} J_\nu\left[b(x^2-a^2)^{1/2}\right],$ $-\infty<x<-a$ или $a<x<\infty$ $a>0,\ b>0$ $-1<\operatorname{Re}\nu<3/2$	$2\pi^{-1}(a^2-y^2)^{\nu/2} K_\nu\left[b(a^2-y^2)^{1/2}\right],$ $-a<y<a$ $-(y^2-a^2)^{\nu/2} Y_\nu\left[b(y^2-a^2)^{1/2}\right],$ $-\infty<y<-a$ или $a<y<\infty$
(38)	$Y_{-\nu}(-ax), \quad -\infty<x<0$ $Y_\nu(ax), \quad 0<x<\infty$ $a>0,\ -1<\operatorname{Re}\nu<1$	$-J_{-\nu}(-ay), \quad -\infty<y<0$ $J_\nu(ay), \quad 0<y<\infty$
(39)	$\sin(\nu\pi/2) J_\nu(a\|x\|)+$ $+\cos(\nu\pi/2) Y_\nu(a\|x\|),$ $a>0,\ -1<\operatorname{Re}\nu<1$	$\operatorname{sgn} y\,[\cos(\nu\pi/2) Y_\nu(a\|y\|)-$ $-\sin(\nu\pi/2) J_\nu(a\|y\|)]$
(40)	$\operatorname{sgn} x\,[\sin(\nu\pi/2) Y_\nu(a\|x\|)-$ $-\cos(\nu\pi/2) J_\nu(a\|x\|)],$ $a>0,\ -1<\operatorname{Re}\nu<1$	$\cos(\nu\pi/2) Y_\nu(a\|y\|)+$ $+\sin(\nu\pi/2) J_\nu(a\|y\|)$
(41)	$\|x\|^\nu Y_\nu(a\|x\|),$ $a>0,\ -1/2<\operatorname{Re}\nu<3/2$	$\operatorname{sgn} y\,\|y\|^\nu J_\nu(a\|y\|)$
(42)	$\operatorname{sgn} x\,\|x\|^{-\mu}\{\sin[2^{-1}(\mu+\nu)\pi]\times$ $\times Y_\nu(a\|x\|)-$ $-\cos[2^{-1}(\mu+\nu)\pi] J_\nu(a\|x\|)\},$ $a>0$ $-1/2<\operatorname{Re}\mu<1-\|\operatorname{Re}\nu\|$	$\|y\|^{-\mu}\{\cos[2^{-1}(\mu+\nu)\pi]\times$ $\times Y_\nu(a\|y\|)+$ $+\sin[2^{-1}(\mu+\nu)\pi] J_\nu(a\|y\|)\}$
(43)	$\|x\|^{-\mu}\{\sin[2^{-1}(\mu+\nu)\pi]\times$ $\times J_\nu(a\|x\|)+$ $+\cos[2^{-1}(\mu+\nu)\pi] Y_\nu(a\|x\|)\},$ $a>0$ $-3/2<\operatorname{Re}\mu<1-\|\operatorname{Re}\nu\|$	$\operatorname{sgn} y\,\|y\|^{-\mu}\times$ $\times\{\cos[2^{-1}(\mu+\nu)\pi] J_\nu(a\|y\|)-$ $-\sin[2^{-1}(\mu+\nu)\pi] Y_\nu(a\|y\|)\}$
(44)	$\operatorname{sgn} x\,\|x\|^{-\mu}\times$ $\times\{\cos[a\|x\|-2^{-1}(\mu+\nu)\pi]\times$ $\times J_\nu(b\|x\|)+$ $+\sin[a\|x\|-2^{-1}(\mu+\nu)\pi]\times$ $\times Y_\nu(b\|x\|)\},$ $a<b, \quad -3/2<\operatorname{Re}\mu<1-\|\operatorname{Re}\nu\|$	$\|y\|^{-\mu}\{\sin[a\|y\|-2^{-1}(\mu+\nu)\pi]\times$ $\times J_\nu(b\|y\|)-$ $-\cos[a\|y\|-2^{-1}(\mu+\nu)\pi]\times$ $\times Y_\nu(b\|y\ \ )\}$

y вещественно, и интеграл понимается в смысле главного значения по Коши.

	$f(x)$	$\pi^{-1} \int_{-\infty}^{\infty} f(x)(x-y)^{-1}\,dx$
(45)	$\lvert x\rvert^{-\mu} \times$ $\times \{\cos[a\lvert x\rvert - 2^{-1}(\mu+\nu)\pi] \times$ $\times Y_\nu(b\lvert x\rvert) -$ $- \sin[a\lvert x\rvert - 2^{-1}(\mu+\nu)\pi] \times$ $\times J_\nu(b\lvert x\rvert)\}$, $a<b$, $-{}^3/_2 < \operatorname{Re}\mu < 1 - \lvert\operatorname{Re}\nu\rvert$	$\operatorname{sgn} y\,\lvert y\rvert^{-\mu} \times$ $\times \{\sin[a\lvert y\rvert - 2^{-1}(\mu+\nu)\pi] \times$ $\times Y_\nu(b\lvert y\rvert) +$ $+ \cos[a\lvert y\rvert - 2^{-1}(\mu+\nu)\pi] \times$ $\times J_\nu(b\lvert y\rvert)\}$
(46)	$0,\quad -\infty<x<0$ $x^\mu\{\cos[(\mu-\nu/2)\pi]\,J_\nu(ax^{1/2}) +$ $+ \sin[(\mu-\nu/2)\pi]\,Y_\nu(ax^{1/2})\}$, $0<x<\infty$ $a>0$, $\lvert\operatorname{Re}\nu\rvert - 1 < \operatorname{Re}\mu < {}^1/_2$	$2\pi^{-1}\lvert y\rvert^\mu K_\nu(a\lvert y\rvert^{1/2})$, $-\infty<y<0$ $y^\mu\{\sin[(\mu-\nu/2)\pi]\,J_\nu(ay^{1/2}) -$ $- \cos[(\mu-\nu/2)\pi]\,Y_\nu(ay^{1/2})\}$, $0<y<\infty$
(47)	$0,\quad -\infty<x<0$ $e^{-ax} I_0(ax),\quad 0<x<\infty$ $a>0$	$\pi^{-1} e^{-ay} K_0(a\lvert y\rvert)$
(48)	$\exp(-a\lvert x\rvert)\,I_0(ax),\quad a>0$	$-2\pi^{-1}\operatorname{sh}(ay)\,K_0(a\lvert y\rvert)$
(49)	$\operatorname{sgn} x \exp(-a\lvert x\rvert)\,I_0(ax)$, $a>0$	$2\pi^{-1}\operatorname{ch}(ay)\,K_0(a\lvert y\rvert)$
(50)	$0,\quad -\infty<x<-a$ $(a^2-x^2)^{\nu/2} e^{-bx} J_\nu[b(a^2-x^2)^{1/2}]$, $-a<x<a$ $2(x^2-a^2)^{\nu/2}\cos(\nu\pi)\,e^{-bx} \times$ $\times I_\nu[b(x^2-a^2)^{1/2}],\quad a<x<\infty$ $a>0,\ b>0$ $-1<\operatorname{Re}\nu<{}^1/_2$	$2\pi^{-1}(y^2-a^2)^{\nu/2} e^{-by} \times$ $\times K_\nu[b(y^2-a^2)^{1/2}]$, $-\infty<y<-a$ $-(a^2-y^2)^{\nu/2} e^{-by} \times$ $\times Y_\nu[b(a^2-y^2)^{1/2}]$, $-a<y<a$ $2(y^2-a^2)^{\nu/2} e^{-by} \times$ $\times \{\pi^{-1}K_\nu[b(y^2-a^2)^{1/2}] +$ $+ \sin(\nu\pi)\,I_\nu[b(y^2-a^2)^{1/2}]\}$, $a<y<\infty$
(51)	$e^{ax} K_0(a\lvert x\rvert),\quad a>0$	$\pi e^{ay} I_0(ay),\quad -\infty<y<0$ $0,\quad 0<y<\infty$

y вещественно, и интеграл понимается в смысле главного значения по Коши.

	$f(x)$	$\pi^{-1}\int\limits_{-\infty}^{\infty} f(x)\,(x-y)^{-1}\,dx$
(52)	$\operatorname{sh}(ax)\,K_0(a\mid x\mid)$, $a>0$	$2^{-1}\pi\exp(-a\mid y\mid)\,I_0(ay)$
(53)	$\operatorname{ch}(ax)\,K_0(a\mid x\mid)$, $a>0$	$-2^{-1}\pi\operatorname{sgn}y\exp(-a\mid y\mid)\,I_0(ay)$
(54)	$\mid x\mid^{-\nu}e^{ax}K_\nu(a\mid x\mid)$, $a>0$, $-1/2<\operatorname{Re}\nu<1/2$	$\frac{\pi e^{ay}}{2\cos(\nu\pi)\mid y\mid^{\nu}}\times[I_\nu(a\mid y\mid)+I_{-\nu}(a\mid y\mid)]$, $-\infty<y<0$ $-\pi\operatorname{tg}(\nu\pi)\,y^{-\nu}e^{ay}K_\nu(ay)$, $0<y<\infty$
(55)	$\mid x\mid^{-\nu}\operatorname{sh}(ax)\,K_\nu(a\mid x\mid)$, $a>0$, $-1/2>\operatorname{Re}\nu<1$	$\frac{1}{\mid y\mid^{\nu}}\left[\frac{\pi\exp(-a\mid y\mid)\,I_\nu(a\mid y\mid)}{2\cos(\nu\pi)}-\operatorname{tg}(\nu\pi)\operatorname{sh}(a\mid y\mid)\,K_\nu(a\mid y\mid)\right]$
(56)	$\mid x\mid^{-\nu}\operatorname{ch}(ax)\,K_\nu(a\mid x\mid)$, $a>0$, $-1/2<\operatorname{Re}\nu<1/2$	$-\frac{\operatorname{sgn}y}{\mid y\mid^{\nu}}\left[\frac{\pi\exp(-a\mid y\mid)\,I_\nu(a\mid y\mid)}{2\cos(\nu\pi)}+\operatorname{tg}(\nu\pi)\operatorname{ch}(ay)\,K_\nu(a\mid y\mid)\right]$
(57)	$\mid x\mid^{2\nu}\exp(-ax^2)\,[K_\nu(ax^2)+\pi\sin(\nu\pi)\,I_\nu(ax^2)]$, $a>0$, $-1/4<\operatorname{Re}\nu<1/4$	$-\pi\cos(\nu\pi)\operatorname{sgn}y\mid y\mid^{2\nu}\times\exp(-ay^2)\,I_\nu(ay^2)$
(58)	$\operatorname{sgn}x\mid x\mid^{2\nu}\exp(-ax^2)\,I_\nu(ax^2)$, $a>0$, $-1/4<\operatorname{Re}\nu<1/2$	$\mid y\mid^{2\nu}\exp(-ay^2)\times\left[\frac{K_\nu(ay^2)}{\pi\cos(\nu\pi)}+\operatorname{tg}(\nu\pi)\,I_\nu(ay^2)\right]$
(59)	$\operatorname{sgn}x\mid x\mid^{-\nu}\mathbf{H}_\nu(a\mid x\mid)$, $a>0$, $\operatorname{Re}\nu>-3/2$	$\mid y\mid^{-\nu}J_\nu(a\mid y\mid)$
(60)	0, $-\infty<x<0$ $G_{pq}^{mn}\left(\alpha x\,\middle\vert\,\begin{matrix}a_1,\ldots,a_p\\ b_1,\ldots,b_q\end{matrix}\right)$, $0<x<\infty$ $p+q<2(m+n)$ $\mid\arg\alpha\mid<(m+n-p/2-q/2)\pi$ $\operatorname{Re}a_j<1$, $j=1,\ldots,n$ $\operatorname{Re}b_j>-1$, $j=1,\ldots,m$	$\pi^{-1}G_{tu}^{rs}\left(\alpha\mid y\mid\,\middle\vert\,\begin{matrix}0,a_1,\ldots,a_p\\ 0,b_1,\ldots,b_q\end{matrix}\right)$, $-\infty<y<0$ $(-1)^k G_{vw}^{rs}\left(\alpha y\,\middle\vert\,\begin{matrix}0,a_1,\ldots,a_p,j\\ 0,b_1,\ldots,b_q,j\end{matrix}\right)$, $0<y<\infty$ $j=1/2+k$, $r=m+1$, $s=n+1$, $t=p+1$, $u=q+1$, $v=p+2$, $w=q+2$ k — целое

y вещественно, и интеграл понимается в смысле главного значения по Коши.

	$f(x)$	$\pi^{-1} \int\limits_{-\infty}^{\infty} f(x)(x-y)^{-1}\,dx$
(61)	$G_{pq}^{mn}\left(\alpha x^2 \middle\vert \begin{matrix} a_1, \ldots, a_p \\ b_1, \ldots, b_q \end{matrix}\right)$, $p+q<2(m+n)$, $\vert\arg\alpha\vert<(m+n-p/2-q/2)\pi$, $\operatorname{Re} a_j<1$, $j=1, \ldots, n$; $\operatorname{Re} b_j>-{}^1/_2$, $j=1, \ldots, m$	$\operatorname{sgn} y\, G_{vw}^{rs}\left(\alpha y^2 \middle\vert \begin{matrix} {}^1/_2, a_1, \ldots, a_p, 1 \\ {}^1/_2, b_1, \ldots, b_q, 1 \end{matrix}\right)$, $r=m+1$, $s=n+1$, $v=p+2$, $w=q+2$
(62)	$x\, G_{pq}^{mn}\left(\alpha x^2 \middle\vert \begin{matrix} a_1, \ldots, a_p \\ b_1, \ldots, b_q \end{matrix}\right)$, $p+q<2(m+n)$, $\vert\arg\alpha\vert<(m+n-p/2-q/2)\pi$, $\operatorname{Re} a_j<{}^1/_2$, $j=1, \ldots, n$; $\operatorname{Re} b_j>-1$, $j=1, \ldots, m$	$\vert y\vert\, G_{vw}^{rs}\left(\alpha y^2 \middle\vert \begin{matrix} -{}^1/_2, a_1, \ldots, a_p, 0 \\ -{}^1/_2, b_1, \ldots, b_q, 0 \end{matrix}\right)$, $r=m+1$, $s=n+1$, $v=p+2$, $w=q+2$

y вещественно, и интеграл понимается в смысле главного значения по Коши.

ИНТЕГРАЛЫ ОТ ВЫСШИХ ТРАНСЦЕНДЕНТНЫХ ФУНКЦИЙ

Эта часть книги содержит в основном интегралы, которые не вошли в таблицы глав I – XV.

ГЛАВА XVI
ОРТОГОНАЛЬНЫЕ МНОГОЧЛЕНЫ

В этой главе мы даем список интегралов, содержащих классические ортогональные многочлены. Относительно теории этих многочленов см. ВТФ, т. II, главу X и указанную там литературу, в особенности книгу Сеге. Обозначения, принятые в данной главе для многочленов Эрмита, отличаются от использованных в ВТФ.

Из указанных здесь формул могут быть получены выражения для других интегралов путем применения методов, указанных во введении к первому тому, применения формулы Родрига (см. ниже) с последующим многократным интегрированием по частям, применения производящих функций (см. ниже), а также путем использования таблиц, данных в других главах этой книги, и применения соотношений (см. ниже) между различными системами ортогональных многочленов и между этими многочленами и функциями Лежандра, гипергеометрическими рядами, вырожденными гипергеометрическими функциями.

Многочлены Чебышева.

$$T_n(x) = (-1)^n T_n(-x) = \cos(n\theta), \qquad x = \cos\theta,$$

$$= \frac{(1-x^2)^{1/2}}{2^n (1/2)_n}\left(-\frac{d}{dx}\right)^n \left[(1-x^2)^{n-1/2}\right],$$

$$= {}_2F_1(-n, n; 1/2; 1/2 - x/2),$$

$$= 2^{-1} n \lim_{\nu\to 0} \Gamma(\nu)\, C_n^{\nu}(x) = \frac{n!}{(1/2)_n} P_n^{(-1/2,\,-1/2)}(x);$$

$$U_n(x) = (-1)^n U_n(-x) = \frac{\sin[(n+1)\theta]}{\sin\theta}, \qquad x = \cos\theta,$$

$$= \frac{(n+1)(1-x^2)^{-1/2}}{2^{n+1}(1/2)_{n+1}}\left(-\frac{d}{dx}\right)^n \left[(1-x^2)^{n+1/2}\right],$$

$$= (n+1)\,{}_2F_1\left(-n, n+1; \frac{3}{2}; \frac{1-x}{2}\right),$$

$$= C_n^1(x) = \frac{(n+1)!}{2(1/2)_{n+1}} P_n^{(1/2,\,1/2)}(x),$$

$$1 + 2\sum_{n=1}^{\infty} T_n(x) z^n = \frac{1-z^2}{1-2xz+z^2},$$

$$\sum_{n=0}^{\infty} U_n(x) z^n = (1-2xz+z^2)^{-1}.$$

Относительно других производящих функций см. ВТФ, т. II, стр. 187.

Многочлены Лежандра.

$$P_n(x) = (-1)^n P_n(-x) = \frac{1}{2^n n!}\left(\frac{d}{dx}\right)^n [(x^2-1)^n],$$

$$= {}_2F_1(-n,\ n+1;\ 1;\ {}^1/_2 - x/2),$$

$$= \frac{2^n ({}^1/_2)_n}{n!} x^n\, {}_2F_1(-n/2,\ {}^1/_2 - n/2;\ {}^1/_2;\ x^{-2}),$$

$$= C_n^{1/2}(x) = P_n^{(0,\ 0)}(x),$$

$$\sum_{n=0}^{\infty} P_n(x) z^n = (1 - 2xz + z^2)^{-1/2}.$$

Относительно связи с функциями Лежандра см. ВТФ, т. I, стр. 151 и далее; относительно других гипергеометрических рядов, выражающих многочлены Лежандра, см. ВТФ, т. I, стр. 128—139 ($\mu = 0$, $\nu = n$) и т. II, стр. 181, а относительно других производящих функций см. ВТФ, т. II, стр. 183.

Относительно определения присоединенных многочленов Лежандра и их свойств см. ВТФ, т. I, стр. 149 и далее.

Многочлены Гегенбауэра. Эти многочлены называют также *ультрасферическими* многочленами и обозначают $P_n^{(\nu)}(x)$.

$$C_n^{\nu}(x) = (-1)^n C_n^{\nu}(-x),$$

$$= \frac{2^{\nu - 1/2}\,\Gamma(2\nu + n)\,\Gamma(\nu + {}^1/_2)}{n!\,\Gamma(2\nu)} (x^2 - 1)^{1/4 - \nu/2} P^{1/2 - \nu}_{n + \nu - 1/2}(x),$$

$$= \frac{(2\nu)_n (1 - x^2)^{1/2 - \nu}}{2^n n! (\nu + {}^1/_2)_n}\left(-\frac{d}{dx}\right)^n [(1 - x^2)^{n + \nu - 1/2}],$$

$$= \frac{(2\nu)_n}{n!}\, {}_2F_1(-n,\ n + 2\nu;\ \nu + {}^1/_2;\ {}^1/_2 - x/2),$$

$$= \frac{2^n (\nu)_n}{n!} (x-1)^n\, {}_2F_1\left(-n,\ \frac{1}{2} - n - \nu;\ 1 - 2n - 2\nu;\ \frac{2}{1-x}\right),$$

$$= \frac{(2\nu)_n}{(\nu + {}^1/_2)_n} P_n^{(\nu - 1/2,\ \nu - 1/2)}(x);$$

$$C_{2n}^{\nu}(x) = (-1)^n \frac{(\nu)_n}{n!}\, {}_2F_1(-n,\ n + \nu;\ {}^1/_2;\ x^2),$$

$$= \frac{(\nu)_n}{({}^1/_2)_n} P_n^{(\nu - 1/2,\ -1/2)}(2x^2 - 1);$$

$$C_{2n+1}^{\nu}(x) = (-1)^n \frac{(\nu)_{n+1}}{n!} 2x\, {}_2F_1(-n,\ n + \nu + 1;\ {}^3/_2;\ x^2),$$

$$= \frac{(\nu)_{n+1}}{({}^1/_2)_{n+1}} x\, P_n^{(\nu - 1/2,\ 1/2)}(2x^2 - 1);$$

$$\sum_{n=0}^{\infty} C_n^{\nu}(x) z^n = (1 - 2xz + z^2)^{-\nu}.$$

Относительно связи с функциями Лежандра и других гипергеометрических разложений см. ВТФ, т. I, стр. 177 и далее, стр. 128—139, т. II, стр. 177. Относительно других производящих функций см. ВТФ, т. II, стр. 178.

Многочлены Якоби.

$$
\begin{aligned}
P_n^{(\alpha,\beta)}(x) &= (-1)^n P_n^{(\beta,\alpha)}(-x),\\
&= \frac{(1-x)^{-\alpha}(1+x)^{-\beta}}{2^n n!}\left(-\frac{d}{dx}\right)^n\left[(1-x)^{\alpha+n}(1+x)^{\beta+n}\right],\\
&= \binom{n+\alpha}{n}\,{}_2F_1\left(-n,\ n+\alpha+\beta+1;\ \alpha+1;\ \frac{1-x}{2}\right),\\
&= (-1)^n\binom{n+\beta}{n}\,{}_2F_1\left(-n,\ n+\alpha+\beta+1;\ \beta+1;\ \frac{1+x}{2}\right),\\
&= \binom{n+\alpha}{n}\left(\frac{1+x}{2}\right)^n{}_2F_1\left(-n,\ -n-\beta;\ \alpha+1;\ \frac{x-1}{x+1}\right),\\
&= \binom{n+\beta}{n}\left(\frac{x-1}{2}\right)^n{}_2F_1\left(-n,\ -n-\alpha;\ \beta+1;\ \frac{x+1}{x-1}\right);
\end{aligned}
$$

$$
\sum_{n=0}^{\infty} P_n^{(\alpha,\beta)}(x)\,z^n = 2^{\alpha+\beta}R^{-1}(1-z+R)^{-\alpha}(1+z+R)^{-\beta},
$$

$$
R=(1-2xz+z^2)^{1/2}
$$

Другие разложения могут быть получены из этих с помощью преобразований, указанных в ВТФ, т. I, п. 2.9.

Многочлены Эрмита.

$$
\begin{aligned}
\mathrm{He}_n(x) &= (-1)^n\,\mathrm{He}_n(-x) = 2^{-n/2}H_n(2^{-1/2}x),\\
&= \exp(2^{-1}x^2)\left(-\frac{d}{dx}\right)^n\left[\exp(-2^{-1}x^2)\right],\\
&= x^n\,{}_2F_0\left(-\frac{n}{2},\ \frac{1-n}{2};\ -\frac{2}{x^2}\right),\\
&= 2^{n/2+1/4}x^{-1/2}\exp(2^{-2}x^2)\,W_{n/2+1/4,\,-1/4}\left(\frac{x^2}{2}\right),\\
&= \exp(2^{-2}x^2)\,D_n(x);
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{He}_{2n}(x) &= (-2)^n\,(1/2)_n\,{}_1F_1(-n;\ 1/2;\ 2^{-1}x^2),\\
&= (-2)^n\,n!\,L_n^{-1/2}(2^{-1}x^2);
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{He}_{2n+1}(x) &= (-2)^n\,(3/2)_n\,x\,{}_1F_1(-n;\ 3/2;\ 2^{-1}x^2),\\
&= (-2)^n\,n!\,x\,L_n^{1/2}(2^{-1}x^2),
\end{aligned}
$$

$$
\sum_{n=0}^{\infty}\mathrm{He}_n(x)\,\frac{z^n}{n!} = \exp(-2^{-1}z^2+xz).
$$

Относительно других производящих функций см. ВТФ, т. II, стр. 194,

Многочлены Лагерра.

$$L_n^\alpha(x)=\frac{1}{n!}e^x x^{-\alpha}\left(\frac{d}{dx}\right)^n\left[e^{-x}x^{n+\alpha}\right],$$
$$=\binom{n+\alpha}{n}{}_1F_1(-n;\ \alpha+1;\ x),$$
$$=\frac{(-1)^n}{n!}x^n\,{}_2F_0(-n,\ -\alpha-n;\ -1/x),$$
$$=\frac{(-1)^n}{n!}x^{-\alpha/2-1/2}\exp(x/2)\,W_{1/2+\alpha/2+n,\ \alpha/2}(x);$$
$$\sum_{n=0}^{\infty}L_n^\alpha(x)z^n=(1-z)^{-\alpha-1}\exp\frac{xz}{z-1}.$$

Относительно других производящих функций см. ВТФ, т. II, стр. 190.

16.1. Многочлены Чебышева

Интегралы в этом пункте могут быть выражены как интегралы от тригонометрических функций.

В этом пункте m и n — неотрицательные целые числа.

(1)	$\int_{-1}^{1}(1-x)^{-1/2}(1+x)^\alpha T_n(x)\,dx=\frac{2^{\alpha+2n+1/2}\pi^{1/2}(n!)^2\,\Gamma(\alpha+1)\,\Gamma(\alpha+3/2)}{(2n)!\,\Gamma(\alpha+n+3/2)\,\Gamma(\alpha-n+3/2)},$ $\operatorname{Re}\alpha>-1$
(2)	$\int_{-1}^{1}(1-x)^\alpha(1+x)^\beta T_n(x)\,dx=\frac{2^{\alpha+\beta+2n+1}(n!)^2\,\Gamma(\alpha+1)\,\Gamma(\beta+1)}{(2n)!\,\Gamma(\alpha+\beta+2)}\times$ $\times\,{}_3F_2(-n,\ n,\ \alpha+1;\ 1/2,\ \alpha+\beta+2;\ 1),\quad \operatorname{Re}\alpha>-1,\ \operatorname{Re}\beta>-1$
(3)	$\fint_{-1}^{1}(x-y)^{-1}(1-x^2)^{-1/2}T_n(x)\,dx=\pi U_{n-1}(y),\qquad -1<y<1$
(4)	$\int_{-1}^{1}\sin(xyz)\cos\left[(1-x^2)^{1/2}(1-y^2)^{1/2}z\right]T_{2n+1}(x)(1-x^2)^{-1/2}\,dx=$ $=(-1)^n\pi T_{2n+1}(y)\,J_{2n+1}(z)$
(5)	$\int_{-1}^{1}\cos(xyz)\cos\left[(1-x^2)^{1/2}(1-y^2)^{1/2}z\right]T_{2n}(x)(1-x^2)^{-1/2}\,dx=$ $=(-1)^n\pi T_{2n}(y)\,J_{2n}(z)$
(6)	$\int_{-1}^{1}[T_n(x)]^2\,dx=1-(4n^2-1)^{-1}$

(6′)	$\int\limits_0^1 (1-x^2)^{-1/2} x^m T_n(x)\, dx = 2^{-m-1}\pi \binom{m}{\frac{m-n}{2}}$	
(7)	$\int\limits_{-1}^1 (1-x^2)^{-1/2} [T_0(x)]^2\, dx = \pi$	
(8)	$\int\limits_{-1}^1 (1-x^2)^{-1/2} [T_n(x)]^2\, dx = \pi/2,$	$n \neq 0$
(9)	$\int\limits_{-1}^1 (1-x^2)^{-1/2} T_m(x)\, T_n(x)\, dx = 0,$	$m \neq n$
(10)	$\int\limits_{-1}^1 (1-x)^{-1/2} (1+x)^{m-n-3/2} T_m(x)\, T_n(x)\, dx = 0,$	$m > n$
(11)	$\int\limits_{-1}^1 (1-x)^{-1/2} (1+x)^{m+n-3/2} T_m(x)\, T_n(x)\, dx = \dfrac{\pi(2m+2n-2)!}{2^{m+n}(2m-1)!\,(2n-1)!},$	$m+n \neq 0$
(12)	$\int\limits_{-1}^1 (1+x)^{-1/2} (1-x)^{\alpha-1} T_m(x)\, T_n(x)\, dx = \dfrac{\pi^{1/2} 2^{\alpha-1/2}\,\Gamma(\alpha)\,\Gamma(n-\alpha+1/2)}{\Gamma(1/2-\alpha)\,\Gamma(\alpha+n+1/2)} \times$ $\times\, {}_4F_3(-m, m, \alpha, \alpha+1/2;\, 1/2, \alpha+n+1/2, \alpha-n+1/2;\, 1),$	$\operatorname{Re}\alpha > 0$
(13)	$\int\limits_0^1 x^{-1/2} (1-x^2)^{-1/2} e^{-2\alpha/x}\, T_n(x)\, dx = \pi^{1/2} D_{n-1/2}(2\alpha^{1/2})\, D_{-n-1/2}(2\alpha^{1/2}),$	$\operatorname{Re}\alpha > 0$
(14)	$\int\limits_{-1}^1 (1-x^2)^{-1/2} T_n(1-x^2 y)\, dx = 2^{-1}\pi\, [P_n(1-y) + P_{n-1}(1-y)]$	
(15)	$\int\limits_0^\infty \dfrac{(1+x^2)^{-n}\, T_{2n}[(1+x^2)^{-1/2}]\, dx}{\operatorname{ch}(\pi x/2)} = (-1)^{n+1} \dfrac{2\pi^{2n}}{(2n)!} (2^{n-1}-1)\, B_{2n}$	

(16)	$\int\limits_0^\infty \frac{(1+x^2)^{-n/2}\, T_n\left[(1+x^2)^{-1/2}\right]}{\operatorname{ch}(\pi x/2)}\, dx = 2^{1-n}\left(1-2^{1-n}\right)\zeta(n)$
(17)	$\int\limits_0^\infty (1+x^2)^{1/2-n}\left[\operatorname{ch}(\pi x/2)\right]^{-2} T_{2n-1}\left[(1+x^2)^{-1/2}\right] dx =$ $= 2(-1)^{n+1}\pi^{2n-1}\frac{2n-1}{(2n)!}B_{2n}$
(18)	$\int\limits_0^\infty (1+x^2)^{-n/2}\left[\operatorname{ch}(\pi x/2)\right]^{-2} T_n\left[(1+x^2)^{-1/2}\right] dx = \pi^{-1}n2^{1-n}\zeta(n+1)$
(19)	$\int\limits_0^\infty \frac{(\alpha^2+x^2)^{-n/2}\, T_n\left[\alpha(\alpha^2+x^2)^{-1/2}\right]}{\operatorname{ch}(\pi x/2)}\, dx =$ $= 2^{1-2n}\left[\zeta\left(n, \frac{\alpha+1}{4}\right) - \zeta\left(n, \frac{\alpha+3}{4}\right)\right] = 2^{1-n}\Phi\left(-1, n, \frac{\alpha+1}{2}\right), \quad \operatorname{Re}\alpha > 0$
(20)	$\int\limits_0^\infty (\alpha^2+x^2)^{-n/2}\left[\operatorname{ch}(\pi x/2)\right]^{-2} T_n\left[\alpha(\alpha^2+x^2)^{-1/2}\right] dx =$ $= \pi^{-1}n2^{1-n}\zeta\left(n+1, \frac{\alpha+1}{2}\right), \quad \operatorname{Re}\alpha > 0$
(21)	$\int\limits_{-1}^1 (1-x)^{1/2}(1+x)^{\alpha}\, U_n(x)\, dx =$ $= \frac{\pi^{1/2}2^{\alpha+2n+3/2}\left[(n+1)!\right]^2\Gamma(\alpha+1/2)\,\Gamma(\alpha+1)}{(2n+2)!\,\Gamma(\alpha+n+5/2)\,\Gamma(\alpha-n+1/2)}, \quad \operatorname{Re}\alpha > -1$
(22)	$\int\limits_{-1}^1 (1-x)^{\alpha}(1+x)^{\beta}\, U_n(x)\, dx = \frac{2^{\alpha+\beta+2n+2}\left[(n+1)!\right]^2\Gamma(\alpha+1)\,\Gamma(\beta+1)}{(2n+2)!\,\Gamma(\alpha+\beta+2)} \times$ $\times\, {}_3F_2\left(-n, n+1, \alpha+1; 3/2, \alpha+\beta+2; 1\right), \quad \operatorname{Re}\alpha > -1, \quad \operatorname{Re}\beta > -1$
(23)	$\fint\limits_{-1}^1 (x-y)^{-1}(1-x^2)^{-1/2}\, U_n(x)\, dx = -\pi\, T_{n+1}(y), \qquad -1 < y < 1$
(24)	$\int\limits_{-1}^1 \cos(xyz)\sin\left[(1-x^2)^{1/2}(1-y^2)^{1/2}z\right] U_{2n}(x)\, dx =$ $= (-1)^n\pi(1-y^2)^{1/2}\, U_{2n}(y)\, J_{2n+1}(z)$

(25)	$\int_{-1}^{1} \sin(xyz) \sin\left[(1-x^2)^{1/2}(1-y^2)^{1/2} z\right] U_{2n+1}(x)\,dx = (-1)^n \pi (1-y^2)^{1/2} U_{2n+1}(y) J_{2n+2}(z)$
(26)	$\int_{-1}^{1} (1-x)^{-1/2}(1+x)^{1/2} [U_n(x)]^2\,dx = (n+1)\pi$
(27)	$\int_{-1}^{1} (1-x^2)^{1/2} [U_n(x)]^2\,dx = \pi/2$
(28)	$\int_{-1}^{1} (1-x^2)^{1/2} U_m(x) U_n(x)\,dx = 0,\qquad m \neq n$
(29)	$\int_{-1}^{1} (1-x)(1+x)^{1/2} U_m(x) U_n(x)\,dx = \dfrac{2^{5/2}(m+1)(n+1)}{(m+n+{}^3/_2)(m+n+{}^5/_2)[1-4(m-n)^2]}$
(30)	$\int_{-1}^{1} (1-x)^{1/2}(1+x)^{m-n-1/2} U_m(x) U_n(x)\,dx = 0,\qquad m > n$
(31)	$\int_{-1}^{1} (1-x)^{1/2}(1+x)^{m+n+3/2} U_m(x) U_n(x)\,dx = \dfrac{\pi(2m+2n+2)!}{2^{m+n+2}(2m+1)!(2n+1)!}$
(32)	$\int_{-1}^{1} (1+x)^{1/2}(1-x)^{\alpha-1} U_m(x) U_n(x)\,dx = \dfrac{\pi^{1/2} 2^{\alpha-1/2}(m+1)(n+1)\Gamma(\alpha)\Gamma(n-\alpha+{}^3/_2)}{\Gamma({}^3/_2-\alpha)\Gamma({}^3/_2+\alpha+n)} \times {}_4F_3(-m, m+2, \alpha, \alpha-{}^1/_2; {}^3/_2, \alpha+n+{}^3/_2, \alpha-n-{}^1/_2; 1),\qquad \operatorname{Re}\alpha > 0$
(33)	$\int_{-1}^{1} (1-x^2)^{-1/2} U_{2n}(xz)\,dx = \pi P_n(2z^2-1)$
(34)	$\int_{-1}^{1} U_n\left[x(1-y^2)^{1/2}(1-z^2)^{1/2}+yz\right] dx = \dfrac{2}{n+1} U_n(y) U_n(z)$

(35)	$\int\limits_0^\infty \frac{x\, U_{2n-1}\left[(1+x^2)^{-1/2}\right]}{(1+x^2)^{n+1/2}\left(e^{\pi x}+1\right)}\, dx = \frac{1}{2\,(2n-1)} + \frac{(-1)^n \pi^{2n}}{2\,(2n)\,!} B_{2n}$	
(36)	$\int\limits_0^\infty \frac{x\, U_n\left[(1+x^2)^{-1/2}\right]}{(1+x^2)^{n/2+1}\left(e^{\pi x}+1\right)}\, dx = \frac{1}{2n} - 2^{-n-1}\zeta\,(n+1)$	
(37)	$\int\limits_0^\infty \frac{x\, U_{2n-1}\left[(1+x^2)^{-1/2}\right]}{(1+x^2)^{n+1/2}\left(e^{2\pi x}-1\right)}\, dx = \frac{(-1)^{n+1}(2\pi)^{2n}}{4\,(2n)\,!} B_{2n} - \frac{1}{4} - \frac{1}{4n-2}$	
(38)	$\int\limits_0^\infty \frac{x\, U_n\left[(1+x^2)^{-1/2}\right]}{(1+x^2)^{n/2+1}\left(e^{2\pi x}-1\right)}\, dx = \frac{1}{2}\zeta\,(n+1) - \frac{1}{4} - \frac{1}{2n}$	
(39)	$\int\limits_0^\infty \frac{x\, U_n\left[\alpha\,(\alpha^2+x^2)^{-1/2}\right]}{(\alpha^2+x^2)^{n/2+1}\left(e^{\pi x}+1\right)}\, dx = \frac{\alpha^{-n}}{2n} - 2^{-n-1}\zeta\left(n+1,\ \frac{\alpha+1}{2}\right),$	$\operatorname{Re}\alpha>0$
(40)	$\int\limits_0^\infty \frac{x\, U_n\left[\alpha\,(\alpha^2+x^2)^{-1/2}\right]}{(\alpha^2+x^2)^{n/2+1}\left(e^{2\pi x}-1\right)}\, dx = \frac{1}{2}\zeta\,(n+1,\ \alpha) - \frac{\alpha^{-n-1}}{4} - \frac{\alpha^{-n}}{2n},$	$\operatorname{Re}\alpha>0$

16.2. Многочлены Лежандра

См. также многочлены Гегенбауэра, функции Лежандра, гипергеометрические ряды.

В этом пункте m и n — неотрнцательные целые числа.

(1)	$\int\limits_0^1 x^\lambda P_{2m}(x)\, dx = \frac{(-1)^m(-\lambda/2)_m}{2\,(1/2+\lambda/2)_{m+1}},$	$\operatorname{Re}\lambda>-1$
(2)	$\int\limits_0^1 x^\lambda P_{2m+1}(x)\, dx = \frac{(-1)^m(1/2-\lambda/2)_m}{2\,(1+\lambda/2)_{m+1}},$	$\operatorname{Re}\lambda>-2$
(3)	$\int\limits_{-1}^1 (1-x)^{-1/2} P_n(x)\, dx = \frac{2^{3/2}}{2n+1}$	

(4)	$\int\limits_{-1}^{1} (1-x^2)^{-1/2} P_{2m}(x)\,dx = \pi \left[\frac{(1/2)_m}{m!}\right]^2$
(5)	$\int\limits_{-1}^{1} x(1-x^2)^{-1/2} P_{2m+1}(x)\,dx = \pi \frac{(1/2)_m (1/2)_{m+1}}{m!\,(m+1)!}$
(6)	$\int\limits_{-1}^{1} (1-x)^{\alpha-1}(1+x)^{\beta-1} P_n(x)\,dx = \frac{2^{\alpha+\beta-1}\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}\, {}_3F_2(-n,\ 1+n,\ \alpha;\ 1,\ \alpha+\beta;\ 1)$, $\operatorname{Re}\alpha>0$, $\operatorname{Re}\beta>0$
(7)	$\int\limits_{-1}^{1} (z-x)^{-1} P_n(x)\,dx = 2Q_n(z)$, z в разрезанной плоскости
(8)	$\int\limits_{-1}^{1} x(z-x)^{-1} P_0(x)\,dx = 2Q_1(z)$, z в разрезанной плоскости
(9)	$\int\limits_{-1}^{1} (z-x)^{-1} x^{n+1} P_n(x)\,dx = 2z^{n+1} Q_n(z) - \frac{2^{n+1}(n!)^2}{(2n+1)!}$, z в разрезанной плоскости
(10)	$\int\limits_{-1}^{1} (z-x)^{-1} x^m P_n(x)\,dx = 2z^m Q_n(z)$, $m\leqslant n$, z в разрезанной плоскости
(11)	$\int\limits_{-1}^{1} (a^2+b^2-2abx)^{-1/2}\sin[\lambda(a^2+b^2-2abx)^{1/2}]\,P_n(x)\,dx = \pi(ab)^{-1/2} J_{n+1/2}(a\lambda) J_{n+1/2}(b\lambda)$, $a,\ b>0$
(12)	$\int\limits_{-1}^{1} (a^2+b^2-2abx)^{-1/2}\cos[\lambda(a^2+b^2-2abx)^{1/2}]\,P_n(x)\,dx = \pi(ab)^{-1/2} J_{n+1/2}(a\lambda) Y_{n+1/2}(b\lambda)$, $0\leqslant a\leqslant b$

Комплексная z-плоскость разрезана вдоль отрезка $[-1,\ 1]$ вещественной оси.

(13)	$\int_{-1}^{1} [P_n(x)]^2\,dx = (n+{}^1/_2)^{-1}$
(14)	$\int_{-1}^{1} P_m(x)\,P_n(x)\,dx = 0, \qquad m \neq n$
(15)	$\int_{-1}^{1} (1+x)^{m+n}\,P_m(x)\,P_n(x)\,dx = \dfrac{2^{m+n+1}\,[(m+n)!]^4}{(m!n!)^2\,(2m+2n+1)!}$
(16)	$\int_{-1}^{1} (1+x)^{m-n-1}\,P_m(x)\,P_n(x)\,dx = 0, \qquad m > n$
(17)	$\int_{-1}^{1} (1-x)^{\alpha-1}\,P_m(x)\,P_n(x)\,dx =$ $= \dfrac{2^{\alpha}\,\Gamma(\alpha)\,\Gamma(n-\alpha+1)}{\Gamma(1-\alpha)\,\Gamma(n+\alpha+1)}\,{}_4F_3(-m,\ m+1,\ \alpha,\ \alpha;\ 1,\ \alpha+n+1,\ \alpha-n;\ 1),$ $\mathrm{Re}\,\alpha > 0$
(18)	$\int_{-1}^{1} (z-x)^{-1}\,P_m(x)\,P_n(x)\,dx = 2\,P_m(z)\,Q_n(z),$ $m \leqslant n$, z в разрезанной плоскости
(19)	$\int_{-1}^{1} (z-x)^{-1}\,P_n(x)\,P_{n+1}(x)\,dx = 2\,P_{n+1}(z)\,Q_n(z) - 2\,(n+1)^{-1},$ z в разрезанной плоскости
(20)	$\int_{-1}^{1} x\,(z-x)^{-1}\,[P_n(x)]^2\,dx = 2z\,P_n(z)\,Q_n(z) - 2\,(2n+1)^{-1},$ z в разрезанной плоскости
(21)	$\int_{-1}^{1} x\,(z-x)^{-1}\,P_m(x)\,P_n(x)\,dx = 2z\,P_m(z)\,Q_n(z),$ $m < n$, z в разрезанной плоскости. Относительно других подобных интегралов см. MacRobert T. M., 1948: Proc. Glasgow Math. Assoc., 1, 10—12.

Комплексная z-плоскость разрезана вдоль отрезка $[-1, 1]$ вещественной оси.

(22)	$\int_0^1 x^{2\mu-1} P_n(1-2x^2)\,dx = \frac{(-1)^n [\Gamma(\mu)]^2}{2\Gamma(\mu+n)\,\Gamma(\mu-n)}$,	$\operatorname{Re}\mu > 0$
(23)	$\int_0^1 x(\alpha^2+x^2)^{-1/2} P_n(1-2x^2)\,dx = \frac{[\alpha+(\alpha^2+1)^{1/2}]^{-2n-1}}{2n+1}$,	$\operatorname{Re}\alpha > 0$
(24)	$\int_{-1}^1 (1-x)^\alpha (1+x)^\beta \mathrm{P}_n^m(x)\,dx$. См. Shabde N. G., 1940: Bull. Calcutta Math. Soc., 32, 121—128.	
(25)	$\int_{-1}^1 (z-x)^{-1}(1-x^2)^{m/2} \mathrm{P}_n^m(x)\,dx = 2^m (z^2-1)^{m/2} Q_n^m(z)$,	$m \leqslant n$, z в разрезанной плоскости
(26)	$\int_{-1}^1 x^k (z-x)^{-1}(1-x^2)^{m/2} \mathrm{P}_n^m(x)\,dx = 2^m z^k (z^2-1)^{m/2} Q_n^m(z)$,	$m \leqslant n$, $k=0, 1, \ldots, n-m$, z в разрезанной плоскости
(27)	$\int_{-1}^1 [\mathrm{P}_n^m(x)]^2\,dx = \frac{2}{2n+1}\,\frac{(n+m)!}{(n-m)!}$,	$m \leqslant n$
(28)	$\int_{-1}^1 \mathrm{P}_n^m(x)\,\mathrm{P}_k^m(x)\,dx = 0$,	$k \neq n$
(29)	$\int_{-1}^1 \mathrm{P}_n^m(x)\,\mathrm{Q}_k^m(x)\,dx = (-1)^m \frac{1-(-1)^{k+n}}{(n-k)(n+k+1)}\,\frac{(k+m)!}{(k-m)!}$	
(30)	$\int_{-1}^1 (1-x^2)^{-1} \mathrm{P}_n^m(x)\,\mathrm{P}_n^k(x)\,dx = 0$,	$k \neq m$
(31)	$\int_{-1}^1 (1-x^2)^{-1} [\mathrm{P}_n^m(x)]^2\,dx = \frac{(n+m)!}{m(n-m)!}$	

Комплексная z-плоскость разрезана вдоль отрезка $[-1, 1]$ вещественной оси.

(32)	$\int\limits_{-1}^{1} (1-x^2)^{m/2} P_k^m(x) P_l^m(x) P_n^m(x)\,dx =$ $= (-1)^m \pi^{-3/2} \frac{(k+m)!\,(l+m)!\,(n+m)!\,(s-m)!}{(k-m)!\,(l-m)!\,(n-m)!\,(s-k)!} \times$ $\times \frac{\Gamma(m+1/2)\,\Gamma(t-k+1/2)\,\Gamma(t-l+1/2)\,\Gamma(t-n+1/2)}{(s-l)!\,(s-n)!\,\Gamma(s+3/2)},$ $2s = k+l+n+m,\quad 2t = k+l+n-m$ четно $l \geqslant m,\quad m \leqslant k-l-m \leqslant n \leqslant k+l+m$
(33)	$\int\limits_{-1}^{1} P_l^p(x) P_m^q(x) P_n^r(x)\,dx.$ См. Gaunt J. A., 1929: Philos. Trans. Royal Soc., A 228, 151—196.

16.3. Многочлены Гегенбауэра (ультрасферические многочлены)

См. также многочлены Якоби, функции Лежандра, гипергеометрические ряды.

В этом пункте m и n — натуральные числа.

(1)	$\int\limits_{-1}^{1} (1-x^2)^{\nu-1/2} C_n^\nu(x)\,dx = 0,\qquad n>0,\quad \operatorname{Re}\nu > -1/2$
(2)	$\int\limits_{0}^{1} x^{n+2\rho} (1-x^2)^{\nu-1/2} C_n^\nu(x)\,dx = \frac{(2\nu)_n (2\rho+1)_n \Gamma(\nu+1/2)\,\Gamma(\rho+1/2)}{2^{n+1} n!\,\Gamma(n+\nu+\rho+1)},$ $\operatorname{Re}\rho > -1/2$
(3)	$\int\limits_{-1}^{1} (1-x)^{\nu-1/2} (1+x)^\beta C_n^\nu(x)\,dx =$ $= \frac{2^{\beta+\nu+1/2}\,\Gamma(\beta+1)\,\Gamma(\nu+1/2)\,\Gamma(2\nu+n)\,\Gamma(\beta-\nu+3/2)}{n!\,\Gamma(2\nu)\,\Gamma(\beta-\nu-n+3/2)\,\Gamma(\beta+\nu+n+3/2)},$ $\operatorname{Re}\beta > -1,\quad \operatorname{Re}\nu > -1/2$
(4)	$\int\limits_{-1}^{1} (1-x)^\alpha (1+x)^\beta C_n^\nu(x)\,dx = \frac{2^{\alpha+\beta+1}\,\Gamma(\alpha+1)\,\Gamma(\beta+1)\,\Gamma(n+2\nu)}{n!\,\Gamma(2\nu)\,\Gamma(\alpha+\beta+2)} \times$ $\times {}_3F_2(-n,\ n+2\nu,\ \alpha+1;\ \nu+1/2,\ \alpha+\beta+2;\ 1),$ $\operatorname{Re}\alpha > -1,\quad \operatorname{Re}\beta > -1$

(5)	$\int_{-1}^{1} x^m (z-x)^{-1} (1-x^2)^{\nu-1/2} C_n^\nu(x)\,dx = \frac{\pi^{1/2} 2^{3/2-\nu}}{\Gamma(\nu)} e^{-(\nu-1/2)\pi i} z^m (z^2-1)^{\nu/2-1/4} Q_{n+\nu-1/2}^{\nu-1/2}(z)$, $m \leqslant n$, $\operatorname{Re}\nu > -1/2$, z в разрезанной плоскости
(6)	$\int_{-1}^{1} x^{n+1} (z-x)^{-1} (1-x^2)^{\nu-1/2} C_n^\nu(x)\,dx = \frac{\pi^{1/2} 2^{3/2-\nu}}{\Gamma(\nu)} e^{-(\nu-1/2)\pi i} z^{n+1} (z^2-1)^{\nu/2-1/4} Q_{n+\nu-1/2}^{\nu-1/2}(z) - \frac{\pi 2^{1-2\nu-n}\, n!}{\Gamma(\nu)\Gamma(\nu+n+1)}$, $\operatorname{Re}\nu > -1/2$, z в разрезанной плоскости
(7)	$\int_{-1}^{1} (1-x^2)^{\nu-1/2} e^{i\alpha x} C_n^\nu(x)\,dx = \frac{\pi 2^{1-\nu} i^n \Gamma(2\nu+n)}{n!\,\Gamma(\nu)} \alpha^{-\nu} J_{\nu+n}(\alpha)$, $\operatorname{Re}\nu > -1/2$
(8)	$\int_{-1}^{1} (1-x^2)^{\nu-1/2} \left[C_n^\nu(x)\right]^2 dx = \frac{\pi 2^{1-2\nu}\Gamma(2\nu+n)}{n!\,(n+\nu)\,[\Gamma(\nu)]^2}$, $\operatorname{Re}\nu > -1/2$
(9)	$\int_{-1}^{1} (1-x)^{\nu-3/2} (1+x)^{\nu-1/2} \left[C_n^\nu(x)\right]^2 dx = \frac{\pi^{1/2}\Gamma(\nu-1/2)\Gamma(2\nu+n)}{n!\,\Gamma(\nu)\Gamma(2\nu)}$, $\operatorname{Re}\nu > 1/2$
(10)	$\int_{-1}^{1} (1-x)^{\nu-1/2} (1+x)^{2\nu-1} \left[C_n^\nu(x)\right]^2 dx = \frac{2^{3\nu-1/2}[\Gamma(2\nu+n)]^2\Gamma(2n+\nu+1/2)}{(n!)^2\Gamma(2\nu)\Gamma(3\nu+2n+1/2)}$, $\operatorname{Re}\nu > 0$
(11)	$\int_{-1}^{1} (1-x)^{3\nu+2n-3/2} (1+x)^{\nu-1/2} \left[C_n^\nu(x)\right]^2 dx = \frac{\pi^{1/2}[\Gamma(\nu+1/2)]^2\Gamma(\nu+2n+1/2)\Gamma(2\nu+2n)\Gamma(3\nu+2n-1/2)}{2^{2\nu+2n}[n!\,\Gamma(\nu+n+1/2)\Gamma(2\nu)]^2\Gamma(2\nu+2n+1/2)}$, $\operatorname{Re}\nu > 1/6$
(12)	$\int_{-1}^{1} (1-x^2)^{\nu-1/2} C_m^\nu(x) C_n^\nu(x)\,dx = 0$, $m \neq n$, $\operatorname{Re}\nu > -1/2$

Комплексная z-плоскость разрезана вдоль отрезка $[-1, 1]$ вещественной оси.

(13)	$\int\limits_{-1}^{1} (1-x)^{\nu-1/2}(1+x)^{\nu+m-n-3/2} C_m^\nu(x) C_n^\nu(x)\,dx =$ $= (-1)^m \frac{2^{2-2\nu-m+n}\pi^{3/2}\,\Gamma(2\nu+n)\,\Gamma(\nu-1/2+m-n)\,\Gamma(1/2-\nu+m-n)}{m!\,(n-m)!\,[\Gamma(\nu)]^2\,\Gamma(1/2+\nu+m)\,\Gamma(1/2-\nu-n)\,\Gamma(1/2+m-n)}$, $\operatorname{Re}\nu > 1/2$
(14)	$\int\limits_{-1}^{1} (1-x)^{2\nu-1}(1+x)^{\nu-1/2} C_m^\nu(x) C_n^\nu(x)\,dx =$ $= \frac{2^{3\nu-1/2}\,\Gamma(\nu+1/2)\,\Gamma(2\nu+m)\,\Gamma(2\nu+n)\,\Gamma(\nu+1/2+m+n)\,\Gamma(1/2-\nu+n-m)}{m!\,n!\,\Gamma(2\nu)\,\Gamma(1/2-\nu)\,\Gamma(\nu+1/2+n-m)\,\Gamma(3\nu+1/2+m+n)}$, $\operatorname{Re}\nu > 1/2$
(15)	$\int\limits_{-1}^{1} (1-x)^{\nu-1/2}(1+x)^{3\nu+m+n-3/2} C_m^\nu(x) C_n^\nu(x)\,dx =$ $= \frac{2^{4\nu+m+n-1}\,[\Gamma(\nu+1/2)\,\Gamma(2\nu+m+n)]^2\,\Gamma(\nu+m+n+1/2)\,\Gamma(3\nu+m+n-1/2)}{\Gamma(\nu+m+1/2)\,\Gamma(\nu+n+1/2)\,\Gamma(2\nu+m)\,\Gamma(2\nu+n)\,\Gamma(4\nu+2m+2n)}$, $\operatorname{Re}\nu > 1/6$
(16)	$\int\limits_{-1}^{1} (1-x)^{\alpha}(1+x)^{\nu-1/2} C_m^\mu(x) C_n^\nu(x)\,dx =$ $= \frac{2^{\alpha+\nu+1/2}\,\Gamma(\alpha+1)\,\Gamma(\nu+1/2)\,\Gamma(\nu-\alpha+n-1/2)\,\Gamma(2\mu+m)\,\Gamma(2\nu+n)}{m!\,n!\,\Gamma(\nu-\alpha-1/2)\,\Gamma(\nu-\alpha+n+3/2)\,\Gamma(2\mu)\,\Gamma(2\nu)} \times$ $\times {}_4F_3(-m, m+2\mu, \alpha+1, \alpha-\nu+3/2;\ \mu+1/2, \nu+\alpha+n+3/2, \alpha-\nu-n+3/2;\ 1)$, $\operatorname{Re}\alpha > -1$, $\operatorname{Re}\nu > -1/2$
(17)	$\int\limits_{-1}^{1} (z-x)^{-1}(1-x^2)^{\nu-1/2} C_m^\nu(x) C_n^\nu(x)\,dx =$ $= \frac{\pi^{1/2}2^{1/2-\nu}}{\Gamma(\nu)} e^{-(\nu-1/2)\pi i}(z^2-1)^{\nu/2-1/4} C_m^\nu(z)\, Q_{n+\nu-1/2}^{\nu-1/2}(z)$, $m \leqslant n$, $\operatorname{Re}\nu > -1/2$, z в разрезанной плоскости
(18)	$\int\limits_{-1}^{1} (1-x^2)^{\nu-1/2} C_m^\nu(x) C_n^\nu(x) C_k^\nu(x)\,dx.$ См. Hsü Hsien-Yü, 1938: Duke Math. J. 4, 374—383.
(19)	$\int\limits_{-1}^{1} (1-x^2)^{\nu/2-1} C_{2n}^\nu(\alpha x)\,dx = \frac{\pi^{1/2}\,\Gamma(\nu/2)}{\Gamma(\nu/2+1/2)} C_n^{\nu/2}(2\alpha^2-1)$, $\operatorname{Re}\nu > 0$

Комплексная z-плоскость разрезана вдоль отрезка $[-1, 1]$ вещественной оси.

(20)	$\int_{-1}^{1} (1-x^2)^{\nu-1} C_n^{\nu} (\cos\alpha\cos\beta + x\sin\alpha\sin\beta)\,dx =$ $= \frac{2^{2\nu-1} n! [\Gamma(\nu)]^2}{\Gamma(2\nu+n)} C_n^{\nu}(\cos\alpha)\, C_n^{\nu}(\cos\beta), \qquad \operatorname{Re}\nu > 0$
(21)	$\int_{0}^{1} x^{2\nu} (1-x^2)^{\sigma-1} C_n^{\nu}(1-x^2 y)\,dx = \frac{(2\nu)_n \Gamma(\nu+1/2)\Gamma(\sigma)}{2\Gamma(n+\nu+\sigma+1/2)} P_n^{(\alpha,\beta)}(1-y),$ $\operatorname{Re}\nu > -1/2, \quad \operatorname{Re}\sigma > 0, \ \alpha = \nu+\sigma-1/2, \quad \beta = \nu-\sigma-1/2$

16.4. Многочлены Якоби

См. также гипергеометрические ряды.

В этом пункте m и n — неотрицательные целые числа.

(1)	$\int_{-1}^{1} (1-x)^{\alpha} (1+x)^{\sigma} P_n^{(\alpha,\beta)}(x)\,dx = \frac{2^{\alpha+\sigma+1}\Gamma(\sigma+1)\Gamma(\alpha+n+1)\Gamma(\sigma-\beta+1)}{n!\,\Gamma(\sigma-\beta-n+1)\Gamma(\alpha+\sigma+n+2)},$ $\operatorname{Re}\alpha > -1, \quad \operatorname{Re}\sigma > -1$
(2)	$\int_{-1}^{1} (1-x)^{\rho} (1+x)^{\beta} P_n^{(\alpha,\beta)}(x)\,dx = \frac{2^{\beta+\rho+1}\Gamma(\rho+1)\Gamma(\beta+n+1)\Gamma(\alpha-\rho+n)}{n!\,\Gamma(\alpha-\rho)\Gamma(\beta+\rho+n+2)},$ $\operatorname{Re}\rho > -1, \ \operatorname{Re}\beta > -1$
(3)	$\int_{-1}^{1} (1-x)^{\rho} (1+x)^{\sigma} P_n^{(\alpha,\beta)}(x)\,dx = \frac{2^{\rho+\sigma+1}\Gamma(\rho+1)\Gamma(\sigma+1)}{\Gamma(\rho+\sigma+2)} \frac{\Gamma(n+\alpha+1)}{n!\,\Gamma(\alpha+1)} \times$ $\times\, {}_3F_2(-n, \alpha+\beta+n+1, \rho+1; \alpha+1, \rho+\sigma+2; 1),$ $\operatorname{Re}\rho > -1, \ \operatorname{Re}\sigma > -1$
(4)	$\int_{-1}^{1} (z-x)^{-1} (1-x)^{\alpha} (1+x)^{\beta} P_n^{(\alpha,\beta)}(x)\,dx =$ $= \frac{2^{\alpha+\beta+n+1}\Gamma(\alpha+n+1)\Gamma(\beta+n+1)}{\Gamma(\alpha+\beta+2n+2)(z-1)^{n+1}} \times$ $\times\, {}_2F_1\left(n+1, \alpha+n+1; \alpha+\beta+2n+2; \frac{2}{1-z}\right),$ $\operatorname{Re}\alpha > -1, \ \operatorname{Re}\beta > -1,$ z в разрезанной плоскости
(5)	$\int_{-1}^{1} (1-x)^{\alpha} (1+x)^{\beta} \left[P_n^{(\alpha,\beta)}(x)\right]^2 dx = \frac{2^{\alpha+\beta+1}\Gamma(\alpha+n+1)\Gamma(\beta+n+1)}{n!\,(\alpha+\beta+2n+1)\Gamma(\alpha+\beta+n+1)},$ $\operatorname{Re}\alpha > -1, \ \operatorname{Re}\beta > -1$

Комплексная z-плоскость разрезана вдоль отрезка $[-1, 1]$ вещественной оси.

(6)	$\int_{-1}^{1} (1-x)^{\alpha-1} (1+x)^{\beta} \left[P_n^{(\alpha,\beta)}(x)\right]^2 dx = \frac{2^{\alpha+\beta}\,\Gamma(\alpha+n+1)\,\Gamma(\beta+n+1)}{n!\,\alpha\,\Gamma(\alpha+\beta+n+1)}$, $\operatorname{Re}\alpha > 0,\ \operatorname{Re}\beta > -1$
(7)	$\int_{-1}^{1} (1-x)^{2\alpha} (1+x)^{\beta} \left[P_n^{(\alpha,\beta)}(x)\right]^2 dx = \frac{2^{4\alpha+\beta+1}\,\Gamma(\alpha+1/2)\,[\Gamma(\alpha+n+1)]^2\,\Gamma(\beta+2n+1)}{\pi^{1/2}\,(n!)^2\,\Gamma(\alpha+1)\,\Gamma(2\alpha+\beta+2n+2)}$, $\operatorname{Re}\alpha > -1/2,\ \operatorname{Re}\beta > -1$
(8)	$\int_{-1}^{1} (1-x)^{2\alpha+\beta+2n} (1+x)^{\beta} \left[P_n^{(\alpha,\beta)}(x)\right]^2 dx = \frac{2^{2\alpha+2\beta+2n+1}\,\Gamma(\beta+2n+1)\,[\Gamma(\alpha+\beta+2n+1)]^2\,\Gamma(2\alpha+\beta+2n+1)}{[n!\,\Gamma(\alpha+\beta+n+1)]^2\,\Gamma(2\alpha+2\beta+4n+2)}$, $\operatorname{Re}\beta > -1,\ \operatorname{Re}(2\alpha+\beta) > -1$
(9)	$\int_{-1}^{1} (1-x)^{\alpha} (1+x)^{\beta} P_n^{(\alpha,\beta)}(x)\, P_m^{(\alpha,\beta)}(x)\, dx = 0$, $m \neq n,\ \operatorname{Re}\alpha > -1,\ \operatorname{Re}\beta > -1$
(10)	$\int_{-1}^{1} (1-x)^{\rho} (1+x)^{\beta} P_n^{(\alpha,\beta)}(x)\, P_n^{(\rho,\beta)}(x)\, dx = \frac{2^{\rho+\beta+1}\,\Gamma(\rho+n+1)\,\Gamma(\beta+n+1)\,\Gamma(\alpha+\beta+2n+1)}{n!\,\Gamma(\rho+\beta+2n+2)\,\Gamma(\alpha+\beta+n+1)}$, $\operatorname{Re}\rho > -1,\ \operatorname{Re}\beta > -1$
(11)	$\int_{-1}^{1} (1-x)^{\rho-1} (1+x)^{\beta} P_n^{(\alpha,\beta)}(x)\, P_n^{(\rho,\beta)}(x)\, dx = \frac{2^{\rho+\beta}\,\Gamma(\alpha+n+1)\,\Gamma(\beta+n+1)\,\Gamma(\rho)}{n!\,\Gamma(\alpha+1)\,\Gamma(\rho+\beta+n+1)}$, $\operatorname{Re}\beta > -1,\ \operatorname{Re}\rho > 0$
(12)	$\int_{-1}^{1} (1-x)^{\alpha} (1+x)^{\sigma} P_n^{(\alpha,\beta)}(x)\, P_m^{(\alpha,\sigma)}(x)\, dx = \frac{2^{\alpha+\sigma+1}\,\Gamma(\alpha+n+1)\,\Gamma(\alpha+\beta+m+n+1)\,\Gamma(\sigma+m+1)\,\Gamma(\sigma-\beta+1)}{m!\,(n-m)!\,\Gamma(\alpha+\beta+n+1)\,\Gamma(\alpha+\sigma+m+n+2)\,\Gamma(\sigma-\beta-n+1)}$, $\operatorname{Re}\alpha > -1,\ \operatorname{Re}\sigma > -1$

(13)	$\displaystyle\int_{-1}^{1} (1-x)^{\alpha} (1+x)^{\beta+\sigma} P_n^{(\alpha,\beta)}(x) P_m^{(\alpha,\sigma)}(x)\,dx = \frac{2^{\alpha+\beta+\sigma+1}\Gamma(\alpha+m+n+1)\Gamma(\beta+n+1)\Gamma(\beta+\sigma+1)\Gamma(\sigma+m+1)}{m!\,n!\,\Gamma(\alpha+\beta+\sigma+m+n+2)\Gamma(\beta-m+n+1)\Gamma(\sigma+m-n+1)},$ $\operatorname{Re}\alpha>-1,\ \operatorname{Re}(\beta+\sigma)>-1$
(14)	$\displaystyle\int_{-1}^{1} (1-x)^{\alpha} (1+x)^{\alpha+\beta+\sigma+m+n} P_n^{(\alpha,\beta)}(x) P_m^{(\alpha,\sigma)}(x)\,dx = \frac{2^{2\alpha+\beta+\sigma+m+n+1}\Gamma(\alpha+\beta+\sigma+m+n+1)\Gamma(\alpha+\sigma+m+n+1)}{m!\,n!\,(\alpha+\beta+n+1)\Gamma(\alpha+\sigma+n+1)}\times \frac{\Gamma(\alpha+m+n+1)\Gamma(\alpha+\beta+m+n+1)}{\Gamma(2\alpha+\beta+\sigma+2m+2n+2)},$ $\operatorname{Re}\alpha>-1,\ \operatorname{Re}(\alpha+\beta+\sigma)>-1$
(15)	$\displaystyle\int_{-1}^{1} (1-x)^{\alpha} (1+x)^{\sigma+m-n-1} P_n^{(\alpha,\beta)}(x) P_m^{(\alpha,\sigma)}(x)\,dx = \frac{2^{\alpha+\sigma+m-n}\Gamma(\alpha+n+1)\Gamma(\beta+n+1)\Gamma(\sigma+m-n)\Gamma(\sigma-\beta+m-n)}{n!\,(n-m)!\,\Gamma(\alpha+\sigma+m+1)\Gamma(\beta-m+n+1)\Gamma(\sigma-\beta+2m-2n)},$ $\operatorname{Re}\alpha>0,\ \operatorname{Re}\sigma>n-m$
(16)	$\displaystyle\int_{-1}^{1} (1-x)^{\rho} (1+x)^{\beta} P_n^{(\alpha,\beta)}(x) P_m^{(\rho,\beta)}(x)\,dx = \frac{2^{\beta+\rho+1}\Gamma(\alpha+\beta+m+n+1)\Gamma(\beta+n+1)\Gamma(\rho+m+1)\Gamma(\alpha-\rho-m+n)}{n!\,(n-m)!\,\Gamma(\alpha+\beta+n+1)\Gamma(\beta+\rho+m+n+2)\Gamma(\alpha-\rho)},$ $\operatorname{Re}\beta>-1,\ \operatorname{Re}\rho>-1$
(17)	$\displaystyle\int_{-1}^{1} (1-x)^{\alpha+\rho} (1+x)^{\beta} P_n^{(\alpha,\beta)}(x) P_m^{(\rho,\beta)}(x)\,dx = \frac{(-1)^{m+n}2^{\alpha+\beta+\rho+1}\Gamma(\alpha+n+1)\Gamma(\alpha+\rho+1)\Gamma(\beta+m+n+1)\Gamma(\rho+m+1)}{m!\,n!\,\Gamma(\alpha-m+n+1)\Gamma(\alpha+\beta+\rho+m+n+2)\Gamma(\rho+m-n+1)},$ $\operatorname{Re}(\alpha+\rho)>-1,\ \operatorname{Re}\beta>-1$
(18)	$\displaystyle\int_{-1}^{1} (1-x)^{\alpha+\beta+\rho+m+n} (1+x)^{\beta} P_n^{(\alpha,\beta)}(x) P_m^{(\rho,\beta)}(x)\,dx = \frac{(-1)^{m+n}2^{\alpha+\beta+\rho+m+n+1}\Gamma(\alpha+\beta+m+n+1)}{m!\,n!\,\Gamma(\alpha+\beta+n+1)}\times \frac{\Gamma(\alpha+\beta+\rho+m+n+1)\Gamma(\beta+m+n+1)\Gamma(\beta+\rho+m+n+1)}{\Gamma(\alpha+2\beta+\rho+2m+2n+2)\Gamma(\beta+\rho+m+1)},$ $\operatorname{Re}\beta>-1,\ \operatorname{Re}(\alpha+\beta+\rho)>-1$

(19)	$\int_{-1}^{1} (1-x)^{\rho+m-n-1} (1+x)^{\beta} P_n^{(\alpha,\beta)}(x) P_m^{(\rho,\beta)}(x)\,dx = \frac{2^{\beta+\rho+m-n}\Gamma(\alpha+n+1)\Gamma(\beta+n+1)\Gamma(\rho+m-n)\Gamma(\alpha-\rho-2m+2n+1)}{n!\,(n-m)!\,\Gamma(\alpha-m+n+1)\Gamma(\alpha-\rho-m+n+1)\Gamma(\beta+\rho+m+1)},$ $\operatorname{Re}\beta > -1,\ \operatorname{Re}\rho > n-m$
(20)	$\int_{-1}^{1} (1-x)^{\tau} (1+x)^{\beta} P_n^{(\alpha,\beta)}(x) P_m^{(\rho,\sigma)}(x)\,dx = \frac{2^{\beta+\tau+1}\Gamma(\alpha-\tau+n)\Gamma(\beta+n+1)\Gamma(\rho+m+1)\Gamma(\tau+1)}{m!\,n!\,\Gamma(\rho+1)\Gamma(\alpha-\tau)\Gamma(\beta+\tau+n+2)} \times {}_4F_3(-m,\ \rho+\sigma+m+1,\ \tau+1,\ \tau-\alpha+1;\ \rho+1,\ \beta+\tau+n+2,\ \tau-\alpha-n+1;\ 1),$ $\operatorname{Re}\beta > -1,\ \operatorname{Re}\tau > -1$

16.5. Многочлены Эрмита

См. также многочлены Лагерра, функции параболического цилиндра, вырожденные гипергеометрические функции.

В этом пункте m и n — неотрицательные целые числа.

(1)	$\int_{-\infty}^{\infty} \exp(-x^2)\,\mathrm{He}_{2n}(x)\,dx = (-1)^n \Gamma(n+{}^1/_2)$
(2)	$\int_{-\infty}^{\infty} \exp[-2^{-1}(x-y)^2]\,\mathrm{He}_n(x)\,dx = (2\pi)^{1/2} y^n$
(3)	$\int_{-\infty}^{\infty} (x \pm ic)^{\nu} \exp(-2^{-1}x^2)\,\mathrm{He}_n(x)\,dx = \frac{\Gamma(\nu+1)}{\Gamma(\nu-n+1)} (2\pi)^{1/2} \exp[\pm 2^{-1}\pi(\nu-n)i + 2^{-2}c^{-2}]\,D_{\nu-n}(c)$
(4)	$\int_{0}^{\infty} x^{-1}(x^2+\alpha^2)^{-1} \exp(-2^{-1}x^2)\,\mathrm{He}_{2n+1}(x)\,dx = (-1)^n (\pi/2)^{1/2} \alpha^{-2} [2^n n! - (2n+1)!\exp(2^{-2}\alpha^2)\,D_{-2n-2}(\alpha)]$
(5)	$\int_{0}^{\infty} \exp(-2^{-1}x^2) \sin(\beta x)\,\mathrm{He}_{2n+1}(x)\,dx = (-1)^n (\pi/2)^{1/2} \beta^{2n+1} \exp(-2^{-1}\beta^2)$
(6)	$\int_{0}^{\infty} \exp(-2^{-1}x^2) \cos(\beta x)\,\mathrm{He}_{2n}(x)\,dx = (-1)^n (\pi/2)^{1/2} \beta^{2n} \exp(-2^{-1}\beta^2)$

(7)	$\int_0^\infty \exp(-2^{-1}x^2)\,\mathrm{sh}(\beta x)\,\mathrm{He}_{2n+1}(x)\,dx = (\pi/2)^{1/2}\beta^{2n+1}\exp(2^{-1}\beta^2)$
(8)	$\int_0^\infty \exp(-2^{-1}x^2)\,\mathrm{ch}(\beta x)\,\mathrm{He}_{2n}(x)\,dx = (\pi/2)^{1/2}\beta^{2n}\exp(2^{-1}\beta^2)$
(9)	$\int_{-\infty}^\infty \exp(-2^{-1}x^2)\,[\mathrm{He}_n(x)]^2\,dx = (2\pi)^{1/2}\,n!$
(10)	$\int_{-\infty}^\infty \exp(-x^2)\,\mathrm{He}_m(x)\,\mathrm{He}_n(x)\,dx = (-1)^{m/2-n/2}\,\Gamma\left(\frac{m+n+1}{2}\right)$, $m+n$ четно
(11)	$\int_{-\infty}^\infty \exp(-2^{-1}x^2)\,\mathrm{He}_m(x)\,\mathrm{He}_n(x)\,dx = 0$, $m \neq n$
(12)	$\int_{-\infty}^\infty \exp(-\alpha^2x^2)\,\mathrm{He}_m(x)\,\mathrm{He}_n(x)\,dx = \alpha^{-m-n-1}(1-2\alpha^2)^{m/2+n/2}\,\Gamma\left(\frac{m+n+1}{2}\right)\times {}_2F_1\left(-m,\,-n;\,\frac{1-m-n}{2};\,\frac{\alpha^2}{2\alpha^2-1}\right)$, $\operatorname{Re}\alpha^2>0$, $m+n$ четно
(13)	$\int_{-\infty}^\infty \exp[-2^{-1}(x-y)^2]\,\mathrm{He}_m(x)\,\mathrm{He}_n(x)\,dx = (2\pi)^{1/2}\,m!\,y^{n-m}\,L_n^{n-m}(-y^2)$, $m \leqslant n$
(14)	$\int_{-\infty}^\infty \exp(-x^2)\,\mathrm{He}_k(x)\,\mathrm{He}_m(x)\,\mathrm{He}_n(x)\,dx = \pi^{-1}\,\Gamma(s-k)\,\Gamma(s-m)\,\Gamma(s-n)$, $k+m+n$ четно, $2s=k+m+n+1$
(15)	$\int_{-\infty}^\infty \exp(-2^{-1}x^2)\,\mathrm{He}_k(x)\,\mathrm{He}_m(x)\,\mathrm{He}_n(x)\,dx = \frac{(2\pi)^{1/2}\,k!\,m!\,n!}{(s-k)!\,(s-m)!\,(s-n)!}$, $k+m+n=2s$ четно

(16)	$\int\limits_{-\infty}^{\infty} \exp(-\alpha^2 x^2)\,\mathrm{He}_m(x)\,\mathrm{He}_n(x)\,\mathrm{He}_k(x)\ldots dx.$ См. Busbridge I. W., 1948: J. London Math. Soc. 23, 135—141.
(17)	$\int\limits_{-\infty}^{\infty} \exp[-2^{-1}(x-y)^2]\,\mathrm{He}_n(\alpha x)\,dx = (2\pi)^{1/2}(1-\alpha^2)^{n/2}\,\mathrm{He}_n\left[\frac{\alpha y}{(1-\alpha^2)^{1/2}}\right]$
(18)	$\int\limits_{0}^{\infty} \exp(-2^{-1}x^2)\sin(\beta x)\,\mathrm{He}_{2n+1}(\alpha x)\,dx =$ $= (-1)^n(\pi/2)^{1/2}(\alpha^2-1)^{n+1/2}\exp(-2^{-1}\beta^2)\,\mathrm{He}_{2n+1}\left[\frac{\alpha\beta}{(\alpha^2-1)^{1/2}}\right]$
(19)	$\int\limits_{0}^{\infty} \exp(-2^{-1}x^2)\cos(\beta x)\,\mathrm{He}_{2n}(\alpha x)\,dx =$ $= (\pi/2)^{1/2}(1-\alpha^2)^n\exp(-2^{-1}\beta^2)\,\mathrm{He}_{2n}\left[\frac{\alpha\beta}{(\alpha^2-1)^{1/2}}\right]$
(20)	$\int\limits_{-\infty}^{\infty} \exp(-2^{-1}x^2)\,H_m(\alpha x)\,H_n(x) = 0,$ $\qquad m<n$
(21)	$\int\limits_{-\infty}^{\infty} \exp(-2^{-1}x^2)\,H_{2m+n}(\alpha x)\,H_n(x)\,dx = \pi^{1/2}\frac{(2m+n)!}{m!\,2^{m-1/2}}(\alpha^2-1)^m\alpha^n$
(22)	$\int\limits_{-\infty}^{\infty} \exp[-2^{-1}(\alpha^2+\beta^2)x^2]\,\mathrm{He}_m(\alpha x)\,\mathrm{He}_n(\beta x)\,dx =$ $= (-1)^{m/2-n/2}2^{m/2+n/2+1/2}\,\Gamma\left(\frac{m+n+1}{2}\right)\alpha^n\beta^m(\alpha^2+\beta^2)^{-m-n-1/2},$ $\mathrm{Re}(\alpha^2+\beta^2)>0$, $m+n$ четно
(23)	$\int\limits_{-\infty}^{\infty} \exp(-2^{-1}x^2)\,\mathrm{He}_m(\alpha x)\,\mathrm{He}_n(\beta x)\,dx =$ $= 2^{1/2-m/2-n/2}\pi^{1/2}\,n!\,(\alpha^2-1)^{-m/2}(\beta^2-1)^{-n/2}(\alpha^2+\beta^2-1)^{m/2+n/2}\,P^n_{m+n}(z),$ $\frac{1}{z^2}=\frac{1}{\alpha^2}+\frac{1}{\beta^2}-\frac{1}{\alpha^2\beta^2}$
(24)	$\int\limits_{-\infty}^{\infty} \exp(-\lambda^2x^2)\,\mathrm{He}_m(\alpha x)\,\mathrm{He}_n(\beta x)\,dx = 0,$ $\qquad m+n$ нечетно

(25)	$\int\limits_{-\infty}^{\infty} x^{\rho} \exp(-x^2) \operatorname{He}_{2n}(\alpha x) \operatorname{He}_{2n}(\beta x)\, dx.$ См. Buchholz Herbert, 1953: Die konfluente hypergeometrische Funktion. Springer-Verlag, Berlin, Göttingen, Heidelberg, Sec. 13.
(26)	$\int\limits_{-\infty}^{\infty} \exp\left[-2^{-1}(x-y)^2\right] \operatorname{He}_m(\alpha x) \operatorname{He}_n(\alpha x)\, dx = (2\pi)^{1/2} \sum\limits_{k=0}^{\min(m,\, n)} k! \binom{m}{k} \binom{n}{k} (1-\alpha^2)^{m/2+n/2-k} \operatorname{He}_{m+n-2k}\left[\frac{\alpha y}{(1-\alpha^2)^{1/2}}\right]$
(27)	$\int\limits_{-\infty}^{\infty} \exp(-\lambda^2 x^2) \operatorname{He}_m(\alpha x) \operatorname{He}_n(\beta x) \operatorname{He}_k(\gamma x)\, dx = 0,\quad m+n+k$ нечетно
(28)	$\int\limits_{-\infty}^{\infty} \exp(-\lambda^2 x^2) \operatorname{He}_k(\alpha x) \operatorname{He}_m(\beta x) \operatorname{He}_n(\gamma x) \ldots dx.$ См. Bailey W. N., 1948: J. London Math. Soc. 23, 291—297; Lord R. D., 1949: J. London Math. Soc. 24, 101—112.
(29)	$\int\limits_{0}^{\infty} x^{\rho-1} \exp(-\lambda^2 x^2) \operatorname{He}_k(\alpha x) \operatorname{He}_m(\beta x) \operatorname{He}_n(\gamma x) \ldots dx.$ См. Appell Paul and M. J. Kampé de Fériet, 1926: Fonctions hypergéométriques et hypersphériques. Polynomes d'Hermite, Gauthier-Villars, стр. 343; Erdélyi Arthur, 1936: Math. Z. 40, 693—702.
(30)	$\int\limits_{-\infty}^{\infty} \exp\left(-2^{-1}x^2\right) \operatorname{He}_m(x+y) \operatorname{He}_n(x+z)\, dx = (2\pi)^{1/2} m!\, z^{n-m} L_m^{n-m}(-yz),\quad m \leqslant n$
(31)	$\int\limits_{0}^{\pi} (\cos x)^n \operatorname{He}_{2n}\left[\alpha\left(\frac{\cos x - 1}{\cos x}\right)^{1/2}\right] dx = \frac{(-1)^n \pi (2n)!}{2^n (n!)^2} [\operatorname{He}_n(\alpha)]^2$

16.6. Многочлены Лагерра

См. также вырожденные гипергеометрические функции.
В этом пункте m и n — неотрицательные целые числа.

(1)	$\int\limits_{0}^{\infty} x^{\beta-1} e^{-x} L_n^{\alpha}(x)\, dx = \frac{\Gamma(\alpha-\beta+n+1)\,\Gamma(\beta)}{n!\,\Gamma(\alpha-\beta+1)},$	$\operatorname{Re}\beta > 0$

(2)	$$\int_0^\infty x^\alpha e^{-x}\left[L_n^\alpha(x)\right]^2 dx = \frac{\Gamma(\alpha+n+1)}{n!},$$	$\operatorname{Re}\alpha>0$
(3)	$$\int_0^\infty x^\alpha e^{-x} L_m^\alpha(x) L_n^\alpha(x)\, dx = 0,$$	$m\neq n.\ \operatorname{Re}\alpha>-1$
(4)	$$\int_0^\infty x^{\alpha+\beta} e^{-x} L_m^\alpha(x) L_n^\beta(x)\, dx = (-1)^{m+n}\binom{\alpha+m}{n}\binom{\beta+n}{m},$$	$\operatorname{Re}(\alpha+\beta)>-1$
(5)	$$\int_0^1 x^\alpha (1-x)^{\beta-\alpha-1} L_n^\alpha(xy)\, dx = \frac{\Gamma(\alpha+n+1)\,\Gamma(\beta-\alpha)}{\Gamma(\beta+n+1)} L_n^\beta(y),$$	$\operatorname{Re}\beta>\operatorname{Re}\alpha>-1$
(6)	$$\int_0^\infty x^\nu e^{-x} L_n^\alpha(\lambda x) L_n^\alpha(\mu x)\, dx.$$ См. Buchholz Herbert, 1953: Die konfluente hypergeometrische Funktion, Springer-Verlag, Berlin, Göttingen, Heidelberg, Sec. 12.	
(7)	$$\int_0^1 x^\alpha (1-x)^\beta L_m^\alpha(xy) L_n^\beta[(1-x)y]\, dx = \frac{(m+n)!\,\Gamma(\alpha+m+1)\,\Gamma(\beta+n+1)}{m!\,n!\,\Gamma(\alpha+\beta+m+n+2)} L_{m+n}^{\alpha+\beta+1}(y),$$	$\operatorname{Re}\alpha>-1,\ \operatorname{Re}\beta>-1$
(8)	$$\int_{-\infty}^\infty x^{m-n}\exp\left[-2^{-1}(x-y)^2\right] L_n^{m-n}(x^2)\, dx = \frac{(2\pi)^{1/2}}{n!} i^{n-m}\,\mathrm{He}_n(iy)\,\mathrm{He}_m(iy)$$	
(9)	$$\int_0^\infty \exp(-2^{-1}x^2)\left[L_n^{-1/4}(2^{-1}x^2)\right]^2\cos(xy)\, dx = (\pi/2)^{1/2}\exp(-2^{-1}y^2)\left[L_n^{-1/4}(2^{-1}y^2)\right]^2$$	
(10)	$$\int_0^\infty x\exp(-2^{-1}x^2)\left[L_n^{1/4}(2^{-1}x^2)\right]^2\sin(xy)\, dx = (\pi/2)^{1/2} y\exp(-2^{-1}y^2)\left[L_n^{1/4}(2^{-1}y^2)\right]^2$$	

(11)	$$\int_0^\infty x \exp(-2^{-1}x^2) L_n^\alpha(2^{-1}x^2) L_n^{1/2-\alpha}(2^{-1}x^2) \sin(xy)\,dx = \\ = (\pi/2)^{1/2}\, y \exp(-2^{-1}y^2) L_n^\alpha(2^{-1}y^2) L_n^{1/2-\alpha}(2^{-1}y^2)$$
(12)	$$\int_0^\infty \exp(-2^{-1}x^2) L_n^\alpha(2^{-1}x^2) L_n^{-1/2-\alpha}(2^{-1}x^2) \cos(xy)\,dx = \\ = (\pi/2)^{1/2} \exp(-2^{-1}y^2) L_n^\alpha(2^{-1}y^2) L_n^{-\alpha-1/2}(2^{-1}y^2)$$
(13)	$$\int_0^\infty \exp(-2^{-1}x^2) L_n(2^{-1}x^2)\, \mathrm{He}_{2n+1}(x/2) \sin(xy)\,dx = \\ = (\pi/2)^{1/2} \exp(-2^{-1}y^2) L_n(2^{-1}y^2)\, \mathrm{He}_{2n+1}(y/2)$$
(14)	$$\int_0^\infty \exp(-2^{-1}x^2) L_n(2^{-1}x^2)\, \mathrm{He}_{2n}(x/2) \cos(xy)\,dx = \\ = (\pi/2)^{1/2} \exp(-2^{-1}y^2) L_n(2^{-1}y^2)\, \mathrm{He}_{2n}(y/2)$$
(15)	$$\int_0^\infty x^{\rho-1} e^{-x} L_{m_1}^{\alpha_1}(\lambda_1 x) \ldots L_{m_n}^{\alpha_n}(\lambda_n x)\,dx.$$ См. Erdélyi Arthur, 1936: Math. Z. 40, 693—702.

ГЛАВА XVII

ГАММА-ФУНКЦИЯ, НЕПОЛНЫЕ ГАММА-ФУНКЦИИ И РОДСТВЕННЫЕ ИМ ФУНКЦИИ

Относительно этих функций см. ВТФ, т. I, главу I и т. II, главу IX. Данные ниже выражения для неполных гамма-функций и родственных функций через вырожденные гипергеометрические функции служат для вычисления интегралов, содержащих эти функции. Поэтому здесь мы дадим лишь небольшую выборку из интегралов, содержащих неполные гамма-функции и их частные случаи.

Функции ошибок и интегралы Френеля.

$$\begin{aligned}
\operatorname{Erf}(x) &= \pi^{-1/2}\gamma(1/2,\ x^2),\\
&= 2\pi^{-1/2}x\ {}_1F_1(1/2;\ 3/2;\ -x^2),\\
&= 2\pi^{-1/2}x\exp(-x^2)\ {}_1F_1(1;\ 3/2;\ x^2),\\
&= 2\pi^{-1/2}x^{-1/2}\exp\left(-2^{-1}x^2\right)M_{-1/4,\,1/4}(x^2),\\
&= 1-\operatorname{Erfc}(x);
\end{aligned}$$

$$\begin{aligned}
\operatorname{Erfc}(x) &= \pi^{-1/2}\Gamma(1/2,\ x^2),\\
&= 2^{1/2}\pi^{-1/2}\exp\left(-2^{-1}x^2\right)D_{-1}\left(2^{1/2}x\right),\\
&= \pi^{-1/2}x^{-1}\exp(-x^2)\ {}_2F_0(1,\ 1/2;\ -x^{-2}),\\
&= \pi^{-1/2}x^{-1/2}\exp\left(-2^{-1}x^2\right)W_{-1/4,\,1/4}(x^2),\\
&= 1-\operatorname{Erf}(x);
\end{aligned}$$

$$\begin{aligned}
C(x)\pm iS(x) &= 2^{-1/2}e^{\pm\pi i/4}\operatorname{Erf}\left(e^{\mp\pi i/4}x^{1/2}\right),\\
&= 2^{1/2}\pi^{-1/2}x^{1/2}\ {}_1F_1(1/2;\ 3/2;\ \pm ix).
\end{aligned}$$

Интегральная экспонента и родственные функции.

$$\begin{aligned}
-\operatorname{Ei}(-x) = E_1(x) &= \Gamma(0,\ x),\\
&= x^{-1}e^{-x}\ {}_2F_0(1,\ 1;\ -x^{-1}),\\
&= x^{-1/2}e^{-x/2}\ W_{-1/2,\,0}(x);
\end{aligned}$$

$$\begin{aligned}
\overline{\operatorname{Ei}}(x) &= 2^{-1}\left[\operatorname{Ei}(x+i0)+\operatorname{Ei}(x-i0)\right],\\
&= x^{-1}e^{x}\ {}_2F_0(1,\ 1;\ x^{-1});
\end{aligned}$$

$$\mathrm{Ci}(x) \pm i\,\mathrm{si}(x) = -\mathrm{ci}(x) \pm i\,\mathrm{si}(x),$$
$$= \mathrm{Ei}(\pm ix) = -\Gamma(0, \mp ix),$$
$$= \mp ix^{-1} e^{\pm ix}\, {}_2F_0(1, 1; \mp ix^{-1}),$$
$$= -x^{-1/2} e^{\pm \pi i/4 \pm xi/2}\, W_{-1/2,\,0}(\mp ix).$$

Неполные гамма-функции.

$$\gamma(\alpha, x) = \alpha^{-1} x^{\alpha}\, {}_1F_1(\alpha; \alpha+1; -x),$$
$$= \alpha^{-1} x^{\alpha/2-1/2} e^{-x/2} M_{\alpha/2-1/2,\,\alpha/2}(x),$$
$$= \Gamma(\alpha) - \Gamma(\alpha, x);$$
$$\Gamma(\alpha, x) = x^{\alpha-1} e^{-x}\, {}_2F_0(1, 1-\alpha; -x^{-1}),$$
$$= x^{\alpha/2-1/2} e^{-x/2} W_{\alpha/2-1/2,\,\alpha/2}(x),$$
$$= \Gamma(\alpha) - \gamma(\alpha, x).$$

17.1. Гамма-функция

(1)	$\int\limits_{-\infty}^{\infty} \Gamma(\alpha+x)\,\Gamma(\beta-x)\,dx = 0,$ $\mathrm{Re}(\alpha+\beta) < 1$ и либо $\mathrm{Im}\,\alpha < 0 < \mathrm{Im}\,\beta$, либо $\mathrm{Im}\,\beta < 0 < \mathrm{Im}\,\alpha$
(2)	$\int\limits_{-\infty}^{\infty} \Gamma(\alpha+x)\,\Gamma(\beta-x)\,dx = i\pi 2^{1-\alpha-\beta}\,\Gamma(\alpha+\beta),$ $\mathrm{Re}(\alpha+\beta) < 1, \quad \mathrm{Im}\,\alpha,\ \mathrm{Im}\,\beta < 0$
(3)	$\int\limits_{-\infty}^{\infty} \Gamma(\alpha+x)\,\Gamma(\beta-x)\,dx = -i\pi 2^{1-\alpha-\beta}\,\Gamma(\alpha+\beta),$ $\mathrm{Re}(\alpha+\beta) < 1, \quad \mathrm{Im}\,\alpha,\ \mathrm{Im}\,\beta > 0$
(4)	$\int\limits_{-\infty}^{\infty} \frac{\Gamma(\alpha+x)}{\Gamma(\beta+x)}\,dx = 0,$ $\quad \mathrm{Im}\,\alpha \neq 0,\ \mathrm{Re}(\alpha-\beta) < -1$
(5)	$\int\limits_{-\infty}^{\infty} \frac{dx}{\Gamma(\alpha+x)\,\Gamma(\beta-x)} = \frac{2^{\alpha+\beta-2}}{\Gamma(\alpha+\beta-1)},$ $\quad \mathrm{Re}(\alpha+\beta) > 1$
(6)	$\int\limits_{-\infty}^{\infty} \frac{\Gamma(\alpha+x)}{\Gamma(\beta+x)} \exp[(2\pi n + \pi - 2\theta)xi]\,dx = 0,$ $\quad \mathrm{Re}(\beta-\alpha) > 0$ $-\pi/2 < \theta < \pi/2$, n — целое, $(n+1/2)\,\mathrm{Im}\,\alpha > 0$

(7)	$\int\limits_{-\infty}^{\infty} \Gamma(\alpha+x)\,\Gamma(\beta-x)\exp[2(\pi n+\theta)xi]\,dx =$ $= 2\pi i\,\Gamma(\alpha+\beta)(2\cos\theta)^{-\alpha-\beta}\exp[(\beta-\alpha)\theta i]\times$ $\times[\eta_n(\beta)\exp(2n\pi\beta i)-\eta_n(-\alpha)\exp(-2n\pi\alpha i)]$, $\operatorname{Re}(\alpha+\beta)<1,\ -\pi/2<\theta<\pi/2,\ n$ — целое, $\eta_n(\zeta)=\begin{cases}0, & \text{если } (^1/_2-n)\operatorname{Im}\zeta>0\\ \operatorname{sgn}(^1/_2-n), & \text{если } (^1/_2-n)\operatorname{Im}\zeta<0\end{cases}$
(8)	$\int\limits_{-\infty}^{\infty}\frac{\Gamma(\alpha+x)}{\Gamma(\beta+x)}\exp[(2\pi n+\pi-2\theta)xi]\,dx =$ $= 2\pi i\operatorname{sgn}(n+{}^1/_2)\frac{(2\cos\theta)^{\beta-\alpha-1}}{\Gamma(\beta-\alpha)}\exp[-(2\pi n+\pi-\theta)\alpha i+\theta(\beta-1)i]$, $\operatorname{Re}(\beta-\alpha)>0,\quad -\pi/2<\theta<\pi/2,\ n$ — целое, $(n+{}^1/_2)\operatorname{Im}\alpha<0$
(9)	$\int\limits_{-\infty}^{\infty}\frac{\sin(cx)\,dx}{\Gamma(\alpha+x)\,\Gamma(\beta-x)}=0,\qquad \operatorname{Re}(\alpha+\beta)>1,\ c>\pi$
(10)	$\int\limits_{-\infty}^{\infty}\frac{\sin(cx)\,dx}{\Gamma(\alpha+x)\,\Gamma(\beta-x)}=\frac{[2\cos(c/2)]^{\alpha+\beta-2}}{\Gamma(\alpha+\beta-1)}\sin[2^{-1}c(\beta-\alpha)]$, $\operatorname{Re}(\alpha+\beta)>1,\ 0<c<\pi$
(11)	$\int\limits_{-\infty}^{\infty}\frac{\sin(2n\pi x)}{\sin(\pi x)}\,\frac{dx}{\Gamma(\alpha+x)\,\Gamma(\beta-x)}=0$, $\operatorname{Re}(\alpha+\beta)>1,\ n$ — целое
(12)	$\int\limits_{-\infty}^{\infty}\frac{\sin[(2n+1)\pi x]}{\sin(\pi x)}\,\frac{dx}{\Gamma(\alpha+x)\,\Gamma(\beta-x)}=\frac{2^{\alpha+\beta-2}}{\Gamma(\alpha+\beta-1)}$, $\operatorname{Re}(\alpha+\beta)>1,\ n$ — целое
(13)	$\int\limits_{-\infty}^{\infty}\frac{\cos(cx)\,dx}{\Gamma(\alpha+x)\,\Gamma(\beta-x)}=0,\qquad \operatorname{Re}(\alpha+\beta)>1,\ c>\pi$
(14)	$\int\limits_{-\infty}^{\infty}\frac{\cos(cx)\,dx}{\Gamma(\alpha+x)\,\Gamma(\beta-x)}=\frac{[2\cos(c/2)]^{\alpha+\beta-2}}{\Gamma(\alpha+\beta-1)}\cos[2^{-1}c(\beta-\alpha)]$, $\operatorname{Re}(\alpha+\beta)>1,\ 0\leqslant c<\pi$

(15)	$\int\limits_{-\infty}^{\infty} \frac{P(x)\, e^{icx}\, dx}{\Gamma(\alpha+x)\,\Gamma(\beta-x)}$, $P(x)$ — многочлен. См. Ramanujan Srinivasa, 1920: Quart. J. Math. 48, 294—310.
(16)	$\int\limits_{-\infty}^{\infty} \frac{\Phi(x)\exp[(2\pi n+\theta)\, xi]\, dx}{\Gamma(\alpha+x)\,\Gamma(\beta-x)} =$ $= \frac{[2\cos(\theta/2)]^{\alpha+\beta-2}}{\Gamma(\alpha+\beta-1)} \exp[2^{-1}\theta(\beta-\alpha)\, i] \int\limits_{0}^{1} \Phi(t)\exp(2\pi nti)\, dt,$ $\operatorname{Re}(\alpha+\beta)>1$, $-\pi<\theta<\pi$, n — целое, $\Phi(x+1)=\Phi(x)$
(17)	$\int\limits_{-\infty}^{\infty} \frac{\Phi(x)\, e^{ixc}\, dx}{\Gamma(\alpha+x)\,\Gamma(\beta-x)}$, $\Phi(x)$ — периодическая функция, имеющая вещественный период. См. Ramanujan Srinivasa, 1920: Quart. J. Math. 48, 294—310.
(18)	$\int\limits_{-\infty}^{\infty} \frac{\Gamma(\gamma+x)\,\Gamma(\delta+x)}{\Gamma(\alpha+x)\,\Gamma(\beta+x)}\, dx = 0,$ $\operatorname{Re}(\alpha+\beta-\gamma-\delta)>1$, $\operatorname{Im}\gamma, \operatorname{Im}\delta>0$
(19)	$\int\limits_{-\infty}^{\infty} \frac{\Gamma(\gamma+x)\,\Gamma(\delta+x)}{\Gamma(\alpha+x)\,\Gamma(\beta+x)}\, dx = \frac{\pm 2\pi^2 i\,\Gamma(\alpha+\beta-\gamma-\delta-1)}{\sin[\pi(\gamma-\delta)]\,\Gamma(\alpha-\gamma)\,\Gamma(\alpha-\delta)\,\Gamma(\beta-\gamma)\,\Gamma(\beta-\delta)},$ $\operatorname{Re}(\alpha+\beta-\gamma-\delta)>1$, $\operatorname{Im}\gamma, \operatorname{Im}\delta<0$ $\pm$ в соответствии с $\operatorname{Im}\gamma \gtrless \operatorname{Im}\delta$
(20)	$\int\limits_{-\infty}^{\infty} \frac{\Gamma(\alpha-\beta-\gamma+x+1)\, dx}{\Gamma(\alpha+x)\,\Gamma(\beta-x)\,\Gamma(\gamma+x)} = \frac{\pi\exp[\pm 2^{-1}\pi(\delta-\gamma)\, i]}{\Gamma(\beta+\gamma-1)\,\Gamma\left(\frac{\alpha+\beta}{2}\right)\Gamma\left(\frac{\gamma-\delta+1}{2}\right)},$ $\operatorname{Re}(\beta+\gamma)>1$, $\delta=\alpha-\beta-\gamma+1$, $\operatorname{Im}\delta\neq 0$ $\pm$ в соответствии с $\operatorname{Im}\delta \gtrless 0$
(21)	$\int\limits_{-\infty}^{\infty} \frac{dx}{\Gamma(\alpha+x)\,\Gamma(\beta-x)\,\Gamma(\gamma+x)\,\Gamma(\delta-x)} =$ $= \frac{\Gamma(\alpha+\beta+\gamma+\delta-3)}{\Gamma(\alpha+\beta-1)\,\Gamma(\beta+\gamma-1)\,\Gamma(\gamma+\delta-1)\,\Gamma(\delta+\alpha-1)},$ $\operatorname{Re}(\alpha+\beta+\gamma+\delta)>3$
(22)	$\int\limits_{-\infty}^{\infty} \frac{\sin(\pi x)\, dx}{\Gamma(\alpha+x)\,\Gamma(\beta-x)\,\Gamma(\gamma+x)\,\Gamma(\delta-x)} = \frac{\sin[2^{-1}\pi(\beta-\alpha)]}{2\,\Gamma\left(\frac{\alpha+\beta}{2}\right)\Gamma\left(\frac{\gamma+\delta}{2}\right)\Gamma(\alpha+\delta-1)},$ $\alpha+\delta=\beta+\gamma$, $\operatorname{Re}(\alpha+\beta+\gamma+\delta)>2$

(23)
$$\int_{-\infty}^{\infty}\frac{\cos(\pi x)\,dx}{\Gamma(\alpha+x)\,\Gamma(\beta-x)\,\Gamma(\gamma+x)\,\Gamma(\delta-x)}=\frac{\cos\left[2^{-1}\pi(\beta-\alpha)\right]}{2\,\Gamma\left(\frac{\alpha+\beta}{2}\right)\Gamma\left(\frac{\gamma+\delta}{2}\right)\Gamma(\alpha+\delta-1)},$$
$\alpha+\delta=\beta+\gamma,\ \mathrm{Re}\,(\alpha+\beta+\gamma+\delta)>2$

(24)
$$\int_{-\infty}^{\infty}\frac{\Phi(x)\,dx}{\Gamma(\alpha+x)\,\Gamma(\beta-x)\,\Gamma(\gamma+x)\,\Gamma(\delta-x)}=$$
$$=\frac{\Gamma(\alpha+\beta+\gamma+\delta-3)}{\Gamma(\alpha+\beta-1)\,\Gamma(\beta+\gamma-1)\,\Gamma(\gamma+\delta-1)\,\Gamma(\delta+\alpha-1)}\int_0^1\Phi(t)\,dt,$$
$\mathrm{Re}\,(\alpha+\beta+\gamma+\delta)>3,\ \Phi(x+1)=\Phi(x)$

(25)
$$\int_{-\infty}^{\infty}\frac{\Phi(x)\,dx}{\Gamma(\alpha+x)\,\Gamma(\beta-x)\,\Gamma(\gamma+x)\,\Gamma(\delta-x)}=\frac{\int_0^1\Phi(t)\cos\left[2^{-1}\pi(2t+\alpha-\beta)\right]dt}{\Gamma\left(\frac{\alpha+\beta}{2}\right)\Gamma\left(\frac{\gamma+\delta}{2}\right)\Gamma(\alpha+\delta-1)},$$
$\alpha+\delta=\beta+\gamma,\ \mathrm{Re}\,(\alpha+\beta+\gamma+\delta)>2,\ \Phi(x+1)=-\Phi(x)$

(26)
$$\int_{-\infty}^{\infty}\Gamma(\alpha+x)\,\Gamma(\beta-x)\,\Gamma(\gamma+kx)\,\Gamma(\delta-kx)]^{-1}\exp(\pi cxi)\,dx=0,$$
$\mathrm{Re}\,(\alpha+\beta+\gamma+\delta)>2,$ $c,\ k$ вещественны, $|c|>|k|+1$

Относительно других интегралов такого вида см. Ramanujan Srinivasa, 1920: Quart. J. Math. 48, 294—310.

(27)
$$\int_0^{\infty}|\Gamma(a+ix)\,\Gamma(b+ix)|^2\,dx=\frac{\pi^{1/2}\,\Gamma(a)\,\Gamma(a+1/2)\,\Gamma(b)\,\Gamma(b+1/2)\,\Gamma(a+b)}{2\,\Gamma(a+b+1/2)},$$
$a>0,\ b>0$

(28)
$$\int_0^{\infty}\left|\frac{\Gamma(a+ix)}{\Gamma(b+ix)}\right|^2dx=\frac{\pi^{1/2}\,\Gamma(a)\,\Gamma(a+1/2)\,\Gamma(b-a-1/2)}{2\,\Gamma(b)\,\Gamma(b-1/2)\,\Gamma(b-a)},$$
$0<a<b-1/2$

(29)
$$(2\pi i)^{-1}\int_{-i\infty}^{i\infty}\Gamma(s-\varkappa-\lambda)\,\Gamma(\lambda+\mu-s+1/2)\,\Gamma(\lambda-\mu-s+1/2)\,z^s\,ds=$$
$$=\Gamma(1/2-\varkappa-\mu)\,\Gamma(1/2-\varkappa+\mu)\,z^{\lambda}e^{z/2}\,W_{\varkappa,\mu}(z),$$
$\mathrm{Re}\,(\varkappa+\lambda)<0,\ \mathrm{Re}\,\lambda>|\mathrm{Re}\,\mu|-1/2,\ |\arg z|<3\pi/2$

(30)	$\dfrac{1}{2\pi i}\displaystyle\int_{-i\infty}^{i\infty} \dfrac{\Gamma(\lambda+\mu-s+{}^1/_2)\,\Gamma(\lambda-\mu-s+{}^1/_2)}{\Gamma(\lambda-\varkappa-s+1)}\, z^s\, ds = z^{\lambda} e^{-z/2}\, W_{\varkappa,\,\mu}(z)$ $\operatorname{Re}\lambda > \lvert \operatorname{Re}\mu \rvert - {}^1/_2,\ \lvert \arg z \rvert < \pi/2$
(31)	$\dfrac{1}{2\pi i}\displaystyle\int_{-i\infty}^{i\infty} \dfrac{\Gamma(\varkappa-\lambda+s)\,\Gamma(\lambda+\mu-s+{}^1/_2)}{\Gamma(\mu-\lambda+s+{}^1/_2)}\, z^s\, ds = \dfrac{\Gamma(\varkappa+\mu+{}^1/_2)}{\Gamma(2\mu+1)}\, z^{\lambda} e^{-z/2} M_{\varkappa,\,\mu}(z),$ $\operatorname{Re}(\varkappa-\lambda) > 0,\ \operatorname{Re}(\lambda+\mu) > -{}^1/_2,\ \lvert \arg z \rvert < \pi/2$
(32)	$\dfrac{1}{2\pi i}\displaystyle\int_{-i\infty}^{i\infty} \Gamma(\alpha+s)\,\Gamma(\beta+s)\,\Gamma(\gamma-s)\,\Gamma(\delta-s)\, ds =$ $= \dfrac{\Gamma(\alpha+\gamma)\,\Gamma(\alpha+\delta)\,\Gamma(\beta+\gamma)\,\Gamma(\beta+\delta)}{\Gamma(\alpha+\beta+\gamma+\delta)},$ $\operatorname{Re}\alpha,\ \operatorname{Re}\beta,\ \operatorname{Re}\gamma,\ \operatorname{Re}\delta > 0$
(33)	$\dfrac{1}{2\pi i}\displaystyle\int_{-i\infty}^{i\infty} \left[\dfrac{\Gamma({}^1/_2-s)}{\Gamma(s)}\right]^2 z^s\, ds = z^{1/2}\left[2\pi^{-1} K_0(4z^{1/4}) - Y_0(4z^{1/4})\right],$ $z > 0$
(34)	$\dfrac{1}{2\pi i}\displaystyle\int_{-i\infty}^{i\infty} \dfrac{\prod\limits_{j=1}^{m}\Gamma(b_j-s)\prod\limits_{j=1}^{n}\Gamma(1-a_j+s)}{\prod\limits_{j=m+1}^{q}\Gamma(1-b_j+s)\prod\limits_{j=n+1}^{p}\Gamma(a_j-s)}\, z^s\, ds = G_{pq}^{mn}\left(z\,\middle\vert\, \begin{matrix} a_1, \ldots, a_p \\ b_1, \ldots, b_q \end{matrix}\right),$ $p+q < 2(m+n),\ \lvert \arg z \rvert < (m+n-p/2-q/2)\,\pi$ $\operatorname{Re} a_j < 1,\ j=1, \ldots, n,\ \operatorname{Re} b_j > 0,\ j=1, \ldots, m$
(35)	$\dfrac{1}{2\pi i}\displaystyle\int_{-i\infty}^{i\infty} \dfrac{\prod\limits_{j=1}^{m}\Gamma(b_j-s)\prod\limits_{j=1}^{n}\Gamma(1-a_j+s)}{\prod\limits_{j=m+1}^{q}\Gamma(1-b_j+s)\prod\limits_{j=n+1}^{p}\Gamma(a_j-s)}\, z^s\, ds = G_{pq}^{mn}\left(z\,\middle\vert\, \begin{matrix} a_1, \ldots, a_p \\ b_1, \ldots, b_q \end{matrix}\right),$ $p+q \leqslant 2(m+n),\ \lvert \arg z \rvert \leqslant (m+n-p/2-q/2)\,\pi$ $\operatorname{Re} a_j < 1,\ j=1, \ldots, n,\ \operatorname{Re} b_j > 0,\ j=1, \ldots, m$ $\operatorname{Re}\left(\sum\limits_{j=1}^{p} a_j - \sum\limits_{j=1}^{q} b_j\right) > p/2 - q/2 + 1$
	Относительно других интегралов этого типа см. п. 7.3.

Пустое произведение считается равным единице.

(36)	$\int\limits_0^1 \sin(2\pi nx) \ln[\Gamma(a+x)]\,dx =$ $= -(2n\pi)^{-1}[\ln a + \cos(2n\pi a)\operatorname{ci}(2n\pi a) - \sin(2n\pi a)\operatorname{si}(2n\pi a)],$ $a>0,\ n=1,\ 2,\ 3,\ \ldots$
(37)	$\int\limits_0^1 \cos(2\pi nx) \ln[\Gamma(a+x)]\,dx =$ $= -(2n\pi)^{-1}[\sin(2n\pi a)\operatorname{ci}(2n\pi a) + \cos(2n\pi a)\operatorname{si}(2n\pi a)],$ $a>0,\ n=1,\ 2,\ 3,\ \ldots$
(38)	$\int\limits_0^1 \exp(2\pi nxi) \ln[\Gamma(a+x)]\,dx =$ $= (2n\pi i)^{-1}[\ln a - \exp(-2\pi nai)\operatorname{Ei}(2n\pi ia)],\quad a>0,\ n=\pm 1,\ \pm 2,\ \ldots$
(39)	$\int\limits_0^1 \ln[\Gamma(x)]\,dx = 2^{-1}\ln(2\pi)$
(40)	$\int\limits_0^1 \ln[\Gamma(a+x)] = a\ln a - a + 2^{-1}\ln(2\pi),$ $a \geqslant 0$
(41)	$\int\limits_0^n \ln[\Gamma(a+x)]\,dx = \sum\limits_{k=0}^{n-1}(a+k)\ln(a+k) - na +$ $+2^{-1}n\ln(2\pi) - 2^{-1}n(n-1),\quad a\geqslant 0,\ n=1,\ 2,\ 3,\ \ldots$
(42)	$\int\limits_0^1 \sin(2\pi nx)\ln[\Gamma(x)]\,dx = (2\pi n)^{-1}\ln(2\pi\gamma n),$ $n=1,\ 2,\ \ldots$
(43)	$\int\limits_0^1 \sin[(2n+1)\pi x]\ln[\Gamma(x)]\,dx =$ $=\frac{1}{(2n+1)\pi}\left[\ln\left(\frac{\pi}{2}\right) + 2\left(1+\frac{1}{3}+\ldots+\frac{1}{2n-1}\right)+\frac{1}{2n+1}\right],$ $n=0,\ 1,\ 2,\ \ldots$
(44)	$\int\limits_0^1 \cos(2\pi nx)\ln[\Gamma(x)]\,dx = \frac{1}{4n},$ $n=1,\ 2,\ 3,\ \ldots$

17.2. ψ-функция

(1)	$\int\limits_0^1 \psi(a+x)\,dx = \ln a,$	$a>0$
(2)	$\int\limits_0^1 e^{2n\pi xi}\,\psi(a+x)\,dx = e^{-2n\pi ai}\,\mathrm{Ei}\,(2n\pi ai),$	$a>0,\ n=\pm 1,\ \pm 2, \ldots$
(3)	$\int\limits_0^1 \sin(2n\pi x)\,\psi(x)\,dx = -\pi/2,$	$n=1,\ 2, \ldots$
(4)	$\int\limits_0^1 \sin(2n\pi x)\,\psi(a+x)\,dx = \sin(2n\pi a)\,\mathrm{ci}\,(2n\pi a) + \cos(2n\pi a)\,\mathrm{si}\,(2n\pi a),$	$a \geqslant 0,\ n=1,\ 2, \ldots$
(5)	$\int\limits_0^1 \cos(2n\pi x)\,\psi(a+x)\,dx = \sin(2n\pi a)\,\mathrm{si}\,(2n\pi a) - \cos(2n\pi a)\,\mathrm{ci}\,(2n\pi a),$	$a>0,\ n=1,\ 2, \ldots$
(6)	$\int\limits_0^\infty x^{-\alpha}\,[C+\psi(1+x)]\,dx = -\frac{\pi\zeta(\alpha)}{\sin(\pi\alpha)},$	$1<\operatorname{Re}\alpha<2$
(7)	$\int\limits_0^\infty x^{-\alpha}\,[\ln x - \psi(1+x)]\,dx = \frac{\pi\zeta(\alpha)}{\sin(\pi\alpha)},$	$0<\operatorname{Re}\alpha<1$
(8)	$\int\limits_0^\infty x^{-\alpha}\,[\ln(1+x) - \psi(1+x)]\,dx = \frac{\pi}{\sin(\pi\alpha)}\left[\zeta(\alpha) - \frac{1}{\alpha-1}\right],$	$0<\operatorname{Re}\alpha<1$
(9)	$\int\limits_0^\infty x^{-\alpha}\left[(1+x)^{-1} - \psi'(1+x)\right]dx = -\frac{\pi\alpha\left[\zeta(1+\alpha)-\alpha^{-1}\right]}{\sin(\pi\alpha)},$	$-1<\operatorname{Re}\alpha<1$
(10)	$\int\limits_0^\infty x^{-\alpha}\,[x^{-1} - \psi'(1+x)]\,dx = -\frac{\pi\alpha\,\zeta(1+\alpha)}{\sin(\alpha\pi)},$	$-2<\operatorname{Re}\alpha<0$

(11)	$\int\limits_0^\infty x^{-\alpha}\,\psi^{(n)}(1+x)\,dx=(-1)^{n-1}\dfrac{\pi\,\Gamma(\alpha+n)}{\Gamma(\alpha)\sin\pi\alpha}\,\zeta(\alpha+n),$ $n=1,\,2,\,\ldots,\quad \psi^{(n)}(z)=\dfrac{d^n\psi}{dz^n},\quad 0<\operatorname{Re}\alpha<1$
(12)	$\int\limits_0^\infty[\psi(x+1)-\ln x]\cos(2\pi xy)\,dx=2^{-1}[\psi(y+1)-\ln y]$

17.3. Неполные гамма-функции и родственные функции

(1)	$\int\limits_0^\infty x^{\rho-1}\operatorname{Erfc}(\alpha x)\,dx=\dfrac{\Gamma(\rho/2+{}^1/_2)}{\pi^{1/2}\rho\,\alpha^{\rho}},\qquad \operatorname{Re}\alpha>0,\quad\operatorname{Re}\rho>0$
(2)	$\int\limits_0^\infty x^{\nu-1}\exp(\beta^2x^2)\operatorname{Erfc}(\alpha x)\,dx=\dfrac{\Gamma\left(\frac{\nu+1}{2}\right)}{\pi^{1/2}\nu\alpha^{\nu}}\,{}_2F_1\left(\dfrac{\nu}{2},\,\dfrac{\nu+1}{2};\,\dfrac{\nu}{2}+1;\,\dfrac{\beta^2}{\alpha^2}\right),$ $\operatorname{Re}\nu>0,\quad 0,\ \operatorname{Re}\beta^2<\operatorname{Re}\alpha^2$
(3)	$\int\limits_0^\infty x^{\nu-1}\sin(\beta x)\operatorname{Erfc}(\alpha x)\,dx=$ $=\dfrac{\Gamma(1+\nu/2)\,\beta}{\pi^{1/2}(\nu+1)\,\alpha^{\nu+1}}\,{}_2F_2\left(\dfrac{\nu+1}{2},\,\dfrac{\nu}{2}+1;\,\dfrac{3}{2},\,\dfrac{\nu+3}{2};\,-\dfrac{\beta^2}{4\alpha^2}\right),$ $\operatorname{Re}\alpha>0,\quad\operatorname{Re}\nu>-1$
(4)	$\int\limits_0^\infty x^{\nu-1}\cos(\beta x)\operatorname{Erfc}(\alpha x)\,dx=$ $=\dfrac{\Gamma({}^1/_2+\nu/2)}{\pi^{1/2}\nu\alpha^{\nu}}\,{}_2F_2\left(\dfrac{\nu}{2},\,\dfrac{\nu+1}{2};\,\dfrac{1}{2},\,\dfrac{\nu}{2}+1;\,-\dfrac{\beta^2}{4\alpha^2}\right),\quad\operatorname{Re}\nu>0,\quad\operatorname{Re}\alpha>0$
(5)	$\int\limits_0^\infty e^{\beta x}\operatorname{Erfc}(\alpha^{1/2}x^{1/2})\,dx=\dfrac{1}{\beta}\left[\dfrac{\alpha^{1/2}}{(\alpha-\beta)^{1/2}}-1\right],\qquad 0,\ \operatorname{Re}\beta<\operatorname{Re}\alpha$
(6)	$\int\limits_0^\infty\sin(\beta x)\operatorname{Erfc}(\alpha^{1/2}x^{1/2})\,dx=\dfrac{1}{\beta}-\left(\dfrac{\alpha/2}{\alpha^2+\beta^2}\right)^{1/2}\left[(\alpha^2+\beta^2)^{1/2}-\alpha\right]^{-1/2},$ $\operatorname{Re}\alpha>\lvert\operatorname{Im}\beta\rvert$

(7)	$\int_0^\infty \cos(\beta x)\operatorname{Erfc}(\alpha^{1/2}x^{1/2})\,dx = \left(\frac{\alpha/2}{\alpha^2+\beta^2}\right)^{1/2}\left[(\alpha^2+\beta^2)^{1/2}+\alpha\right]^{-1/2},$	$\operatorname{Re}\alpha > \lvert\operatorname{Im}\beta\rvert$
(8)	$\int_0^\infty \sin(bx)\operatorname{Erf}(\alpha^{1/2}x^{-1/2})\,dx = -b^{-1}\exp\left[-(2\alpha b)^{1/2}\right]\cos\left[(2\alpha b)^{1/2}\right]+b^{-1},$	$\operatorname{Re}\alpha>0,\quad b>0$
(9)	$\int_0^\infty \cos(bx)\operatorname{Erf}(\alpha^{1/2}x^{-1/2})\,dx = b^{-1}\exp\left[-(2\alpha b)^{1/2}\right]\sin\left[(2\alpha b)^{1/2}\right],$	$\operatorname{Re}\alpha>0,\quad b>0$
(10)	$\int_0^\infty \operatorname{ch}(2\nu t)\exp\left[(\alpha\operatorname{ch}t)^2\right]\operatorname{Erfc}(\alpha\operatorname{ch}t)\,dt = \frac{\exp(2^{-1}\alpha^2)K_\nu(\alpha^2)}{2\cos(\nu\pi)},$	$\operatorname{Re}\alpha>0,\quad -1/2<\operatorname{Re}\nu<1/2$

Относительно интегралов, содержащих произведения функций ошибок и других вырожденных гипергеометрических функций, см. Bock Philipp, 1939: Compositio Math. 7, 123—134. Отметим, что функция, которую Бок обозначает Erfc, в наших обозначениях равна $2^{-1}\pi^{1/2}\operatorname{Erfc}$.

(11)	$\int_0^a e^x\operatorname{Ei}(-x)\,dx = -\ln(a\gamma)+e^a\operatorname{Ei}(-a)$	
(12)	$\int_0^c e^{-\beta x}\operatorname{Ei}(-\alpha x)\,dx = -\beta^{-1}\left\{e^{-\beta c}\operatorname{Ei}(-\alpha c)+\ln(1+\beta/\alpha)-\operatorname{Ei}\left[-(\alpha+\beta)c\right]\right\}$	
(13)	$\int_0^\infty x^\nu e^x\operatorname{Ei}(-x)\,dx = \frac{\pi\,\Gamma(\nu+1)}{\sin(\pi\nu)},$	$-1<\operatorname{Re}\nu<0$
(14)	$\int_0^\infty x^{\nu-1}e^{-\beta x}\operatorname{Ei}(-\alpha x)\,dx = -\frac{\Gamma(\nu)}{\nu(\alpha+\beta)^\nu}\,{}_2F_1\left(1,\ \nu;\ \nu+1;\ \frac{\beta}{\alpha+\beta}\right),$	$\lvert\arg\alpha\rvert<\pi,\quad \operatorname{Re}(\alpha+\beta)>0,\quad \operatorname{Re}\nu>0$

Относительно других интегралов, содержащих $-\operatorname{Ei}(-x)=E_1(x)$, см. LeCaine J., 1948: National Research Council of Canada, Division of Atomic Energy, Document No. MT-131 (NRC 1553), стр. 45 и след. и Busbridge I. W., 1950: Quart. J. Math. Oxford Ser. (2), 1, 176—184.

(15)	$\int\limits_0^\infty x^{\mu-1} e^{-\beta x} \gamma(\nu, \alpha x)\, dx = \frac{\alpha^\nu \Gamma(\mu+\nu)}{\nu (\alpha+\beta)^{\mu+\nu}} \, {}_2F_1\left(1, \mu+\nu; \nu+1; \frac{\alpha}{\alpha+\beta}\right),$ $\operatorname{Re}(\alpha+\beta) > 0, \quad \operatorname{Re}\beta > 0, \quad \operatorname{Re}(\mu+\nu) > 0$
(16)	$\int\limits_0^\infty x^{\mu-1} e^{-\beta x} \Gamma(\nu, \alpha x)\, dx = \frac{\alpha^\nu \Gamma(\mu+\nu)}{\mu (\alpha+\beta)^{\mu+\nu}} \, {}_2F_1\left(1, \mu+\nu; \mu+1; \frac{\beta}{\alpha+\beta}\right),$ $\operatorname{Re}(\alpha+\beta) > 0, \quad \operatorname{Re}\mu > 0, \quad \operatorname{Re}(\mu+\nu) > 0$
(17)	$\int\limits_0^\infty e^{-\beta x} \gamma(\nu, \alpha x^2)\, dx = 2^{1-\nu}\beta^{-1}\, \Gamma(2\nu) \exp\left(\frac{\beta^2}{8\alpha}\right) D_{-2\nu}\left[\frac{\beta}{(2\alpha)^{1/2}}\right],$ $\operatorname{Re}\alpha > 0, \quad \operatorname{Re}\beta > 0, \quad \nu \neq 0, \quad \operatorname{Re}\nu > -1/2$
(18)	$\int\limits_0^\infty x^{1-2\nu} \exp(\alpha x^2) \sin(bx)\, \Gamma(\nu, \alpha x^2)\, dx =$ $= \pi^{1/2} 2^{-\nu} \alpha^{\nu-1}\, \Gamma\left(\frac{3}{2} - \nu\right) \exp\left(\frac{b^2}{8\alpha}\right) D_{2\nu-2}\left[\frac{b}{(2\alpha)^{1/2}}\right],$ $\lvert \arg\alpha \rvert < 3\pi/2, \quad 0 < \operatorname{Re}\nu < 1$

Относительно других интегралов, содержащих

$$E_n(x) = x^{n-1}\, \Gamma(1-n, x),$$

см. LeCaine J., 1948: National Research Council of Canada, Division of Atomic Energy, Document No. MT-131 (NRC 1553), стр. 45 и след., а также Busbridge I. W., 1950: Quart. J. Math. Oxford Ser. (2), 1, 176—184.

(19)	$\int\limits_0^\infty e^{-\beta x} \gamma(\nu, \alpha x^{1/2})\, dx = 2^{-\nu/2} \alpha^\nu \beta^{-\nu/2-1}\, \Gamma(\nu) \exp\left(\frac{\alpha^2}{8\beta}\right) D_{-\nu}\left[\frac{\alpha}{(2\beta)^{1/2}}\right],$ $\operatorname{Re}\beta > 0, \quad \operatorname{Re}\nu > 0$

ГЛАВА XVIII

ФУНКЦИИ ЛЕЖАНДРА

Относительно теории этих функций см. ВТФ, т. I, главу III и указанную там литературу, в особенности книги Гобсона, Уиттекера и Ватсона, Мак-Роберта (MacRobert). Многочисленные разложения функций Лежандра в гипергеометрические ряды перечислены в ВТФ, т. I, стр. 128—139. Эти разложения можно использовать для преобразования интегралов, содержащих функции Лежандра, в интегралы, содержащие гипергеометрические функции.

18.1. Функции Лежандра переменного $\alpha x+\beta$: конечные промежутки интегрирования

(1)	$$\int_0^1 x^{\lambda-1} P_\nu(x)\,dx=\frac{\pi^{1/2}2^{-\lambda}\,\Gamma(\lambda)}{\Gamma(1/2+\lambda/2-\nu/2)\,\Gamma(1+\lambda/2+\nu/2)},\qquad \operatorname{Re}\lambda>0$$
(2)	$$\int_0^1 x^{\lambda-1} P_\nu^m(x)\,dx=\frac{(-1)^m\pi^{1/2}2^{-2m-1}\,\Gamma(\lambda/2)\,\Gamma(1+m+\nu)}{\Gamma(1/2+m/2)\,\Gamma(1+\lambda/2+m)\,\Gamma(1-m+\nu)}\times$$ $$\times\,{}_3F_2\left(\frac{m+\nu+1}{2},\ \frac{m-\nu}{2},\ \frac{m}{2}+1;\ m+1,\ \frac{\lambda+m}{2}+1;\ 1\right),$$ $$\operatorname{Re}\lambda>0,\ m=0,\ 1,\ 2,\ \ldots$$
(3)	$$\int_0^1 x^{\lambda-1} P_\nu^\mu(x)\,dx=\frac{\pi^{1/2}2^{2\mu-1}\,\Gamma(\lambda/2)}{\Gamma(1/2-\mu/2)\,\Gamma(1+\lambda/2-\mu/2)}\times$$ $$\times\,{}_3F_2\left(\frac{\nu-\mu+1}{2},\ -\frac{\mu+\nu}{2},\ 1-\frac{\mu}{2},\ 1-\mu,\ \frac{\lambda-\mu}{2}+1;\ 1\right),$$ $$\operatorname{Re}\lambda>0,\ \operatorname{Re}\mu<2$$
(4)	$$\int_0^1 x^{\lambda-1}(1-x^2)^{m/2} P_\nu^m(x)\,dx=$$ $$=\frac{(-1)^m\pi^{1/2}2^{-\lambda-m}\,\Gamma(\lambda)\,\Gamma(1+m+\nu)}{\Gamma(1/2+\lambda/2+m/2-\nu/2)\,\Gamma(1+\lambda/2+m/2+\nu/2)\,\Gamma(1-m+\nu)},$$ $$\operatorname{Re}\lambda>0,\ m=0,\ 1,\ 2,\ \ldots$$

(5)	$\int\limits_0^1 x^{\lambda-1}(1-x^2)^{-\mu/2}\,\mathrm{P}_\nu^\mu(x)\,dx=\frac{\pi^{1/2}2^{\mu-\lambda}\,\Gamma(\lambda)}{\Gamma(1/2+\lambda/2-\mu/2-\nu/2)\,\Gamma(1+\lambda/2-\mu/2+\nu/2)}$, $\operatorname{Re}\lambda>0,\ \operatorname{Re}\mu<1$
(6)	$\int\limits_0^1 x^{\lambda-1}(1-x^2)^{\varkappa}\,\mathrm{P}_\nu^\mu(x)\,dx=\frac{2^{\mu-1}\,\Gamma(1+\varkappa-\mu/2)\,\Gamma(\lambda/2)}{\Gamma(1-\mu)\,\Gamma(1+\varkappa+\lambda/2-\mu/2)}\times$ $\times\,{}_3F_2\left(\frac{\nu-\mu+1}{2},\ -\frac{\mu+\nu}{2},\ 1+\varkappa-\frac{\mu}{2};\ 1-\mu,\ 1+\frac{\lambda-\mu}{2}+\varkappa;\ 1\right)$, $\operatorname{Re}(\varkappa-\mu/2)>-1,\ \operatorname{Re}\lambda>0$
(7)	$\int\limits_0^1 x^{\lambda-1}(1-x^2)^{-\mu/2}\sin(\alpha x)\,\mathrm{P}_\nu^\mu(x)\,dx=$ $=\frac{\pi^{1/2}2^{\mu-\lambda-1}\,\Gamma(\lambda+1)\,\alpha}{\Gamma[1+2^{-1}(\lambda-\mu-\nu)]\,\Gamma[2^{-1}(3+\lambda-\mu+\nu)]}\times$ $\times\,{}_2F_3\left(\frac{1+\lambda}{2},\ 1+\frac{\lambda}{2};\ \frac{3}{2},\ 1+\frac{\lambda-\mu-\nu}{2},\ \frac{3+\lambda-\mu+\nu}{2};\ -\frac{\alpha^2}{4}\right)$, $\operatorname{Re}\lambda>-1,\ \operatorname{Re}\mu<1$
(8)	$\int\limits_0^1 x^{\lambda-1}(1-x^2)^{-\mu/2}\cos(\alpha x)\,\mathrm{P}_\nu^\mu(x)\,dx=$ $=\frac{\pi^{1/2}\,2^{\mu-\lambda}\,\Gamma(\lambda)}{\Gamma[1+2^{-1}(\lambda-\mu+\nu)]\,\Gamma[2^{-1}(1+\lambda-\mu-\nu)]}\times$ $\times\,{}_2F_3\left(\frac{\lambda}{2},\ \frac{\lambda+1}{2};\ \frac{1}{2},\ \frac{1+\lambda-\mu-\nu}{2},\ 1+\frac{\lambda-\mu+\nu}{2};\ -\frac{\alpha^2}{4}\right)$, $\operatorname{Re}\lambda>0,\ \operatorname{Re}\mu<1$
(9)	$\int\limits_0^1 (1-x^2)^{-1}\left[\mathrm{P}_\nu^{n-\nu}(x)\right]^2 dx=-\frac{n!}{2(n-\nu)\,\Gamma(1-n+2\nu)}$, $n=0,\ 1,\ 2,\ \ldots,\ \operatorname{Re}\nu>n$
(10)	$\int\limits_0^1 \mathrm{P}_\lambda(x)\,\mathrm{P}_\nu(x)\,dx=2\,[\pi(\lambda-\nu)(\lambda+\nu+1)]^{-1}\times$ $\times\left[A\sin(2^{-1}\lambda\pi)\cos(2^{-1}\nu\pi)-A^{-1}\cos(2^{-1}\lambda\pi)\sin(2^{-1}\nu\pi)\right]$, $A=\frac{\Gamma(1/2+\nu/2)\,\Gamma(1+\lambda/2)}{\Gamma(1/2+\lambda/2)\,\Gamma(1+\nu/2)}$
(11)	$\int\limits_0^1 \mathrm{P}_\nu(x)\,\mathrm{Q}_\lambda(x)\,dx=[(\lambda-\nu)(\lambda+\nu+1)]^{-1}\left\{A^{-1}\cos[2^{-1}(\nu-\lambda)\pi]-1\right\}$, $A=\frac{\Gamma(1+\lambda/2)\,\Gamma(1/2+\nu/2)}{\Gamma(1/2+\lambda/2)\,\Gamma(1+\nu/2)}$

(12)	$\int\limits_0^1 Q_\lambda(x)\,Q_\nu(x)\,dx = [(\lambda-\nu)(\lambda+\nu+1)]^{-1}\{\psi(\nu+1)-\psi(\lambda+1)-$ $-2^{-1}\pi(A-A^{-1})\sin[2^{-1}(\lambda+\nu)\pi]+2^{-1}\pi(A+A^{-1})\sin[2^{-1}(\lambda-\nu)\pi]\},$ $A=\dfrac{\Gamma(1+\lambda/2)\,\Gamma(^1/_2+\nu/2)}{\Gamma(^1/_2+\lambda/2)\,\Gamma(1+\nu/2)}$
(13)	$\int\limits_0^1 [P_\nu^\mu(x)]^2\,dx,\quad \int\limits_0^1 P_\nu^\mu(x)\,Q_\nu^\mu(x)\,dx.$ См. Barnes E. W., 1908: Quart. J. Math. 39, 97—204. Отметим, что применяемое Бернсом определение функций Лежандра второго рода отличается от используемого в этой книге.
(14)	$\int\limits_0^1 P_\nu^m(x)\,P_\lambda^m(x)\,dx.$ См. Shabde N. G., 1937: Bull. Calcutta Math. Soc. 29, 33—40.
(15)	$\int\limits_{-1}^1 (1+x)^{\lambda-1}P_\nu(x)\,dx=\dfrac{2^\lambda[\Gamma(\lambda)]^2}{\Gamma(\lambda+\nu+1)\,\Gamma(\lambda-\nu)},\quad \operatorname{Re}\lambda>0$
(16)	$\int\limits_{-1}^1 (1-x^2)^{\lambda-1}P_\nu^\mu(x)\,dx=$ $=\dfrac{\pi 2^\mu\,\Gamma(\lambda+\mu/2)\,\Gamma(\lambda-\mu/2)}{\Gamma(\lambda+\nu/2+^1/_2)\,\Gamma(\lambda-\nu/2)\,\Gamma(-\mu/2+\nu/2+1)\,\Gamma(-\mu/2-\nu/2+^1/_2)},$ $2\operatorname{Re}\lambda>\lvert\operatorname{Re}\mu\rvert$
(17)	$\int\limits_{-1}^1 (1-x^2)^{-\mu/2}(z-x)^{-1}P_{\mu+n}^\mu(x)\,dx=2\,e^{-i\mu\pi}(z^2-1)^{-\mu/2}Q_{\mu+n}^\mu(z),$ $n=0,\ 1,\ 2,\ \ldots,\ \operatorname{Re}\mu+n>-1,$ z пробегает разрезанную комплексную плоскость
(18)	$\int\limits_{-1}^1 (1-x)^{-\mu/2}(1+x)^{\mu/2-1/2}(z+x)^{\mu-1/2}P_\nu^\mu(x)\,dx=$ $=\dfrac{2\,e^{-2\mu\pi i}\,\Gamma(^1/_2+\mu)}{\pi^{1/2}\,\Gamma(\mu-\nu)\,\Gamma(\mu+\nu+1)}(z-1)^\mu Q_\nu^\mu\left[\left(\dfrac{1+z}{2}\right)^{1/2}\right]Q_{-\nu-1}^\mu\left[\left(\dfrac{1+z}{2}\right)^{1/2}\right],$ $-^1/_2<\operatorname{Re}\mu<1,$ z пробегает разрезанную комплексную плоскость

Комплексная z-плоскость разрезана вдоль отрезка $[-1,1]$ вещественной оси.

(19)	$\int_{-1}^{1}(1-x)^{-\mu/2-1}(1+x)^{\mu/2-1/2}(z+x)^{\mu-1/2}\times$ $\times\left[(\nu-\mu)P_\nu^\mu(x)-(\nu+\mu)P_{\nu-1}^\mu(x)\right]dx=\frac{e^{-2\mu\pi i}\Gamma(\mu+1/2)(z-1)^{\mu}}{(\pi/2)^{1/2}\Gamma(\mu+\nu)\Gamma(\mu-\nu)(z+1)^{1/2}}\times$ $\times\left\{Q_\nu^\mu\left[\left(\frac{1+z}{2}\right)^{1/2}\right]Q_{-\nu}^\mu\left[\left(\frac{1+z}{2}\right)^{1/2}\right]+Q_{\nu-1}^\mu\left[\left(\frac{1+z}{2}\right)^{1/2}\right]Q_{-\nu-1}^\mu\left[\left(\frac{1+z}{2}\right)^{1/2}\right]\right\},$ $-1/2<\operatorname{Re}\mu<0,$ z пробегает разрезанную комплексную плоскость
(20)	$\int_{-1}^{1}(1-x)^{-\mu/2}(1+x)^{\mu/2-1/2}(z+x)^{\mu-3/2}P_\nu^\mu(x)\,dx=$ $=-\frac{\Gamma(\mu-1/2)(z-1)^{\mu-1/2}(z+1)^{-1/2}}{\pi^{1/2}e^{2\mu\pi i}\Gamma(\mu+\nu)\Gamma(\mu-\nu-1)}\times$ $\times\left\{Q_\nu^\mu\left[\left(\frac{1+z}{2}\right)^{1/2}\right]Q_{-\nu-1}^{\mu-1}\left[\left(\frac{1+z}{2}\right)^{1/2}\right]+Q_\nu^{\mu-1}\left[\left(\frac{1+z}{2}\right)^{1/2}\right]Q_{-\nu-1}^\mu\left[\left(\frac{1+z}{2}\right)^{1/2}\right]\right\},$ $-1/2<\operatorname{Re}\mu<1,$ z пробегает разрезанную комплексную плоскость
(21)	$\int_{-1}^{1}(1-x)^{-\mu/2}(1+x)^{\mu/2+\nu-1}\exp\left(-\frac{1-x}{1+x}y\right)P_\nu^\mu(x)\,dx=$ $=2^\nu y^{\mu/2+\nu-1/2}e^{y/2}W_{\mu/2-\nu-1/2,\,\mu/2}(y),\qquad \operatorname{Re}y>0$
(22)	$\int_{-1}^{1}P_\nu(x)P_\lambda(x)\,dx=\frac{4\sin(\nu\pi)\sin(\lambda\pi)[\psi(\nu+1)-\psi(\lambda+1)]+2\pi\sin[(\lambda-\nu)\pi]}{\pi^2(\lambda-\nu)(\lambda+\nu+1)}$
(23)	$\int_{-1}^{1}[P_\nu(x)]^2\,dx=\frac{\pi^2-2[\sin(\nu\pi)]^2\psi'(\nu+1)}{\pi^2(\nu+1/2)}$
(24)	$\int_{-1}^{1}P_\nu(x)P_\lambda(x)(1+x)^{\lambda+\nu}\,dx=\frac{2^{\lambda+\nu+1}[\Gamma(\lambda+\nu+1)]^4}{[\Gamma(\lambda+1)\Gamma(\nu+1)]^2\Gamma(2\lambda+2\nu+2)},$ $\operatorname{Re}(\lambda+\nu)>-1$
(25)	$\int_{-1}^{1}P_\nu(x)Q_\nu(x)\,dx=-\frac{\sin(2\pi\nu)\psi'(\nu+1)}{\pi(2\nu+1)}$

Комплексная z-плоскость разрезана вдоль отрезка $[-1, 1]$ вещественной оси.

(26)	$\int\limits_{-1}^{1} \mathrm{P}_\nu(x)\,\mathrm{Q}_\lambda(x)\,dx = [(\nu-\lambda)(\nu+\lambda+1)]^{-1}\{1-\cos[(\lambda-\nu)\pi] - 2\pi^{-1}\sin(\nu\pi)\cos(\lambda\pi)[\psi(\nu+1)-\psi(\lambda+1)]\}$
(27)	$\int\limits_{-1}^{1} [\mathrm{Q}_\nu(x)]^2\,dx = \dfrac{2^{-1}\pi^2 - \{1+[\cos(\nu\pi)]^2\}\,\psi'(\nu+1)}{2\nu+1}$
(28)	$\int\limits_{-1}^{1} \mathrm{Q}_\nu(x)\,\mathrm{Q}_\lambda(x)\,dx = [(\lambda-\nu)(\lambda+\nu+1)]^{-1}\{2^{-1}\pi\sin[(\lambda-\nu)\pi] + [\psi(\nu+1)-\psi(\lambda+1)]\,[1+\cos(\lambda\pi)\cos(\nu\pi)]\}$
(29)	$\int\limits_{-1}^{1} (1-x^2)^{m/2-M-1/2}\left(\dfrac{1-x}{1+x}\right)^{\nu/2} \mathrm{P}_\mu^M(x)\,\mathrm{P}_\lambda^M(x)\,dx.$ См. Shabde N. G., 1940: Bull. Calcutta Math. Soc. 32, 121—128.
(30)	$\int\limits_{-1}^{1} \mathrm{P}_\lambda^\rho(x)\,\mathrm{P}_\mu^\sigma(x)\,\mathrm{P}_\nu^\tau(x)\,dx.$ См. Gaunt J. A., 1929: Philos. Trans. Royal Soc. 228, 151—196.
(31)	$\int\limits_{-1}^{1} (1-x^2)^{\lambda-1}(1-a^2x^2)^{\mu/2}\,\mathrm{P}_\nu(ax)\,dx = \dfrac{\pi 2^\mu\,\Gamma(\lambda)}{\Gamma(1/2+\lambda)\,\Gamma(1/2-\mu/2-\nu/2)\,\Gamma(1-\mu/2+\nu/2)} \times {}_2F_1\left(-\dfrac{\mu+\nu}{2},\ \dfrac{1-\mu+\nu}{2};\ \dfrac{1}{2}+\lambda;\ a^2\right),$ $\operatorname{Re}\lambda>0,\ -1<a<1$
(32)	$\int\limits_{0}^{1} x^{-\mu/2-1/2}(1-x)^{-\mu-1/2}(1+\alpha x)^{\mu/2}\,P_\nu^\mu(1+2\alpha x)\,dx = \pi^{1/2}\,\Gamma(1/2-\mu)\,\alpha^{\mu/2}\left\{P_\nu^\mu[(1+\alpha)^{1/2}]\right\}^2,$ $\operatorname{Re}\mu<1/2,\ \lvert\arg\alpha\rvert<\pi$
(33)	$\int\limits_{0}^{1} x^{-\mu/2-1/2}(1-x)^{-\mu-3/2}(1+\alpha x)^{\mu/2}\,P_\nu^\mu(1+2\alpha x)\,dx = \pi^{1/2}\,\Gamma(-\mu-1/2)\,(1+\alpha)^{-1/2}\,\alpha^{\mu/2+1/2}\,P_\nu^{\mu+1}[(1+\alpha)^{1/2}]\,P_\nu^\mu[(1+\alpha)^{1/2}],$ $\operatorname{Re}\mu<-1/2,\ \lvert\arg\alpha\rvert<\pi$

(34)	$\int_0^1 x^{\mu/2-1/2}(1-x)^{\mu-1/2}(1+\alpha x)^{-\mu/2} P_\nu^\mu(1+2\alpha x)\,dx =$ $= \pi^{1/2}\Gamma(1/2+\mu)\,\alpha^{-\mu/2} P_\nu^\mu[(1+\alpha)^{1/2}]\, P_\nu^{-\mu}[(1+\alpha)^{1/2}],$ $\operatorname{Re}\mu > -1/2,\ \lvert\arg\alpha\rvert < \pi$
(35)	$\int_0^1 x^{\mu/2-1/2}(1-x)^{\mu-3/2}(1+\alpha x)^{-\mu/2} P_\nu^\mu(1+2\alpha x)\,dx =$ $= 2^{-1}\pi^{1/2}\Gamma(\mu-1/2)(1+\alpha)^{-1/2}\alpha^{1/2-\mu/2}\{P_\nu^{1-\mu}[(1+\alpha)^{1/2}]\,P_\nu^\mu[(1+\alpha)^{1/2}] +$ $+(\mu+\nu)(1-\mu+\nu)\,P_\nu^{-\mu}[(1+\alpha)^{1/2}]\,P_\nu^\mu[(1+\alpha)^{1/2}]\},$ $\operatorname{Re}\mu > 1/2,\ \lvert\arg\alpha\rvert < \pi$
(36)	$\int_0^1 x^{-\mu/2-1}(1-x)^{-\mu-1/2}(1+\alpha x)^{\mu/2-1/2}\bigl[(1-\nu-\mu)\,P_{\nu-1}^{\mu-1}(1+2\alpha x) +$ $+(1+\nu-\mu)\,P_\nu^{\mu-1}(1+2\alpha x)\bigr]\,dx =$ $= 2\pi^{1/2}\Gamma(1/2-\mu)(1+\alpha)^{-1/2}\alpha^{\mu/2+1/2} P_\nu^\mu[(1+\alpha)^{1/2}]\,P_{\nu-1}^\mu[(1+\alpha)^{1/2}],$ $\operatorname{Re}\mu < 1/2,\ \lvert\arg\alpha\rvert < \pi$
(37)	$\int_0^1 x^{-\mu/2-1}(1-x)^{-\mu-1/2}(1+\alpha x)^{\mu/2-1/2}\bigl[P_{\nu-1}^{1-\mu}(1+2\alpha x) -$ $- P_\nu^{1-\mu}(1+2\alpha x)\bigr]\,dx = \pi^{1/2}\Gamma(1/2-\mu)(1+\alpha)^{-1/2}\alpha^{\mu/2+1/2}\times$ $\times\{(\mu-\nu)\,P_\nu^\mu[(1+\alpha)^{1/2}]\,P_{\nu-1}^{-\mu}[(1+\alpha)^{1/2}] -$ $-(\mu+\nu)\,P_{\nu-1}^\mu[(1+\alpha)^{1/2}]\,P_\nu^{-\mu}[(1+\alpha)^{1/2}]\},\quad \operatorname{Re}\mu < 1/2,\ \lvert\arg\alpha\rvert < \pi$
(38)	$\int_0^1 x^{-\mu/2-1/2}(1-x)^{-\mu-1/2}(1+\alpha x)^{\mu/2} Q_\nu^\mu(1+2\alpha x)\,dx =$ $= \pi^{1/2}\Gamma(1/2-\mu)\,\alpha^{\mu/2} P_\nu^\mu[(1+\alpha)^{1/2}]\,Q_\nu^\mu[(1+\alpha)^{1/2}],\quad \operatorname{Re}\mu < 1/2,\ \lvert\arg\alpha\rvert < \pi$
(39)	$\int_0^1 x^{-\mu/2-1/2}(1-x)^{-\mu-3/2}(1+\alpha x)^{\mu/2} Q_\nu^\mu(1+2\alpha x)\,dx =$ $= 2^{-1}\pi^{1/2}\Gamma(-\mu-1/2)(1+\alpha)^{-1/2}\alpha^{\mu/2+1/2}\times$ $\times\{P_\nu^{\mu+1}[(1+\alpha)^{1/2}]\,Q_\nu^\mu[(1+\alpha)^{1/2}] + P_\nu^\mu[(1+\alpha)^{1/2}]\,Q_\nu^{\mu+1}[(1+\alpha)^{1/2}]\},$ $\operatorname{Re}\mu < -1/2,\ \lvert\arg\alpha\rvert < \pi$

18.2. Функции Лежандра переменного $\alpha x+\beta$: бесконечные промежутки интегрирования

(1)	$\int_0^\infty (x^2-1)^{\mu/2}\sin(ax)\,P_\nu^\mu(x)\,dx = \frac{2^\mu\pi^{1/2}a^{-\mu-1/2}}{\Gamma(1/2-\mu/2-\nu/2)\,\Gamma(1-\mu/2+\nu/2)}\,S_{\mu+1/2,\,\nu+1/2}(a)$, $a>0$, $\operatorname{Re}\mu<3/2$, $\operatorname{Re}(\mu+\nu)<1$
(2)	$\int_1^\infty (x^2-1)^{\lambda-1}P_\nu^\mu(x)\,dx = \frac{2^{\mu-1}\Gamma(\lambda-\mu/2)\,\Gamma(1-\lambda+\nu/2)\,\Gamma(1/2-\lambda-\nu/2)}{\Gamma(1-\mu/2+\nu/2)\,\Gamma(1/2-\mu/2-\nu/2)\,\Gamma(1-\lambda-\mu/2)}$, $\operatorname{Re}\lambda>\operatorname{Re}\mu$, $\operatorname{Re}(1-2\lambda-\nu)>0$, $\operatorname{Re}(2-2\lambda+\nu)>0$
(3)	$\int_1^\infty x^{-\rho}(x^2-1)^{-\mu/2}P_\nu^\mu(x)\,dx = \frac{2^{\rho+\mu-2}\,\Gamma\left(\frac{\rho+\mu+\nu}{2}\right)\Gamma\left(\frac{\rho+\mu-\nu-1}{2}\right)}{\pi^{1/2}\Gamma(\rho)}$, $\operatorname{Re}\mu<1$, $\operatorname{Re}(\rho+\mu+\nu)>0$, $\operatorname{Re}(\rho+\mu-\nu)>1$
(4)	$\int_1^\infty (x-1)^{\lambda-1}(x^2-1)^{\mu/2}P_\nu^\mu(x)\,dx = \frac{2^{\lambda+\mu}\Gamma(\lambda)\,\Gamma(-\lambda-\mu-\nu)\,\Gamma(1-\lambda-\mu+\nu)}{\Gamma(1-\mu+\nu)\,\Gamma(-\mu-\nu)\,\Gamma(1-\lambda-\mu)}$, $\operatorname{Re}\lambda>0$, $\operatorname{Re}(\lambda+\mu+\nu)<0$, $\operatorname{Re}(\lambda+\mu-\nu)<1$
(5)	$\int_1^\infty (x-1)^{\lambda-1}(x^2-1)^{-\mu/2}P_\nu^\mu(x)\,dx = -\frac{2^{\lambda-\mu}\sin(\nu\pi)\,\Gamma(\lambda-\mu)\,\Gamma(-\lambda+\mu-\nu)\,\Gamma(1-\lambda+\mu+\nu)}{\pi\,\Gamma(1-\lambda)}$, $\operatorname{Re}(\lambda-\mu)>0$, $\operatorname{Re}(\mu-\lambda-\nu)>0$, $\operatorname{Re}(\mu-\lambda+\nu)>-1$
(6)	$\int_1^\infty (x-1)^{-\mu/2}(x+1)^{\mu/2-1/2}(z+x)^{\mu-1/2}P_\nu^\mu(x)\,dx = \pi^{1/2}\frac{\Gamma(-\mu-\nu)\,\Gamma(1-\mu+\nu)}{\Gamma(1/2-\mu)}(z-1)^\mu\left\{P_\nu^\mu\left[\left(\frac{1+z}{2}\right)^{1/2}\right]\right\}^2$, $\operatorname{Re}(\mu+\nu)<0$, $\operatorname{Re}(\mu-\nu)<1$, $\|\arg(z+1)\|<\pi$
(7)	$\int_1^\infty (x-1)^{-\mu/2}(x+1)^{\mu/2-1/2}(z+x)^{\mu-3/2}P_\nu^\mu(x)\,dx = \pi^{1/2}\frac{\Gamma(1-\mu-\nu)\,\Gamma(2-\mu+\nu)}{\Gamma(3/2-\mu)}(z-1)^{\mu-1/2}(z+1)^{-1/2}\times P_\nu^\mu\left[\left(\frac{1+z}{2}\right)^{1/2}\right]P_\nu^{\mu-1}\left[\left(\frac{1+z}{2}\right)^{1/2}\right]$, $\operatorname{Re}\mu<1$, $\operatorname{Re}(\mu+\nu)<1$, $\operatorname{Re}(\mu-\nu)<2$, $\|\arg(1+z)\|<\pi$

(8)	$\int\limits_1^\infty (x-1)^{-\mu/2-1}(x+1)^{\mu/2-1/2}(z+x)^{\mu-1/2}\left[(\nu-\mu)P_\nu^\mu(x)-(\nu+\mu)P_{\nu-1}^\mu(x)\right]dx=(2\pi)^{1/2}\frac{\Gamma(1-\mu-\nu)\,\Gamma(1-\mu+\nu)}{\Gamma(1/2-\mu)}(z-1)^\mu(z+1)^{-1/2}\times P_\nu^\mu\left[\left(\frac{1+z}{2}\right)^{1/2}\right]P_{\nu-1}^\mu\left[\left(\frac{1+z}{2}\right)^{1/2}\right],$ $\operatorname{Re}\mu<0,\ \operatorname{Re}\mu<1-\lvert\operatorname{Re}\nu\rvert,\ \lvert\arg(z+1)\rvert<\pi$
(9)	$\int\limits_1^\infty (x-1)^{\lambda-1}(x^2-1)^{\mu/2}(x+z)^{-\rho}P_\nu^\mu(x)\,dx=\frac{2^{\lambda+\mu-\rho}\,\Gamma(\lambda-\rho)\,\Gamma(\rho-\lambda-\mu-\nu)\,\Gamma(\rho-\lambda-\mu+\nu+1)}{\Gamma(1-\mu+\nu)\,\Gamma(-\mu-\nu)\,\Gamma(1+\rho-\lambda-\mu)}\times{}_3F_2(\rho,\rho-\lambda-\mu-\nu,\rho-\lambda-\mu+\nu+1;\rho-\lambda+1,\rho-\lambda-\mu+1;1/2-z/2)+\frac{\Gamma(\rho-\lambda)\,\Gamma(\lambda)}{\Gamma(\rho)\,\Gamma(1-\mu)}2^\mu(z+1)^{\lambda-\rho}\times{}_3F_2(\lambda,-\mu-\nu,1-\mu+\nu;1-\mu,1-\rho+\lambda;1/2+z/2),$ $\operatorname{Re}\lambda>0$, $\operatorname{Re}(\rho-\lambda-\mu-\nu)>0,\ \operatorname{Re}(\rho-\lambda-\mu+\nu+1)>0,\ \lvert\arg(z+1)\rvert<\pi$
(10)	$\int\limits_1^\infty (x-1)^{\lambda-1}(x^2-1)^{-\mu/2}(x+z)^{-\rho}P_\nu^\mu(x)\,dx=\frac{\sin(\nu\pi)\,\Gamma(\lambda-\mu-\rho)\,\Gamma(\rho-\lambda+\mu-\nu)\,\Gamma(\rho-\lambda+\mu+\nu+1)}{2^{\rho-\lambda+\mu}\pi\,\Gamma(1+\rho-\lambda)}\times{}_3F_2(\rho,\rho-\lambda+\mu-\nu,\rho-\lambda+\mu+\nu+1;1+\rho-\lambda,1+\rho-\lambda+\mu;1/2+z/2)+\frac{\Gamma(\lambda-\mu)\,\Gamma(\rho-\lambda+\mu)}{\Gamma(\rho)\,\Gamma(1-\mu)}(z+1)^{\lambda-\rho-\mu}\times{}_3F_2(\lambda-\mu,-\nu,\nu+1;1+\lambda-\mu-\rho,1-\mu;1/2+z/2),$ $\operatorname{Re}(\lambda-\mu)>0$, $\operatorname{Re}(\rho-\lambda+\mu-\nu)>0,\ \operatorname{Re}(\rho-\lambda+\mu+\nu+1)>0,\ \lvert\arg(z+1)\rvert<\pi$
(11)	$\int\limits_1^\infty e^{-\alpha x}(x^2-1)^{-\mu/2}P_\nu^\mu(x)\,dx=2^{1/2}\pi^{-1/2}\alpha^{\mu-1/2}K_{\nu+1/2}(\alpha),$ $\operatorname{Re}\alpha>0,\ \operatorname{Re}\mu<1$
(12)	$\int\limits_1^\infty\left(\frac{x+1}{x-1}\right)^{\mu/2}e^{-\alpha x}P_\nu^\mu(x)\,dx=\alpha^{-1}W_{\mu,\nu+1/2}(2\alpha),$ $\operatorname{Re}\alpha>0,\ \operatorname{Re}\mu<1$
(13)	$\int\limits_1^\infty (x-1)^{\lambda-1}(x^2-1)^{\mu/2}e^{-\alpha x}P_\nu^\mu(x)\,dx=\frac{\alpha^{-\lambda-\mu}e^{-\alpha}}{\Gamma(1-\mu+\nu)\,\Gamma(-\mu-\nu)}G_{23}^{31}\left(2\alpha\,\middle\vert\,\begin{matrix}1+\mu,\ 1\\ \lambda+\mu,\ -\nu,\ 1+\nu\end{matrix}\right).$ $\operatorname{Re}\alpha>0,\ \operatorname{Re}\lambda>0$

(14)	$\int_1^\infty (x-1)^{\lambda-1}(x^2-1)^{-\mu/2}e^{-\alpha x}Q_\nu^\mu(x)\,dx=$ $=2^{-1}e^{\mu\pi i}\alpha^{\mu-\lambda}e^{-\alpha}G_{23}^{22}\left(2\alpha\,\middle\|\,\begin{matrix}1-\mu,\ 1\\ \lambda-\mu,\ \nu+1,\ -\nu\end{matrix}\right),$ $\operatorname{Re}\alpha>0,\ \operatorname{Re}\lambda>0,\ \operatorname{Re}(\lambda-\mu)>0$
(15)	$\int_1^\infty (x-1)^{\lambda-1}(x^2-1)^{-\mu/2}e^{-\alpha x}P_\nu^\mu(x)\,dx=$ $=-\pi^{-1}\sin(\nu\pi)\,\alpha^{\mu-\lambda}e^{-\alpha}G_{23}^{31}\left(2\alpha\,\middle\|\,\begin{matrix}1,\ 1-\mu\\ \lambda-\mu,\ 1+\nu,\ -\nu\end{matrix}\right),$ $\operatorname{Re}\alpha>0,\ \operatorname{Re}\lambda>\operatorname{Re}\mu$
(16)	$\int_1^\infty (\alpha^2+\beta^2+2\alpha\beta x)^{-1/2}\exp\left[-(\alpha^2+\beta^2+2\alpha\beta x)^{1/2}\right]P_\nu(x)\,dx=$ $=2\pi^{-1}(\alpha\beta)^{-1/2}K_{\nu+1/2}(\alpha)\,K_{\nu+1/2}(\beta),\quad \operatorname{Re}\alpha>0,\ \operatorname{Re}\beta>0$
(17)	$\int_1^\infty (x^2-1)^{-\mu/2}\exp(\alpha^2x^2)\operatorname{Erfc}(\alpha x)\,P_\nu^\mu(x)\,dx=$ $=\pi^{-1}2^{\mu-1}\Gamma\left(\frac{1+\mu+\nu}{2}\right)\Gamma\left(\frac{\mu-\nu}{2}\right)\alpha^{\mu-3/2}\exp\left(\frac{\alpha^2}{2}\right)W_{1/4-\mu/2,\ 1/4+\nu/2}(\alpha^2),$ $\operatorname{Re}\alpha>0,\ \operatorname{Re}\mu<1,\ \operatorname{Re}(\mu+\nu)>-1,\ \operatorname{Re}(\mu-\nu)>0$
(18)	$\int_1^\infty Q_\nu(x)\,dx=[\nu(\nu+1)]^{-1},\qquad \operatorname{Re}\nu>0$
(19)	$\int_1^\infty P_\nu(x)\,Q_\lambda(x)\,dx=[(\lambda-\nu)(\lambda+\nu+1)]^{-1},$ $\operatorname{Re}(\lambda-\nu)>0,\ \operatorname{Re}(\lambda+\nu)>-1$
(20)	$\int_1^\infty [Q_\nu(x)]^2\,dx=(2\nu+1)^{-1}\psi'(\nu+1),\qquad \operatorname{Re}\nu>-1/2$
(21)	$\int_1^\infty Q_\nu(x)\,Q_\lambda(x)\,dx=\frac{\psi(\lambda+1)-\psi(\nu+1)}{(\lambda-\nu)(\lambda+\nu+1)},\qquad \operatorname{Re}(\lambda+\nu)>-1$
(22)	$\int_1^\infty \left[Q_\nu^\mu(x)\right]^2dx.$ См. Barnes E. W., 1908: Quart. J. Math. 39, 97—204. Отметим, что применяемое Бернсом определение функций Лежандра второго рода отличается от используемого в этой книге.

(23)	$\int_1^\infty (x^2-1)^{\lambda-1} Q_\nu^\mu(x)\,dx = e^{\mu\pi i}\dfrac{\Gamma(1/2+\nu/2+\mu/2)\,\Gamma(1-\lambda+\nu/2)\,\Gamma(\lambda+\mu/2)\,\Gamma(\lambda-\mu/2)}{2^{2\lambda-\mu}\,\Gamma(1+\nu/2-\mu/2)\,\Gamma(1/2+\lambda+\nu/2)}$, $\|\operatorname{Re}\mu\| < 2\operatorname{Re}\lambda < \operatorname{Re}\nu+2$
(24)	$\int_1^\infty (x-1)^{\lambda-1}(x^2-1)^{\mu/2} e^{-\alpha x} Q_\nu^\mu(x)\,dx = \dfrac{\Gamma(\nu+\mu+1)}{2\,\Gamma(\nu-\mu+1)}\, e^{\mu\pi i}\alpha^{-\lambda-\mu} e^{-\alpha} G_{23}^{22}\left(2\alpha \middle\| \begin{matrix} 1+\mu,\ 1 \\ \lambda+\mu,\ \nu+1,\ -\nu \end{matrix}\right)$, $\operatorname{Re}\alpha>0,\ \operatorname{Re}\lambda>0,\ \operatorname{Re}(\lambda+\mu)>0$
(25)	$\int_1^\infty (x^2-1)^{\lambda-1}(\alpha^2x^2-1)^{\mu/2} P_\nu^\mu(\alpha x)\,dx = \dfrac{\Gamma(\lambda)\,\Gamma(1-\lambda-\mu/2+\nu/2)\,\Gamma(1/2-\lambda-\mu/2-\nu/2)}{\Gamma(1-\mu/2+\nu/2)\,\Gamma(1/2-\nu/2-\mu/2)\,\Gamma(1-\lambda-\mu)} \times 2^{\mu-1}\alpha^{\mu-\nu-1}\,{}_2F_1\left(\dfrac{1-\mu+\nu}{2},\ 1-\lambda-\dfrac{\mu-\nu}{2};\ 1-\lambda-\mu;\ 1-\dfrac{1}{\alpha^2}\right)$, $\operatorname{Re}\alpha>0,\ \operatorname{Re}\lambda>0,\ \operatorname{Re}(\nu-\mu-2\lambda)>-2,\ \operatorname{Re}(2\lambda+\mu+\nu)<1$
(26)	$\int_1^\infty x^{\mu-1} Q_\nu(\alpha x)\,dx = e^{\mu\pi i}\,\Gamma(\mu)\,\alpha^{-\mu}(\alpha^2-1)^{\mu/2} Q_\nu^{-\mu}(\alpha)$, $\|\arg(\alpha-1)\|<\pi,\ \operatorname{Re}\mu>0,\ \operatorname{Re}(\nu-\mu)>-1$
(27)	$\int_1^\infty (x^2-1)^{\lambda-1}(\alpha^2x^2-1)^{-\mu/2} Q_\nu^\mu(\alpha x)\,dx = \dfrac{\Gamma\left(\dfrac{\mu+\nu+1}{2}\right)\Gamma(\lambda)\,\Gamma\left(1-\lambda+\dfrac{\mu+\nu}{2}\right)}{\Gamma(\nu+3/2)}\, 2^{\mu-2} e^{\mu\pi i}\alpha^{-\mu-\nu-1} \times {}_2F_1\left(\dfrac{\mu+\nu+1}{2},\ 1-\lambda+\dfrac{\mu+\nu}{2};\ \nu+\dfrac{3}{2};\ \alpha^{-2}\right)$, $\|\arg(\alpha-1)\|<\pi,\ \operatorname{Re}\lambda>0,\ \operatorname{Re}(2\lambda-\mu-\nu)<2$
(28)	$\int_1^\infty x^{-\mu/2-1/2}(x-1)^{-\mu-1/2}(1+\alpha x)^{\mu/2} Q_\nu^\mu(1+2\alpha x)\,dx = \pi^{-1/2} e^{-\mu\pi i}\,\Gamma(1/2-\mu)\,\alpha^{\mu/2}\left\{Q_\nu^\mu\left[(1+\alpha)^{1/2}\right]\right\}^2$, $\|\arg\alpha\|<\pi,\ \operatorname{Re}\mu<1/2,\ \operatorname{Re}(\mu+\nu)>-1$

(29)	$\int\limits_1^\infty x^{-\mu/2-1/2}(x-1)^{-\mu-3/2}(1+\alpha x)^{\mu/2}Q_\nu^\mu(1+2\alpha x)\,dx=$ $=-\pi^{-1/2}e^{-\mu\pi i}\Gamma(-\mu-1/2)\,\alpha^{\mu/2+1/2}(1+\alpha^2)^{-1/2}\times$ $\times Q_\nu^{\mu+1}[(1+\alpha)^{1/2}]\,Q_\nu^\mu[(1+\alpha)^{1/2}],$ $\lvert\arg\alpha\rvert<\pi,\ \operatorname{Re}\mu<-1/2,\ \operatorname{Re}(\mu+\nu+2)>0$
(30)	$\int\limits_1^\infty (x-1)^{\mu-1}P_\nu(\alpha x)\,Q_\lambda(\alpha x)\,dx,\qquad \int\limits_1^\infty (x-1)^{\mu-1}Q_\nu(\alpha x)\,Q_\lambda(\alpha x)\,dx.$ См. Shabde N. G., 1937: Bull. Calcutta Math. Soc. 29, 33—40.

18.3. Функции Лежандра других переменных

(1)	$\int\limits_a^\infty P_\nu(2x^2a^{-2}-1)\sin(bx)\,dx=$ $=-\dfrac{\pi a}{4\cos(\nu\pi)}\left\{\left[J_{\nu+1/2}\left(\dfrac{ab}{2}\right)\right]^2-\left[J_{-\nu-1/2}\left(\dfrac{ab}{2}\right)\right]^2\right\},$ $a,\ b>0,\ -1<\operatorname{Re}\nu<0$
(2)	$\int\limits_a^\infty P_\nu(2x^2a^{-2}-1)\cos(bx)\,dx=$ $=-2^{-2}\pi a\left[J_{\nu+1/2}(2^{-1}ab)\,J_{-\nu-1/2}(2^{-1}ab)-Y_{\nu+1/2}(2^{-1}ab)\,Y_{-\nu-1/2}(2^{-1}ab)\right],$ $a,\ b>0,\ -1<\operatorname{Re}\nu<0$
(3)	$\int\limits_0^\infty (\alpha+x)^{-\mu-\nu-2}P_\mu\left(\dfrac{\alpha-x}{\alpha+x}\right)P_\nu\left(\dfrac{\alpha-x}{\alpha+x}\right)dx=$ $=\dfrac{\alpha^{-\mu-\nu-1}[\Gamma(\mu+\nu+1)]^4}{[\Gamma(\mu+1)\,\Gamma(\nu+1)]^2\,\Gamma(2\mu+2\nu+2)},$ $\lvert\arg\alpha\rvert<\pi,\ \operatorname{Re}(\mu+\nu)>-1$
(4)	$\int\limits_0^1 x^{-1}\cos(ax)\,P_\nu(2x^{-2}-1)\,dx=$ $=-\dfrac{\pi}{2\sin(\nu\pi)}\,{}_1F_1(\nu+1;\,1;\,ai)\,{}_1F_1(\nu+1;\,1;\,-ai),$ $a>0,\ -1<\operatorname{Re}\nu<0$
(5)	$\int\limits_0^\infty x^{-1}e^{-\beta x}Q_{-1/2}(1+2x^{-2})\,dx=\dfrac{\pi^2}{8}\{[J_0(\beta/2)]^2+[Y_0(\beta/2)]^2\},\qquad \operatorname{Re}\beta>0$

(6)	$\int_0^\infty x^{-1} e^{-\alpha x} Q_\nu (1 + 2x^{-2})\, dx = 2^{-1} [\Gamma(\nu+1)]^2 \alpha^{-1} \times$ $\times W_{-\nu-1/2,\,0}(\alpha i)\, W_{-\nu-1/2,\,0}(-\alpha i),\quad \operatorname{Re}\alpha > 0,\ \operatorname{Re}\nu > -1$
(7)	$\int_0^\infty x^{\lambda-1} (x^2+a^2)^{\nu/2} e^{-\beta x}\, \mathrm{P}_\nu^\mu \left[\frac{x}{(x^2+a^2)^{1/2}}\right] dx =$ $= \frac{2^{-\nu-2} a^{\lambda+\nu}}{\pi\,\Gamma(-\mu-\nu)}\, G_{24}^{32}\left(\frac{a^2\beta^2}{4}\,\middle\vert\, \begin{matrix} 1-\frac{\lambda}{2},\ \frac{1-\lambda}{2} \\ 0,\ \frac{1}{2},\ -\frac{\lambda+\mu+\nu}{2},\ -\frac{\lambda-\mu+\nu}{2} \end{matrix}\right),$ $a > 0,\quad \operatorname{Re}\beta > 0,\quad \operatorname{Re}\lambda > 0$
(8)	$\int_0^\infty x^{1/2} \sin(bx) \left[P_\nu^{-1/4}(y)\right]^2 dx = \frac{(\pi/2)^{-1/2} \alpha^{-1} b^{-1/2}}{\Gamma(5/4+\nu)\,\Gamma(1/4-\nu)} \left[K_{\nu+1/2}\left(\frac{b}{2\alpha}\right)\right]^2,$ $\operatorname{Re}\alpha > 0,\quad b > 0,\quad -5/4 < \operatorname{Re}\nu < 1/4$
(9)	$\int_0^\infty x^{1/2} \sin(bx)\, P_\nu^{-1/4}(y)\, Q_\nu^{-1/4}(y)\, dx =$ $= \frac{(\pi/2)^{1/2} \exp(-\pi i/4)\, \Gamma(\nu+5/4)}{\alpha b^{1/2}\, \Gamma(\nu+3/4)} I_{\nu+1/2}\left(\frac{b}{2\alpha}\right) K_{\nu+1/2}\left(\frac{b}{2\alpha}\right),$ $\operatorname{Re}\alpha > 0,\quad b > 0,\quad \operatorname{Re}\nu > -5/4$
(10)	$\int_0^\infty x^{1/2} y^{-1} \sin(bx)\, P_\nu^{-1/4}(y)\, P_{\nu-1}^{-1/4}(y)\, dx =$ $= \frac{(2\pi)^{-1/2} \alpha^{-2} b^{1/2}}{\Gamma(5/4+\nu)\,\Gamma(5/4-\nu)} K_{\nu-1/2}\left(\frac{b}{2\alpha}\right) K_{\nu+1/2}\left(\frac{b}{2\alpha}\right),$ $\operatorname{Re}\alpha > 0,\quad b > 0,\quad -5/4 < \operatorname{Re}\nu < 5/4$
(11)	$\int_0^\infty x^{1/2} y^{-1} \sin(bx)\, P_\nu^{1/4}(y)\, P_\nu^{-3/4}(y)\, dx = \frac{(2\pi)^{-1/2} \alpha^{-2} b^{1/2}}{\Gamma(7/4+\nu)\,\Gamma(3/4-\nu)} \left[K_{\nu+1/2}\left(\frac{b}{2\alpha}\right)\right]^2,$ $\operatorname{Re}\alpha > 0,\quad b > 0,\quad -7/4 < \operatorname{Re}\nu < 3/4$
(12)	$\int_0^\infty x^{1/2} \cos(bx) \left[P_\nu^{1/4}(y)\right]^2 dx = -\frac{\alpha^{-1} (2^{-1}\pi b)^{-1/2}}{\Gamma(3/4+\nu)\,\Gamma(-1/4-\nu)} \left[K_{\nu+1/2}\left(\frac{b}{2\alpha}\right)\right]^2,$ $\operatorname{Re}\alpha > 0,\quad b > 0,\quad -3/4 < \operatorname{Re}\nu < -1/4$

$y = (1 + \alpha^2 x^2)^{1/2}$

(13)	$\int_0^\infty x^{1/2}\cos(bx)\,P_\nu^{1/4}(y)\,Q_\nu^{1/4}(y)\,dx = \dfrac{(\pi/2)^{1/2}\exp(\pi i/4)\,\Gamma(\nu+3/4)}{ab^{1/2}\,\Gamma(\nu+5/4)}\,I_{\nu+1/2}\left(\dfrac{b}{2a}\right)K_{\nu+1/2}\left(\dfrac{b}{2a}\right)$, $\operatorname{Re} a>0,\quad b>0,\quad \operatorname{Re}\nu>-3/4$
(14)	$\int_0^\infty x^{1/2}y^{-1}\cos(bx)\,P_\nu^{-1/4}(y)\,P_\nu^{3/4}(y)\,dx = \dfrac{(2\pi)^{-1/2}a^{-2}b^{1/2}}{\Gamma(5/4+\nu)\,\Gamma(1/4-\nu)}\left[K_{\nu+1/2}\left(\dfrac{b}{2a}\right)\right]^2$, $\operatorname{Re} a>0,\quad b>0,\quad -5/4<\operatorname{Re}\nu<1/4$
(15)	$\int_0^\infty x^{1/2}y^{-1}\cos(bx)\,P_\nu^{1/4}(y)\,P_{\nu-1}^{1/4}(y)\,dx = \dfrac{(2\pi)^{-1/2}a^{-2}b^{1/2}}{\Gamma(3/4+\nu)\,\Gamma(3/4-\nu)}\,K_{\nu-1/2}\left(\dfrac{b}{2a}\right)K_{\nu+1/2}\left(\dfrac{b}{2a}\right)$, $\operatorname{Re} a>0,\quad b>0,\quad -3/4<\operatorname{Re}\nu<3/4$
(16)	$\int_0^a \left[\dfrac{\sin(a-x)}{\sin x}\right]^{\varkappa} \mathrm{P}_\nu^{-\mu}(\cos x)\,\mathrm{P}_\nu^{-\varkappa}[\cos(a-x)]\,\dfrac{dx}{\sin x} = \dfrac{2^{\varkappa}\,\Gamma(\mu-\varkappa)\,\Gamma(\varkappa+1/2)}{\pi^{1/2}\,\Gamma(\varkappa+\mu+1)}(\sin a)^{\varkappa}\,\mathrm{P}_\nu^{-\mu}(\cos a)$, $\operatorname{Re}\mu>\operatorname{Re}\varkappa>-1/2$
(17)	$\int_0^a (\sin x)^{\rho}\,[\sin(a-x)]^{\sigma}\,\mathrm{P}_\nu^{\mu}(\cos x)\,\mathrm{P}_\varkappa^{\lambda}[\cos(a-x)]\,dx$. Относительно этого интеграла и многих частных случаев см. Bailey W. N., 1931: Proc. Cambridge Philos. Soc. 27, 184—189 и 381—386.
(18)	$\int_0^\infty \cos(ax)\,P_\nu(\operatorname{ch} x)\,dx = -\dfrac{\sin(\nu\pi)}{4\pi^2}\,\Gamma\left(\dfrac{1+\nu+ia}{2}\right)\Gamma\left(\dfrac{1+\nu-ia}{2}\right)\Gamma\left(-\dfrac{\nu+ia}{2}\right)\Gamma\left(-\dfrac{\nu-ia}{2}\right)$, $a>0,\quad -1<\operatorname{Re}\nu<0$
(19)	$\int_0^\infty \mathrm{P}_{-x-1/2}(\cos\theta)\,dx = \dfrac{1}{2\sin(\theta/2)}$, $\quad 0<\theta<\pi$
(20)	$\int_{-\infty}^\infty \mathrm{P}_x(\cos\theta)\,dx = \dfrac{1}{\sin(\theta/2)}$, $\quad 0<\theta<\pi$

$y=(1+a^2x^2)^{1/2}$

(21)	$\int\limits_0^\infty \cos(bx)\, P^{\mu}_{-1/2+ix}(\operatorname{ch} a)\,dx = \begin{cases} 0, \\ \dfrac{(\pi/2)^{1/2}(\operatorname{sh} a)^{\mu}}{\Gamma(1/2-\mu)(\operatorname{ch} a-\operatorname{ch} b)^{\mu+1/2}}, \end{cases}$	$0<a<b$ $0<b<a$
(22)	$\int\limits_0^\infty x^{-1}\operatorname{th}(\pi x)\, P_{-1/2+ix}(\operatorname{ch} a)\,dx = 2\,e^{-a/2}K(e^{-a}),$	$a>0$
(23)	$\int\limits_0^\infty \dfrac{x\operatorname{th}(\pi x)}{\alpha^2+x^2}\, P_{-1/2+ix}(\operatorname{ch} b)\,dx = Q_{\alpha-1/2}(\operatorname{ch} b),$	$\operatorname{Re}\alpha>0$
(24)	$\int\limits_0^\infty \cos(bx)\,\Gamma(\mu+ix)\,\Gamma(\mu-ix)\, P^{1/2-\mu}_{-1/2+ix}(\operatorname{ch} a)\,dx =$ $= \dfrac{(\pi/2)^{1/2}\,\Gamma(\mu)\,(\operatorname{sh} a)^{\mu-1/2}}{(\operatorname{ch} a+\operatorname{ch} b)^{\mu}},$	$a,\ b>0,\ \operatorname{Re}\mu>0$

ГЛАВА XIX

ФУНКЦИИ БЕССЕЛЯ

Относительно функций Бесселя и родственных им функций см. ВТФ т. II, главу VII и указанную там литературу, в особенности книги Ватсона (основной трактат по данному вопросу), Грея и Метьюза, Мак-Лахлана (Mc Lachlan, второе, пересмотренное издание) и Вейрнха (Weyrich). Интегралы, содержащие функции Бесселя, встречаются почти в каждой главе этой книги, и ядра преобразований, перечисленных в главах VIII—XII, являются функциями Бесселя. Данная глава содержит в основном интегралы, которые не попали в предыдущие главы, но некоторые из приведенных выше интегралов включены здесь, чтобы облегчить ссылки.

Функции Бесселя являются частными случаями вырожденных гипергеометрических функций, и выражения, которые указаны ниже, могут быть использованы для того, чтобы привести интегралы, содержащие функции Бесселя, к интегралам, содержащим гипергеометрические функции.

$$J_\nu(z)=\frac{(z/2)^\nu}{\Gamma(\nu+1)}\,{}_0F_1\left(\nu+1;\ -2^{-2}z^2\right),$$

$$=\frac{(z/2)^\nu}{\Gamma(\nu+1)}\,e^{-iz}\,{}_1F_1(\nu+{}^1/_2;\ 2\nu+1;\ 2iz),$$

$$=\frac{z^{-1/2}\exp\left[-2^{-1}(\nu+{}^1/_2)\pi i\right]}{2^{2\nu+1/2}\,\Gamma(\nu+1)}\,M_{0,\nu}(2iz);$$

$$H_\nu^{(1)}(z)=\left(2^{-1}\pi z\right)^{-1/2}\exp\left[-2^{-1}(\nu+{}^1/_2)\pi i\right]W_{0,\nu}(-2iz),$$

$$H_\nu^{(2)}(z)=\left(2^{-1}\pi z\right)^{-1/2}\exp\left[2^{-1}(\nu+{}^1/_2)\pi i\right]W_{0,\nu}(2iz);$$

$$I_\nu(z)=\frac{(z/2)^\nu}{\Gamma(\nu+1)}\,{}_0F_1\left(\nu+1;\ \frac{z^2}{4}\right),$$

$$=\frac{(z/2)^\nu}{\Gamma(\nu+1)}\,e^{-z}\,{}_1F_1(\nu+{}^1/_2;\ 2\nu+1;\ 2z),$$

$$=\frac{z^{-1/2}2^{-2\nu-1/2}}{\Gamma(\nu+1)}\,M_{0,\nu}(2z);$$

$$K_\nu(z)=\left(\frac{\pi}{2z}\right)^{1/2}W_{0,\nu}(2z),$$

$$\mathbf{H}_\nu(z)=\frac{2\,(z/2)^{\nu+1}}{\pi^{1/2}\,\Gamma(\nu+{}^3/_2)}\,{}_1F_2\left(1;\ \nu+{}^3/_2;\ {}^3/_2;\ -\frac{z^2}{4}\right),$$

$$\mathbf{L}_\nu(z)=\frac{2\,(z/2)^{\nu+1}}{\pi^{1/2}\,\Gamma(\nu+{}^3/_2)}\,{}_1F_2\left(1;\ \nu+{}^3/_2,\ {}^3/_2;\ \frac{z^2}{4}\right),$$

$$s_{\mu,\nu}(z)=\frac{z^{\mu+1}}{(\mu-\nu+1)(\mu+\nu+1)}\,{}_1F_2\left(1;\ \frac{\mu+\nu+3}{2},\ \frac{\mu-\nu+3}{2};\ -\frac{z^2}{4}\right),$$

Выражения различиых комбинаций функций Бесселя через G-функцию Мейера приведены в Приложении. Многие интегралы, содержащие функции Бесселя, могут быть получены путем выбора частных значений параметров в небольшом числе известных сейчас интегралов, содержащих G-функцию. Далее, в приведенных ниже таблицах многие весьма общие интегралы, содержащие функции Бесселя, выражены через вырожденные гипергеометрические функции или G-функции. Для частных значений параметров эти выражения заметно упрощаются. Во многих случаях эти частные случаи не отмечены особо, и тем, кто будет пользоваться таблицами, придется самим выполнить соответствующие преобразования. Необходимые для этого формулы даны в Приложении.

19.1. Функции Бесселя аргумента x: конечные промежутки интегрирования

(1)	$\int_0^a J_\nu(x)\,dx = 2\sum_{n=0}^{\infty} J_{\nu+2n+1}(a),$	$\operatorname{Re}\nu > -1$
(2)	$\int_0^a x^\nu J_\nu(x)\,dx = 2^{\nu-1}\pi^{1/2}\,\Gamma(\nu+1/2)\,a\,[J_\nu(a)\,\mathbf{H}_{\nu-1}(a) - \mathbf{H}_\nu(a)\,J_{\nu-1}(a)],$	$\operatorname{Re}\nu > -1/2$
(3)	$\int_0^a x^{\nu+1} J_\nu(x)\,dx = a^{\nu+1} J_{\nu+1}(a),$	$\operatorname{Re}\nu > -1$
(4)	$\int_0^a x^{1-\nu} J_\nu(x)\,dx = \frac{1}{2^{\nu-1}\,\Gamma(\nu)} - a^{1-\nu} J_{\nu-1}(a)$	
(5)	$\int_0^a x^\mu J_\nu(x)\,dx = (\mu+\nu-1)\,a\,J_\nu(a)\,S_{\mu-1,\,\nu-1}(a) - a\,J_{\nu-1}(a)\,S_{\mu,\,\nu}(a) + 2^\mu\,\frac{\Gamma\left(\frac{1+\mu+\nu}{2}\right)}{\Gamma\left(\frac{1-\mu+\nu}{2}\right)},$	$\operatorname{Re}(\mu+\nu) > -1$
(6)	$\int_0^a (a-x)^{-1/2} J_\nu(x)\,dx = \pi\,(a/2)^{1/2} J_{\nu/2+1/4}(a/2)\,J_{\nu/2-1/4}(a/2),$	$\operatorname{Re}\nu > -1$
(7)	$\int_0^a x^{-\nu}(a^2-x^2)^{-\nu-1/2} J_\nu(x)\,dx = \pi^{1/2}2^{-\nu-1}a^{-2\nu}\,\Gamma(1/2-\nu)\,J_\nu(c/2)\,J_{-\nu}(a/2),$	$\operatorname{Re}\nu < 1/2.$

Относительно подобных интегралов см. пп. 8.5 и 13.1.

(8)	$\int\limits_0^a x^{\varrho}(a^2-2abx+b^2)^{-1/2} J_\nu(x)\,dx.$ См. Bose S. K., 1946: Bull. Calcutta Math. Soc. 38, 177—180.
(9)	$\int\limits_0^a x^\nu \sin x\, J_\nu(x)\,dx = \frac{a^{\nu+1}}{2\nu+1}\sin a\, J_\nu(a) - \cos a\, J_{\nu+1}(a),$ $\quad \operatorname{Re}\nu > -1$
(10)	$\int\limits_0^a \sin(a-x)\, J_{2n}(x)\,dx =$ $= a\,J_{2n+1}(a) + (-1)^n\, 2n\left[\cos a - J_0(a) - 2\sum_{m=1}^{n}(-1)^m J_{2m}(a)\right],$ $\quad n=0,\ 1,\ 2,\ \ldots$
(11)	$\int\limits_0^a \sin(a-x)\, J_{2n+1}(x)\,dx =$ $= a\,J_{2n+2}(a) + (-1)^n(2n+1)\left[\sin a - 2\sum_{m=0}^{n}(-1)^m J_{2m+1}(a)\right],$ $\quad n=0,\ 1,\ 2,\ \ldots$
(12)	$\int\limits_0^a \sin(a-x)\, J_\nu(x)\,dx = a\,J_{\nu+1}(a) - 2\nu\sum_{n=0}^{\infty}(-1)^n J_{\nu+2n+2}(a),$ $\quad \operatorname{Re}\nu > -1$
(13)	$\int\limits_0^a x^{-1}\sin(a-x)\, J_\nu(x)\,dx = 2\nu^{-1}\sum_{n=0}^{\infty}(-1)^n J_{\nu+2n+1}(a),$ $\quad \operatorname{Re}\nu > 0$
(14)	$\int\limits_0^a x^{-3/2}\sin(a-x)\, J_\nu(x)\,dx = (\nu^2-1/4)^{-1}\, a^{1/2} J_\nu(a),$ $\quad \operatorname{Re}\nu > 1/2$
(15)	$\int\limits_0^a x^\nu \sin(a-x)\, J_\nu(x)\,dx = \frac{a^{\nu+1}}{2\nu+1} J_{\nu+1}(a),$ $\quad \operatorname{Re}\nu > -1/2$
(16)	$\int\limits_0^a x^\lambda \sin(a-x)\, J_\nu(x)\,dx =$ $= 2a^{\lambda+1}\sum_{n=0}^{\infty}\frac{(-1)^n(\nu-\lambda)_{2n}}{(\nu+\lambda+1)_{2n+2}}(\nu+2n+1)\, J_{\nu+2n+1}(a),$ $\quad \operatorname{Re}(\lambda+\nu) > -1.$ См. также Bailey W. N., 1930: Proc. London Math. Soc. (2), 31, 200—208.

(17)	$\int_0^a (a^2-x^2)^{-1/2}\sin(\beta x)\,J_\nu(x)\,dx = \pi\sum_{n=0}^{\infty}(-1)^n J_{2n+1}(a\beta)\,J_{\nu/2+n+1/2}(a/2)\,J_{\nu/2-n+1/2}(a),\qquad \operatorname{Re}\nu > -2$
(18)	$\int_0^a x^{\nu+1}\sin\left[2^{-1}\beta(a^2-x^2)\right]J_\nu(x)\,dx = \beta^{-\nu-1}\,U_{\nu+2}(a^2\beta,\ a),\qquad \operatorname{Re}\nu > -1$
(19)	$\int_0^a x^{\nu+1}\sin\left[b(a^2-x^2)^{1/2}\right]J_\nu(x)\,dx = (\pi/2)^{1/2}\,a^{\nu+3/2}b\,(1+b^2)^{-\nu/2-3/4}\,J_{\nu+3/2}\left[a(1+b^2)^{1/2}\right],\qquad \operatorname{Re}\nu > -1$
(20)	$\int_0^a x^\nu\cos x\,J_\nu(x)\,dx = \frac{a^{\nu+1}}{2\nu+1}\left[\cos a\,J_\nu(a)+\sin a\,J_{\nu+1}(a)\right],\qquad \operatorname{Re}\nu > -1/2$
(21)	$\int_0^a \cos(a-x)\,J_{2n}(x)\,dx = a\,J_{2n}(a)-(-1)^n\,2n\left[\sin a-2\sum_{m=0}^{n-1}(-1)^m J_{2m+1}(a)\right],\qquad n=0,\ 1,\ 2,\ \ldots$
(22)	$\int_0^a \cos(a-x)\,J_{2n+1}(x)\,dx = a\,J_{2n+1}(a)+(-1)^n(2n+1)\left[\cos a-J_0(a)-2\sum_{m=1}^{n}(-1)^m J_{2m}(a)\right],\quad n=0,\ 1,\ 2,\ \ldots$
(23)	$\int_0^a \cos(a-x)\,J_\nu(x)\,dx = a\,J_\nu(a)-2\nu\sum_{n=0}^{\infty}(-1)^n J_{\nu+2n+1}(a),\qquad \operatorname{Re}\nu > -1$
(24)	$\int_0^a x^{-1}\cos(a-x)\,J_\nu(x)\,dx = \nu^{-1}J_\nu(a)+2\nu^{-1}\sum_{n=1}^{\infty}(-1)^n J_{\nu+2n}(a),\qquad \operatorname{Re}\nu > 0$
(25)	$\int_0^a x^\nu\cos(a-x)\,J_\nu(x)\,dx = \frac{a^{\nu+1}}{2\nu+1}\,J_\nu(a),\qquad \operatorname{Re}\nu > -1/2$

(26)	$\int_0^a x^{\lambda} \cos(a-x) J_\nu(x)\,dx = \dfrac{a^{\lambda+1} J_\nu(a)}{\lambda+\nu+1} + 2a^{\lambda+1} \sum_{n=1}^{\infty} \dfrac{(-1)^n (\nu-\lambda)_{2n-1}}{(\nu+\lambda+1)_{2n+1}} (\nu+2n) J_{\nu+2n}(a)$, $\operatorname{Re}(\lambda+\nu) > -1$. См. также Bailey W. N., 1930: Proc. London Math. Soc. (2), 31, 200–208.
(27)	$\int_0^a (a^2-x^2)^{-1/2} \cos(\beta x) J_\nu(x)\,dx = 2^{-1}\pi J_0(a\beta)\,[J_{\nu/2}(a/2)]^2 + \pi \sum_{n=1}^{\infty} (-1)^n J_{2n}(a\beta) J_{\nu/2+n}(a/2) J_{\nu/2-n}(a/2)$, $\operatorname{Re}\nu > -1$
(28)	$\int_0^a x^{\nu+1} \cos[2^{-1}\beta(a^2-x^2)] J_\nu(x)\,dx = \beta^{-\nu-1} U_{\nu+1}(a^2\beta, a)$, $\operatorname{Re}\nu > -1$
(29)	$\int_0^a x(a^2-x^2)^{-1/2} \cos[\beta(a^2-x^2)^{1/2}] J_0(x)\,dx = (\beta^2+1)^{-1/2} \sin[a(\beta^2+1)^{1/2}]$
(30)	$\int_0^a (a^2-x^2)^{-1/2} \cos[\beta(a^2-x^2)^{1/2}] J_\nu(x)\,dx = 2^{-1}\pi J_{\nu/2}(2^{-1}a e^{u}) J_{\nu/2}(2^{-1}a e^{-u})$, $\beta = \operatorname{sh} u$, $\operatorname{Re}\nu > -1$
(31)	$\int_0^a x^{\mu-1} P_n(x/a) J_\nu(x)\,dx$, $\int_0^a x^{\mu-1} P_n(x/a) J_\mu(x) J_\nu(x)\,dx$. См. Bose S. K., 1946: Bull. Calcutta Math. Soc. 38, 177–180.
(32)	$\int_0^a x^{\mu-1} P_n(2x^2a^{-2}-1) J_\nu(x)\,dx = \dfrac{2^{-\nu-1} a^{\mu+\nu} [\Gamma(\mu/2+\nu/2)]^2}{\Gamma(\nu+1)\,\Gamma(\mu/2+\nu/2+n+1)\,\Gamma(1/2+\nu/2-n)} \times {}_2F_3\left(\dfrac{\mu+\nu}{2}, \dfrac{\mu+\nu}{2}; \nu+1, \dfrac{\mu+\nu}{2}+n+1, \dfrac{\mu+\nu}{2}-n; -\dfrac{a^2}{4}\right)$, $\operatorname{Re}(\mu+\nu) > 0$. Относительно частных случаев см. Bose B. N., 1944: Bull. Calcutta Math. Soc. 36, 125–132.

(33)	$\int\limits_0^a x^{1/2-\mu}(a^2-x^2)^{-\mu/2}\, \mathrm{P}_\nu^\mu(x/a)\, J_{\nu+1/2}(x)\, dx =$ $= (\pi/2)^{1/2} a^{1-\mu} J_{1/2-\mu}(a/2)\, J_{\nu+1/2}(a/2), \quad \operatorname{Re}\mu < 1, \ \operatorname{Re}(\mu-\nu) < 2$
(34)	$\int\limits_0^a x^{1/2}(a^2-x^2)^{-\nu/2-1/4}\, \mathrm{P}_\mu^{\nu+1/2}(2x^2a^{-2}-1)\, J_\nu(x)\, dx =$ $= \pi^{1/2} 2^{-\nu-1} a\, J_{\mu+1/2}(a/2)\, J_{-\mu-1/2}(a/2), \quad -1 < \operatorname{Re}\nu < 1/2$
(35)	$\int\limits_0^a J_0(x)\, J_1(x)\, dx = 1/2 - 2^{-1}[J_0(a)]^2$
(36)	$\int\limits_0^a J_n(x)\, J_{n+1}(x)\, dx = 1/2 - 2^{-1}[J_0(a)]^2 - \sum\limits_{m=1}^{n}[J_m(a)]^2, \quad n = 1, 2, 3, \ldots$
(37)	$\int\limits_0^a J_\nu(x)\, J_{\nu+1}(x)\, dx = \sum\limits_{n=0}^{\infty}[J_{\nu+n+1}(a)]^2, \quad \operatorname{Re}\nu > -1$
(38)	$\int\limits_0^a x^{\rho-1}(a^2-x^2)^{\sigma-1} J_\mu(x)\, J_\nu(x)\, dx.$ См. Bailey W. N., 1938: Quart. J. Math. Oxford Ser. 9, 141—147.
(39)	$\int\limits_0^a x\, P_n(1-2x^2a^{-2})\, [J_0(x)]^2\, dx = \frac{a^2}{2(2n+1)}\{[J_n(a)]^2 + [J_{n+1}(a)]^2\},$ $n = 0, 1, 2, \ldots$
(40)	$\int\limits_0^a x^{2\nu+1} P_n(2x^2a^{-2}-1)\, [J_\nu(x)]^2\, dx.$ См. Bose B. N., 1944: Bull. Calcutta Math. Soc. 36, 125—132.
(41)	$\int\limits_a^b \frac{dx}{x\,[J_\nu(x)]^2} = \frac{\pi}{2}\left[\frac{Y_\nu(b)}{J_\nu(b)} - \frac{Y_\nu(a)}{J_\nu(a)}\right]$
(42)	$\int\limits_a^b \frac{dx}{x\, J_\nu(x)\, J_{-\nu}(x)} = \frac{\pi}{2\sin(\nu\pi)} \ln\left[\frac{J_{-\nu}(a)\, J_\nu(b)}{J_\nu(a)\, J_{-\nu}(b)}\right]$

(43)	$\int_0^a x^{\nu} Y_{\nu}(x)\,dx = 2^{\nu-1}\pi^{1/2}\,\Gamma(\nu+1/2)\,a\,[Y_{\nu}(a)\,\mathbf{H}_{\nu-1}(a) - \mathbf{H}_{\nu}(a)\,Y_{\nu-1}(a)]$, $\quad \operatorname{Re}\nu > -1/2$
(44)	$\int_0^a x^{\nu+1} Y_{\nu}(x)\,dx = a^{\nu+1}\,Y_{\nu+1}(a) + 2^{\nu+1}\,\Gamma(\nu+1)$, $\quad \operatorname{Re}\nu > -1$
(45)	$\int_0^a x^{1-\nu} Y_{\nu}(x)\,dx = \dfrac{\operatorname{ctg}(\nu\pi)}{2^{\nu-1}\,\Gamma(\nu)} - a^{1-\nu}\,Y_{\nu-1}(a)$, $\quad \operatorname{Re}\nu < 1$
(46)	$\int_a^b Y_{\nu}(x)\,dx = 2\sum_{n=0}^{\infty}[Y_{\nu+2n+1}(b) - Y_{\nu+2n+1}(a)]$
(47)	$\int_0^a (a^2-x^2)^{-1/2}\cos\left[\beta(a^2-x^2)^{1/2}\right] Y_{2\nu}(x)\,dx =$ $= 2^{-2}\pi\,[J_{\nu}(u)\,Y_{\nu}(v) + J_{\nu}(v)\,Y_{\nu}(u)] -$ $- 2^{-2}\pi\operatorname{tg}(\nu\pi)\,[J_{\nu}(u)\,J_{\nu}(v) + Y_{\nu}(u)\,Y_{\nu}(v)]$, $u = 2^{-1}a\left[(\beta^2+1)^{1/2}+\beta\right]$, $\quad v = 2^{-1}a\left[(\beta^2+1)^{1/2}-\beta\right]$, $\quad -1/2 < \operatorname{Re}\nu < 1/2$
(48)	$\int_0^a x\,P_n(1-2x^2a^{-2})\,J_0(x)\,Y_0(x)\,dx =$ $= \dfrac{a^2}{2(2n+1)}\,[J_n(a)\,Y_n(a) + J_{n+1}(a)\,Y_{n+1}(a)]$, $\quad n = 0, 1, 2, \ldots$
(49)	$\int_a^b \dfrac{dx}{x\,[Y_{\nu}(x)]^2} = \dfrac{\pi}{2}\left[\dfrac{J_{\nu}(a)}{Y_{\nu}(a)} - \dfrac{J_{\nu}(b)}{Y_{\nu}(b)}\right]$
(50)	$\int_a^b \dfrac{dx}{x\,J_{\nu}(x)\,Y_{\nu}(x)} = \dfrac{\pi}{2}\ln\left[\dfrac{J_{\nu}(a)\,Y_{\nu}(b)}{J_{\nu}(b)\,Y_{\nu}(a)}\right]$

19.2. Функции Бесселя аргумента x: бесконечные промежутки интегрирования

(1)	$\int_0^{\infty} J_{\nu}(x)\,dx = 1$, $\quad \operatorname{Re}\nu > -1$

(2)	$\int\limits_0^\infty \frac{J_\nu(x)}{x^2+\alpha^2}\,dx = \frac{\pi\,[\mathbf{J}_\nu(\alpha) - J_\nu(\alpha)]}{\alpha \sin(\nu\pi)}$,	$\operatorname{Re}\alpha > 0,\ \operatorname{Re}\nu > -1$
(3)	$\int\limits_0^\infty x^{-1}(\alpha^2+x^2)^{-1/2}\,[J_0(x)-1]\,dx = \alpha^{-1}\,[\operatorname{Ei}(-\alpha) - \ln(\gamma\alpha)]$,	$\operatorname{Re}\alpha > 0$
(4)	$\int\limits_0^\infty x\,[(x^2+\alpha^2)(x^2+\beta^2)]^{-1/2}\,\left[(x^2+\alpha^2)\right]^{1/2} + (x^2+\beta^2)^{1/2}\right]^{-2\nu} J_0(x)\,dx =$ $= [\alpha^2-\beta^2)^{-\nu}\, I_\nu\left(\frac{\alpha-\beta}{2}\right) K_\nu\left(\frac{\alpha+\beta}{2}\right)$,	$\operatorname{Re}\alpha > 0,\ \operatorname{Re}\beta > 0,\ \operatorname{Re}\nu > -3/4$
(5)	$\int\limits_0^\infty x^3\,(x^4-a^4)^{-1}\,J_0(x)\,dx = 2^{-1}K_0(a) - 2^{-2}\pi\,Y_0(a)$,	$a > 0$
(6)	$\int\limits_0^\infty x^\nu\,(e^{\alpha x}-1)^{-1} J_\nu(x)\,dx = 2^\nu \pi^{-1/2}\,\Gamma(\nu+1/2) \sum\limits_{n=1}^\infty (1+\alpha^2 n^2)^{-\nu-1/2}$,	$\operatorname{Re}\alpha > 0,\ \operatorname{Re}\nu > 0$
(7)	$\int\limits_0^\infty x^{-1}\exp\left(-\frac{2\alpha}{x}\right) J_\nu(x)\,dx = 2\,J_\nu(2\alpha^{1/2})\,K_\nu(2\alpha^{1/2})$,	$\operatorname{Re}\alpha > 0$
(8)	$\int\limits_0^\infty \frac{x^\nu}{x+\beta}\sin(x+\beta)\,J_\nu(x)\,dx = \frac{\pi\beta^\nu J_{-\nu}(\beta)}{2\cos(\nu\pi)}$,	$\lvert\arg\beta\rvert < \pi,\ \lvert\operatorname{Re}\nu\rvert < 1/2$
(9)	$\int\limits_0^\infty \frac{x^\nu}{x+\beta}\cos(x+\beta)\,J_\nu(x)\,dx = -\frac{\pi\beta^\nu Y_{-\nu}(\beta)}{2\cos(\nu\pi)}$,	$\lvert\arg\beta\rvert < \pi,\ \lvert\operatorname{Re}\nu\rvert < 1/2$
(10)	$\int\limits_0^\infty x^{1/4}\sin(2ax^{1/2})\,J_{-1/4}(x)\,dx = \pi^{1/2}a^{3/2}J_{3/4}(a^2)$,	$a > 0$
(11)	$\int\limits_0^\infty x^{1/4}\sin(2ax^{1/2})\,J_{3/4}(x)\,dx = \pi^{1/2}a^{3/2}J_{-1/4}(a^2)$,	$a > 0$
(12)	$\int\limits_0^\infty x^{1/4}\cos(2ax^{1/2})\,J_{1/4}(x)\,dx = \pi^{1/2}a^{3/2}J_{-3/4}(a^2)$,	$a > 0$

(13)	$\int\limits_0^\infty x^{1/4}\cos\left(2ax^{1/2}\right) J_{-3/4}(x)\,dx = \pi^{1/2} a^{3/2} J_{1/4}(a^2),$	$a>0$
(14)	$\int\limits_0^\infty x^{-1/4} e^{-\alpha x}\sin\left(2\beta x^{1/2}\right) J_{-1/4}(x)\,dx = \pi^{1/2}\left(\frac{\beta}{\alpha^2+1}\right)^{1/2}\exp\left(-\frac{\alpha\beta^2}{\alpha^2+1}\right) J_{-1/4}\left(\frac{\beta^2}{\alpha^2+1}\right),$	$\operatorname{Re}\alpha>0$
(15)	$\int\limits_0^\infty x^{-1/4} e^{-\alpha x}\sin\left(2\beta x^{1/2}\right) J_{1/4}(x)\,dx = \pi^{1/2}\left(\frac{\beta}{\alpha^2+1}\right)^{1/2}\exp\left(-\frac{\alpha\beta^2}{\alpha^2+1}\right) J_{1/4}\left(\frac{\beta^2}{\alpha^2+1}\right),$	$\operatorname{Re}\alpha>0$
(16)	$\int\limits_0^\infty x^{-1/2}\sin x\sin\left(4ax^{1/2}\right) J_0(x)\,dx = (\pi/2)^{1/2}\cos(a^2+\pi/4)\, J_0(a^2),$	$a>0$
(17)	$\int\limits_0^\infty x^{-1/3}\sin x\sin\left(4ax^{1/2}\right) J_{1/3}(x)\,dx = -2^{-5/2}\pi^{1/2}a^{1/3}\left[\sin(a^2+\pi/12)\, J_{1/3}(a^2) + \cos(a^2+\pi/12)\, Y_{1/3}(a^2)\right],$	$a>0$
(18)	$\int\limits_0^\infty x^{-1/2}\sin x\cos\left(4ax^{1/2}\right) J_0(x)\,dx = -2^{-3/2}\pi^{1/2}\left[\cos(a^2-\pi/4)\, J_0(a^2) - \sin(a^2-\pi/4)\, Y_0(a^2)\right],$	$a>0$
(19)	$\int\limits_0^\infty x^{-1/3}\sin x\cos\left(4ax^{1/2}\right) J_{-1/3}(x)\,dx = -2^{-3/2}\pi^{1/2}a^{1/3}\sin(a^2-\pi/12)\, J_{-1/3}(a^2),$	$a>0$
(20)	$\int\limits_0^\infty x^{-1/2}\cos x\sin\left(4ax^{1/2}\right) J_0(x)\,dx = (\pi/2)^{1/2}\cos(a^2-\pi/4)\, J_0(a^2),$	$a>0$
(21)	$\int\limits_0^\infty x^{-1/3}\cos x\sin\left(4ax^{1/2}\right) J_{1/3}(x)\,dx = 2^{-5/2}\pi^{1/2}a^{1/3}\left[\cos(a^2+\pi/12)\, J_{1/3}(a^2) - \sin(a^2+\pi/12)\, Y_{1/3}(a^2)\right],$	$a>0$

(22)
$$\int_0^\infty x^{-1/2}\cos x\cos\left(4ax^{1/2}\right)J_0(x)\,dx = -2^{-3/2}\pi^{1/2}\left[\sin(a^2-\pi/4)\,J_0(a^2)+\cos(a^2-\pi/4)\,Y_0(a^2)\right], \qquad a>0$$

(23)
$$\int_0^\infty x^{-1/3}\cos x\cos\left(4ax^{1/2}\right)J_{-1/3}(x)\,dx = 2^{-3/2}\pi^{1/2}a^{1/3}\cos(a^2-\pi/12)\,J_{-1/3}(a^2), \qquad a>0$$

(24)
$$\int_0^\infty x^{-\rho}J_\mu(x)\,J_\nu(x)\,dx = \frac{2^{-\rho}\,\Gamma(\rho)\,\Gamma\left[2^{-1}(\mu+\nu-\rho+1)\right]}{\Gamma\left[2^{-1}(\rho+\mu+\nu+1)\right]\Gamma\left[2^{-1}(\rho-\mu+\nu+1)\right]\Gamma\left[2^{-1}(\rho+\mu-\nu+1)\right]},$$
$$0<\operatorname{Re}\rho<\operatorname{Re}(\mu+\nu)+1$$

(25)
$$\int_0^\infty x^{1-2\nu}\left[J_\nu(x)\right]^4dx = \frac{\Gamma(\nu)\,\Gamma(2\nu)}{2\pi\left[\Gamma(\nu+1/2)\right]^2\Gamma(3\nu)}, \qquad \operatorname{Re}\nu>0$$

(26)
$$\int_0^\infty \frac{x}{x^2+\alpha^2}\left[J_\nu(x)\right]^2dx = I_\nu(\alpha)\,K_\nu(\alpha), \qquad \operatorname{Re}\alpha>0,\ \operatorname{Re}\nu>-1$$

(27)
$$\int_0^\infty (x^2+\alpha^2)^{-1/2}J_\mu(x)\,J_\nu(x)\,dx.$$

См. Bouwkamp C. S., 1950: Nederl. Akad. Wetensch., Proc. 53, 654—661.

(28)
$$\int_0^\infty x^{\rho-1}(x^2+\alpha^2)^{-\sigma}J_\mu(x)\,J_\nu(x)\,dx.$$

См. Ватсон Г. Н., 1949, стр. 479.

(29)
$$\int_0^\infty x^{\mu+\nu}e^{-\alpha x}J_\mu(x)\,J_\nu(x)\,dx.$$

См. Ватсон Г. Н., 1949, стр. 427.

(30)
$$\int_0^\infty \sin(2ax)\left[J_\nu(x)\right]^2dx = \begin{cases} 2^{-1}\,P_{\nu-1/2}(1-2a^2), & 0<a<1\\ \pi^{-1}\cos(\nu\pi)\,Q_{\nu-1/2}(2a^2-1), & a>1\end{cases}$$
$$\operatorname{Re}\nu>-1$$

(31)
$$\int_0^\infty \sin(2ax)\,[x^\nu J_\nu(x)]^2\,dx =$$
$$= \begin{cases} \dfrac{a^{-2\nu}\,\Gamma(1/2+\nu)}{2\pi^{1/2}\,\Gamma(1-\nu)}\,{}_2F_1(1/2+\nu,\ 1/2;\ 1-\nu;\ a^2), & 0<a<1 \\ \dfrac{a^{-4\nu-1}\,\Gamma(1/2+\nu)}{2\,\Gamma(1+\nu)\,\Gamma(1/2-2\nu)}\,{}_2F_1(1/2+\nu,\ 1/2+2\nu;\ 1+\nu;\ a^{-2}), & a>1 \end{cases}$$
$$-1/2<\operatorname{Re}\nu<1/2$$

(32)
$$\int_0^\infty \cos(2ax)\,[J_\nu(x)]^2\,dx =$$
$$= \begin{cases} \pi^{-1}\,Q_{\nu-1/2}(1-2a^2), & 0<a<1 \\ -\pi^{-1}\sin(\nu\pi)\,Q_{\nu-1/2}(2a^2-1), & a>1 \end{cases}$$
$$\operatorname{Re}\nu>-1/2$$

(33)
$$\int_0^\infty \cos(2ax)\,[x^\nu J_\nu(x)]^2\,dx =$$
$$= \begin{cases} \dfrac{a^{-2\nu}\,\Gamma(\nu)}{2\pi^{1/2}\,\Gamma(1/2-\nu)}\,{}_2F_1(\nu+1/2,\ 1/2;\ 1-\nu;\ a^2)\ + \\ \quad +\dfrac{\Gamma(-\nu)\,\Gamma(1/2+2\nu)}{2\pi\,\Gamma(1/2-\nu)}\,{}_2F_1(1/2+\nu,\ 1/2+2\nu;\ 1+\nu;\ a^2), & 0<a<1 \\ -\dfrac{\sin(\nu\pi)\,a^{-4\nu-1}\,\Gamma(1/2+2\nu)}{\Gamma(1+\nu)\,\Gamma(1/2-\nu)}\,{}_2F_1(1/2+\nu,\ 1/2+2\nu;\ 1+\nu;\ a^{-2}), & a>1 \end{cases}$$
$$-1/4<\operatorname{Re}\nu<1/2$$

(34)
$$\int_a^\infty (x^2-a^2)^{-1/2}\,J_\nu(x)\,dx = -2^{-1}\pi\,J_{\nu/2}(a/2)\,Y_{\nu/2}(a/2),$$
$$\operatorname{Re}\nu>-1$$

(35)
$$\int_a^\infty x^{-1}(x^2-a^2)^{-1/2}\,J_0(x)\,dx = -\operatorname{si}(a)$$

(36)
$$\int_a^\infty x^{1/2}\,P_{\nu-1/2}(x/a)\,J_\nu(x)\,dx =$$
$$= -(a/2)^{1/2}\,[\cos(a/2)\,Y_\nu(a/2)+\sin(a/2)\,J_\nu(a/2)],$$
$$-1/2<\operatorname{Re}\nu<1/2$$

(37)
$$\int_a^\infty x^{1/2-\mu}\,(x^2-a^2)^{-\mu/2}\,P^{\mu}_{\nu-1/2}(x/a)\,J_\nu(x)\,dx =$$
$$= -2^{-3/2}\pi^{1/2}a^{1-\mu}\,[J_{\mu-1/2}(a/2)\,Y_\nu(a/2)+Y_{\mu-1/2}(a/2)\,J_\nu(a/2)],$$
$$-1/4<\operatorname{Re}\mu<1,\quad |\operatorname{Re}\nu|<1/2+2\operatorname{Re}\mu$$

(38)
$$\int_a^\infty x^\nu (x^2-a^2)^{\lambda/2-1/2} P_\lambda^{\lambda-1}(x/a)\, J_\nu(x)\,dx = \frac{2^{\lambda+\nu} a^\nu\, \Gamma(1/2+\nu)}{\pi^{1/2}\,\Gamma(1-\lambda)}\, S_{\lambda-\nu,\,\lambda+\nu}(a),$$
$\operatorname{Re}\nu < 5/2,\ \operatorname{Re}(2\lambda+\nu) < 3/2$

(39)
$$\int_a^\infty x^{1/2} (x^2-a^2)^{\nu/2-1/4} P_\mu^{1/2-\nu}(2x^2a^{-2}-1)\, J_\nu(x)\,dx = -\frac{2^{\nu-2}\pi^{1/2}a}{\cos(\mu\pi)}\left\{[J_{\mu+1/2}(a/2)]^2 - [J_{-\mu-1/2}(a/2)]^2\right\},$$
$\operatorname{Re}\nu > -1/2,\ \operatorname{Re}\nu - 3/2 < 2\operatorname{Re}\mu < 1/2 - \operatorname{Re}\nu$

(40)
$$\int_a^\infty x^{1-2\nu} (x^2-a^2)^{\nu-3/2} [J_\nu(x)]^2\,dx = \frac{\Gamma(\nu-1/2)}{2\pi^{1/2}a^{\nu+1}}\, \mathbf{H}_\nu(2a),$$
$\operatorname{Re}\nu > 1/2$

(41)
$$\int_a^\infty x^{2\nu+1} (a^2-x^2)^{-\nu-3/2} \left\{[J_\nu(x)]^2 + [J_{-\nu}(x)]^2\right\} dx = \pi^{-1/2} a^{\nu-1}\, \Gamma(-\nu-1/2)\sin(\nu\pi)\, J_\nu(2a),$$
$\operatorname{Re}\nu < -1/2$

(42)
$$\int_{-\infty}^\infty \frac{\sin[a(x+\beta)]}{x+\beta} J_0(x)\,dx = \begin{cases} 2\displaystyle\int_0^a (1-u^2)^{-1/2}\cos(\beta u)\,du, & 0 \leqslant a \leqslant 1 \\ \pi J_0(\beta), & 1 \leqslant a < \infty \end{cases}$$

(43)
$$\int_{-\infty}^\infty \frac{|x|}{x+\beta} \sin[a(x+\beta)]\, J_0(x)\,dx = 0, \qquad 0 \leqslant a < 1$$

(44)
$$\int_{-\infty}^\infty \frac{\sin[a(x+\beta)]}{x^\nu (x+\beta)} J_{\nu+2n}(x)\,dx = \pi\beta^{-\nu} J_{\nu+2n}(\beta), \qquad 1 \leqslant a < \infty$$
$n = 0, 1, 2, \ldots,\ \operatorname{Re}\nu > -3/2$

(45)
$$\int_{-\infty}^\infty \frac{\sin[a(x+\beta)]}{x+\beta} [J_{n+1/2}(x)]^2\,dx = \pi[J_{n+1/2}(\beta)]^2, \qquad 2 \leqslant a < \infty$$
$n = 0, 1, 2, \ldots$

(46)	$\int\limits_{-\infty}^{\infty} \frac{\sin[a(x+\beta)]}{x+\beta} J_{n+1/2}(x) J_{-n-1/2}(x)\,dx = \pi J_{n+1/2}(\beta) J_{-n-1/2}(\beta),$ $n=0,\ 1,\ 2,\ \ldots,\ 2 \leqslant a < \infty$
(47)	$\int\limits_{-\infty}^{\infty} \frac{\sin[a(x+\beta)]}{x^{2\nu}(x+\beta)} [J_{\nu+n}(x)]^2\,dx = \pi\beta^{-2\nu} [J_{\nu+n}(\beta)]^2,$ $2 \leqslant a < \infty$ $n=0,\ 1,\ 2,\ \ldots,\ \operatorname{Re}\nu > -1$
(48)	$\int\limits_{0}^{\infty} Y_\nu(x)\,dx = -\operatorname{tg}(\nu\pi/2)$ $-1 < \operatorname{Re}\nu < 1$
(49)	$\int\limits_{0}^{\infty} x^\rho (x^2+a^2)^{-\mu} Y_\nu(x)\,dx = \frac{a^{\rho-2\mu}}{\Gamma(\mu)} G_{24}^{31}\left(\frac{a^2}{4}\middle\vert \begin{matrix} 1-\frac{\rho}{2},\ -\frac{\nu}{2} \\ \mu-\frac{\rho}{2},\ \frac{1+\nu}{2},\ \frac{1-\nu}{2},\ -\frac{\nu}{2} \end{matrix}\right),$ $\vert\operatorname{Re}\nu\vert - 1 < \operatorname{Re}\rho < 2\operatorname{Re}\mu + 1/2$
(50)	$\int\limits_{0}^{\infty} x^{-1} \exp\left(-\frac{2\alpha}{x}\right) Y_\nu(x)\,dx = 2 Y_\nu(2\alpha^{1/2}) K_\nu(2\alpha^{1/2}),$ $\operatorname{Re}\alpha > 0$
(51)	$\int\limits_{0}^{\infty} x^{-1} \sin\left(\frac{a}{2x}\right) [\sin x\, J_0(x) + \cos x\, Y_0(x)]\,dx = \pi J_0(a^{1/2}) Y_0(a^{1/2}),$ $a > 0$
(52)	$\int\limits_{0}^{\infty} x^{-1} \cos\left(\frac{a}{2x}\right) [\sin x\, Y_0(x) - \cos x\, J_0(x)]\,dx = \pi J_0(a^{1/2}) Y_0(a^{1/2}),$ $a > 0$
(53)	$\int\limits_{0}^{\infty} x^{1/4} \sin(2ax^{1/2}) Y_{3/4}(x)\,dx = -\pi^{1/2} a^{3/2} \mathbf{H}_{-1/4}(a^2),$ $a > 0$
(54)	$\int\limits_{0}^{\infty} x^{1/4} \cos(2ax^{1/2}) Y_{1/4}(x)\,dx = -\pi^{1/2} a^{3/2} \mathbf{H}_{-3/4}(a^2),$ $a > 0$

(55)	$\int\limits_0^\infty x^{-1/2} \sin x \cos(4ax^{1/2})\, Y_0(x)\, dx =$ $= 2^{-3/2}\pi^{1/2}\,[3\sin(a^2-\pi/4)\, J_0(a^2) - \cos(a^2-\pi/4)\, Y_0(a^2)],$ $a>0$
(56)	$\int\limits_0^\infty x^{-1/2} \cos x \cos(4ax^{1/2})\, Y_0(x)\, dx =$ $= -2^{-3/2}\pi^{1/2}\,[3\cos(a^2-\pi/4)\, J_0(a^2) + \sin(a^2-\pi/4)\, Y_0(a^2)],$ $a>0$
(57)	$\int\limits_0^\infty J_{\nu+n}(x)\, Y_{\nu-n}(x)\, dx = \frac{(-1)^{n+1}}{2},$ $\operatorname{Re}\nu > -1/2,\ n=0,\ 1,\ 2,\ \ldots$
(58)	$\int\limits_0^\infty e^{-2\alpha x} J_0(x)\, Y_0(x)\, dx = \frac{K[\alpha(\alpha^2+1)^{-1/2}]}{\pi(\alpha^2+1)^{1/2}},$ $\operatorname{Re}\alpha>0$
(59)	$\int\limits_0^\infty x^{2\nu+1} \exp(-\alpha x^2)\, J_\nu(x)\, Y_\nu(x)\, dx =$ $= -2^{-1}\pi^{-1/2}\alpha^{-3\nu/2-1/2} \exp\left(-\frac{1}{2\alpha}\right) W_{\nu/2,\ \nu/2}\left(\frac{1}{\alpha}\right),$ $\operatorname{Re}\alpha>0,\ \operatorname{Re}\nu>-1/2$
(60)	$\int\limits_0^\infty \sin(2ax)\, J_0(x)\, Y_0(x)\, dx = \begin{cases} 0, & 0<a<1 \\ -\dfrac{K[(1-a^{-2})^{1/2}]}{\pi a}, & a>1 \end{cases}$
(61)	$\int\limits_0^\infty \cos(2ax)\, J_0(x)\, Y_0(x)\, dx = \begin{cases} -\pi^{-1} K(a), & 0<a<1 \\ -(\pi a)^{-1} K(a^{-1}), & a>1 \end{cases}$
(62)	$\int\limits_0^\infty \cos(2ax)\, [Y_0(x)]^2\, dx = \begin{cases} \pi^{-1} K[(1-a^2)^{1/2}], & 0<a<1 \\ 2(\pi a)^{-1} K[(1-a^{-2})^{1/2}], & a>1 \end{cases}$

(63)	$\int_0^\infty x^{1-2\nu}\sin(2ax)\,J_\nu(x)\,Y_\nu(x)\,dx = -\dfrac{\Gamma(^3/_2-\nu)\,a}{2\,\Gamma(2\nu-{}^1/_2)\,\Gamma(2-\nu)}\,{}_2F_1\left(\frac{3}{2}-\nu,\ \frac{3}{2}-2\nu;\ 2-\nu;\ a^2\right)$, $0<\operatorname{Re}\nu<{}^3/_2,\ 0<a<1$
(64)	$\int_0^\infty x^{1-2\nu}\sin(2ax)\,\{[J_\nu(x)]^2-[Y_\nu(x)]^2\}\,dx = \dfrac{\sin(2\nu\pi)\,\Gamma(^3/_2-\nu)\,\Gamma(^3/_2-2\nu)\,a}{\pi\,\Gamma(2-\nu)}\,{}_2F_1\left(\frac{3}{2}-\nu,\ \frac{3}{2}-2\nu;\ 2-\nu;\ a^2\right)$, $0<\operatorname{Re}\nu<{}^3/_4,\ 0<a<1$
(65)	$\int_0^\infty x^{2-2\nu}\sin(2ax)\,[J_\nu(x)\,J_{\nu-1}(x)-Y_\nu(x)\,Y_{\nu-1}(x)]\,dx = -\dfrac{\sin(2\nu\pi)\,\Gamma(^3/_2-\nu)\,\Gamma(^5/_2-2\nu)\,a}{\pi\,\Gamma(2-\nu)}\,{}_2F_1\left(\frac{3}{2}-\nu,\ \frac{5}{2}-2\nu;\ 2-\nu;\ a^2\right)$, ${}^1/_2<\operatorname{Re}\nu<{}^5/_4,\ 0<a<1$
(66)	$\int_0^\infty x^{2-2\nu}\sin(2ax)\,[J_\nu(x)\,Y_{\nu-1}(x)+Y_\nu(x)\,J_{\nu-1}(x)]\,dx = -\dfrac{\Gamma(^3/_2-\nu)\,a}{\Gamma(2\nu-{}^3/_2)\,\Gamma(2-\nu)}\,{}_2F_1\left(\frac{3}{2}-\nu,\ \frac{5}{2}-2\nu;\ 2-\nu;\ a^2\right)$, ${}^1/_2<\operatorname{Re}\nu<{}^5/_2,\ 0<a<1$
(67)	$\int_0^\infty x^{1/2-\mu}\,(x^2-a^2)^{-\mu/2}\,P^{\mu}_{\nu-1/2}(x/a)\,Y_\nu(x)\,dx = 2^{-3/2}\pi^{1/2}a^{1-\mu}\left[J_\nu(a/2)\,J_{\mu-1/2}(a/2)-Y_\nu(a/2)\,Y_{\mu-1/2}(a/2)\right]$, $-{}^1/_4<\operatorname{Re}\mu<1,\ \operatorname{Re}(2\mu-\nu)>-{}^1/_2$

19.3. Функции Бесселя аргументов $\alpha x+\beta$, x^2, x^{-1}

(1)	$\int_0^\infty x^{\rho-1}J_\mu(ax)\,J_\nu(bx)\,dx = \dfrac{2^{\rho-1}a^\mu b^{-\mu-\rho}\,\Gamma(\mu/2+\nu/2+\rho/2)}{\Gamma(\mu+1)\,\Gamma(1-\mu/2+\nu/2-\rho/2)}\,{}_2F_1\left(\frac{\mu+\nu+\rho}{2},\ \frac{\mu-\nu+\rho}{2};\ \mu+1;\ \frac{a^2}{b^2}\right)$, $\operatorname{Re}(\mu+\nu+\rho)>0,\quad \operatorname{Re}\rho<2,\quad 0<a<b$

(2)
$$\int_0^\infty x^{1/2}(x^2+\lambda^2)^{-1/2} J_\mu(ax)\,J_\nu(bx)\,dx.$$

См. Bouwkamp C. S., 1950: Nederl. Akad. Wetensch., Proc. 53, 654—661.

(3)
$$\int_0^\infty x^{1+\nu}\,[J_\nu(ax)]^2\,J_\nu(2bx)\,dx=$$
$$=\begin{cases} -\dfrac{\Gamma(1/2+\nu)}{2\pi^{3/2}}\sin(\nu\pi)\,a^{2\nu}b^{-\nu-1}(b^2-a^2)^{-\nu-1/2}, & 0<a<b\\[2mm] \dfrac{\Gamma(1/2+\nu)}{2\pi^{3/2}}\,a^{2\nu}b^{-\nu-1}(a^2-b^2)^{-\nu-1/2}, & 0<b<a \end{cases}$$
$$-1/2<\operatorname{Re}\nu<1/2$$

(4)
$$\int_0^\infty x^{1+\nu}\,J_\nu(ax)\,J_{-\nu}(ax)\,J_\nu(2bx)\,dx=$$
$$=\begin{cases} 0, & 0<a<b\\[2mm] \dfrac{a^{2\nu}b^{-\nu-1}}{2\pi^{1/2}\,\Gamma(1/2-\nu)}(a^2-b^2)^{-\nu-1/2}, & 0<b<a \end{cases}$$
$$-1<\operatorname{Re}\nu<1/2$$

(5)
$$\int_0^\infty x\,J_{\nu/2-1/4}(ax)\,J_{\nu/2+1/4}(ax)\,J_\nu(2bx)\,dx=$$
$$=\begin{cases} 0, & 0<a<b\\ 2^{-3/2}\pi^{-1}a^{-1/2}b^{-1}(a-b)^{-1/2}, & 0<b<a \end{cases}$$
$$\operatorname{Re}\nu>-1$$

(6)
$$\int_0^\infty x^{1+\nu}\,J_\mu(ax)\,J_{-\mu}(ax)\,J_\nu(2bx)\,dx=$$
$$=\begin{cases} 0, & 0<a<b\\[2mm] \dfrac{(a^2-b^2)^{-\nu/2-1/4}}{2\pi^{1/2}ab^{1/2}}\,P^{\nu+1/2}_{\mu-1/2}(2b^2a^{-2}-1), & 0<b<a \end{cases}$$
$$-1<\operatorname{Re}\nu<1/2$$

(7)
$$\int_0^\infty x^{\rho-1}\,J_\lambda(ax)\,J_\mu(ax)\,J_\nu(2bx)\,dx=$$
$$=\frac{a^{\lambda+\mu}b^{-\lambda-\mu-\rho}\,\Gamma(\lambda/2+\mu/2+\nu/2+\rho/2)}{2^{\lambda+\mu}\,\Gamma(\lambda+1)\,\Gamma(\mu+1)\,\Gamma(1-\lambda/2-\mu/2+\nu/2-\rho/2)}\times$$
$$\times\,{}_4F_3\left(\frac{\lambda+\mu+1}{2},\ \frac{\lambda+\mu}{2}+1,\ \frac{\lambda+\mu+\nu+\rho}{2},\ \frac{\lambda+\mu-\nu+\rho}{2};\right.$$
$$\left.\lambda+1,\ \mu+1,\ \lambda+\mu+1;\ \frac{a^2}{b^2}\right),$$
$$\operatorname{Re}(\lambda+\mu+\nu+\rho)>0,\qquad 0<a<b$$

(8)	$$\int_0^\infty x^{1-\nu} J_\nu(ax) J_\nu(bx) J_\nu(cx)\,dx = \begin{cases} \dfrac{2^{\nu-1}\Delta^{2\nu-1}}{\pi^{1/2}(abc)^\nu \Gamma(\nu+1/2)}, & \text{если существует треугольник со сторонами } a, b, c;\ \Delta \text{ — площадь этого треугольника,} \\ 0, & \text{если } a, b, c \text{ не являются сторонами некоторого треугольника} \end{cases}$$ $a, b, c>0$, $\operatorname{Re}\nu>-1/2$
(9)	$$\int_0^\infty x^{\rho-1} J_\lambda(ax) J_\mu(bx) J_\nu(cx)\,dx = \frac{2^{\rho-1}a^\lambda b^\mu c^{-\lambda-\mu-\rho}\,\Gamma(\lambda/2+\mu/2+\nu/2+\rho/2)}{\Gamma(\lambda+1)\Gamma(\mu+1)\Gamma(1-\lambda/2-\mu/2+\nu/2-\rho/2)} \times F_4\left(\frac{\lambda+\mu-\nu+\rho}{2}, \frac{\lambda+\mu+\nu+\rho}{2}; \lambda+1, \mu+1; \frac{a^2}{c^2}, \frac{b^2}{c^2}\right),$$ $\operatorname{Re}(\lambda+\mu+\nu+\rho)>0$, $\operatorname{Re}\rho<5/2$, $a, b, c>0$, $c>a+b$. Относительно частных случаев см. Ватсон Г. Н., 1949, п. 13.46; Bailey W. N., 1936; Proc. London Math. Soc. (2), 40, 37—48.
(10)	$$\int_0^\infty x^{1-2\nu}[J_\nu(ax) J_\nu(bx)]^2\,dx = \frac{a^{2\nu-1}b^{-1}\Gamma(\nu)}{2\pi\Gamma(\nu+1/2)\Gamma(2\nu+1/2)}\,{}_2F_1\left(\nu, 1/2-\nu, 2\nu+1/2; \frac{a^2}{b^2}\right),$$ $\operatorname{Re}\nu>0$, $0<a<b$
(11)	$$\int_0^\infty x^{1-\nu}[J_\nu(x)Y_{-\nu}(x)+Y_\nu(x)J_{-\nu}(x)]\,J_\nu(2ax)\,dx = \begin{cases} 0, & 0<a<1 \\ \dfrac{a^{\nu-1}(a^2-1)^{\nu-1/2}}{\pi^{1/2}\Gamma(\nu+1/2)}, & a>1 \end{cases}$$ $-1/2<\operatorname{Re}\nu<1$
(12)	$$\int_0^\infty x^{\mu+1}[J_\nu(x)Y_\mu(x)+J_\mu(x)Y_\nu(x)]\,J_\nu(2ax)\,dx = \begin{cases} 0, & 0<a<1 \\ -\dfrac{a^{-\mu-1}(a^2-1)^{-\mu/2-1/4}}{\pi^{1/2}2^{\mu+1/2}}P_{\nu-1/2}^{\mu+1/2}(a), & a>1 \end{cases}$$ $-1<\operatorname{Re}\mu<1/2$, $\operatorname{Re}\nu>-1$, $\operatorname{Re}(\mu+\nu)>-1$
(13)	$$\int_0^\infty x^{1+\mu}Y_\mu(ax)J_\nu(bx)J_\nu(cx)\,dx = 0,$$ $0<b<c$, $0<a<c-b$

(14)	$\int_0^\infty x J_\nu^2(ax) J_\nu(bx) Y_\nu(bx)\,dx = \begin{cases} 0, \\ -(2\pi ab)^{-1}, \end{cases}$	$0<a<b$ $0<b<a$ $\operatorname{Re}\nu > -1/2$
(15)	$\int_0^\infty x^{2\nu+1} J_\nu(ax) Y_\nu(ax) J_\nu(bx) Y_\nu(bx)\,dx = \frac{a^{2\nu}\,\Gamma(3\nu+1)}{2\pi b^{4\nu+2}\,\Gamma(1/2-\nu)\,\Gamma(2\nu+3/2)}\,{}_2F_1\left(\nu+\frac{1}{2},\ 3\nu+1;\ 2\nu+\frac{3}{2};\ \frac{a^2}{b^2}\right),$	$0<a<b,\ -1/3<\operatorname{Re}\nu<1/2$
(16)	$\int_0^\infty \frac{J_\nu(ax)Y_\nu(bx)-J_\nu(bx)Y_\nu(ax)}{x\{[J_\nu(bx)]^2+[Y_\nu(bx)]^2\}}\,dx = -\frac{\pi}{2}\left(\frac{b}{a}\right)^\nu,$	$0<b<a$

Относительно других интегралов такого вида см. п. 6.8.

(17)	$\int_0^\infty \frac{[J_\nu(ax)Y_\nu(bx)-J_\nu(bx)Y_\nu(ax)]}{[J_\nu(bx)]^2+[Y_\nu(bx)]^2}\times [J_{\nu+1}(cx)Y_\nu(bx)-J_\nu(bx)Y_{\nu+1}(cx)]\,dx = \begin{cases} -\frac{b^{2\nu}}{a^\nu c^{\nu+1}}, \\ \frac{a^\nu}{c^{\nu+1}}-\frac{b^{2\nu}}{a^\nu c^{\nu+1}}, \end{cases}$	$0<b<c<a$ $0<b<a<c$
(18)	$\int_0^\infty \frac{[J_\nu(ax)Y_\nu(bx)-J_\nu(bx)Y_\nu(ax)]}{[J_\nu(bx)]^2+[Y_\nu(bx)]^2}\times [J_{\nu+1}(ax)Y_\nu(bx)-J_\nu(bx)Y_{\nu+1}(ax)]\,dx = \frac{1}{2a}-\frac{b^{2\nu}}{a^{2\nu+1}},$	$0<b<a$
(19)	$\int_0^\infty \frac{J_0(ax)Y_0(bx)-J_0(bx)Y_0(ax)}{[J_0(bx)]^2+[Y_0(bx)]^2}\,\frac{x\,dx}{\lambda^2+x^2} = -\frac{\pi}{2}\,\frac{K_0(a\lambda)}{K_0(b\lambda)},$	$\operatorname{Re}\lambda>0,\ 0<b<a$
(20)	$\int_0^\infty \frac{J_0(ax)Y_0(bx)-J_0(bx)Y_0(ax)}{[J_0(bx)]^2+[Y_0(bx)]^2}\,\frac{x\,dx}{c^2-x^2} = \frac{\pi}{2}\,\frac{J_0(ac)J_0(bc)+Y_0(ac)Y_0(bc)}{[J_0(bc)]^2+[Y_0(bc)]^2},$	$0<b<a,\quad c>0$

(21)	$\int\limits_0^a J_\nu(x)\,J_{-\nu}(a-x)\,dx = \sin a,$	$-1<\operatorname{Re}\nu<1$
(22)	$\int\limits_0^a J_\nu(x)\,J_{1-\nu}(a-x)\,dx = J_0(a)-\cos a,$	$-1<\operatorname{Re}\nu<2$
(23)	$\int\limits_0^a J_\mu(x)\,J_\nu(a-x)\,dx = 2\sum\limits_{m=0}^{\infty}(-1)^m J_{\mu+\nu+2m+1}(a),$	$\operatorname{Re}\mu>-1,\quad \operatorname{Re}\nu>-1$
(24)	$\int\limits_0^a x^{-1} J_\mu(x)\,J_\nu(a-x)\,dx = \mu^{-1}J_{\mu+\nu}(a),$	$\operatorname{Re}\mu>0,\quad \operatorname{Re}\nu>-1$
(25)	$\int\limits_0^a x^{\lambda-1} J_\mu(x)\,J_\nu(a-x)\,dx = 2^\lambda\sum\limits_{m=0}^{\infty}\frac{(-1)^m\,\Gamma(\lambda+\mu+m)\,(\lambda)_m}{m!\,\Gamma(\mu+m+1)}J_{\lambda+\mu+\nu+2m}(a),$	$\operatorname{Re}(\lambda+\mu)>0,\quad \operatorname{Re}\nu>-1$
(26)	$\int\limits_0^a x^{-1}(a-x)^{-1} J_\mu(x)\,J_\nu(a-x)\,dx = (\mu^{-1}+\nu^{-1})\,a^{-1}J_{\mu+\nu}(a),$	$\operatorname{Re}\mu>0,\ \operatorname{Re}\nu>0$
(27)	$\int\limits_0^a x^{\lambda-1}(a-x)^{-1} J_\mu(x)\,J_\nu(a-x)\,dx = \frac{2^\lambda}{\nu a}\sum\limits_{m=0}^{\infty}\frac{(-1)^m\,\Gamma(\lambda+\mu+m)\,(\lambda)_m}{m!\,\Gamma(\mu+m+1)}(\lambda+\mu+\nu+2m)\,J_{\lambda+\mu+\nu+2m}(a),$	$\operatorname{Re}(\lambda+\mu)>0,\ \operatorname{Re}\nu>0$
(28)	$\int\limits_0^a x^\mu(a-x)^\nu J_\mu(x)\,J_\nu(a-x)\,dx = \frac{\Gamma(\mu+1/2)\,\Gamma(\nu+1/2)}{2^{1/2}\pi^{1/2}\,\Gamma(\mu+\nu+1)}a^{\mu+\nu+1/2}J_{\mu+\nu+1/2}(a),$	$\operatorname{Re}\mu>-1/2,\quad \operatorname{Re}\nu>-1/2$
(29)	$\int\limits_0^a x^\mu(a-x)^{\nu+1} J_\mu(x)\,J_\nu(a-x)\,dx = \frac{\Gamma(\mu+1/2)\,\Gamma(\nu+3/2)}{2^{1/2}\pi^{1/2}\,\Gamma(\mu+\nu+2)}a^{\mu+\nu+3/2}J_{\mu+\nu+1/2}(a),$	$\operatorname{Re}\nu>-1,\quad \operatorname{Re}\mu>-1/2$

(30)	$\int\limits_0^a x^{\mu}(a-x)^{-\mu-1} J_{\mu}(x) J_{\nu}(a-x)\,dx = \frac{2^{\mu}\,\Gamma(\mu+1/2)\,\Gamma(\nu-\mu)}{\pi^{1/2}\,\Gamma(\mu+\nu+1)}\,a^{\mu} J_{\nu}(a),$ $\operatorname{Re}\nu > \operatorname{Re}\mu > -1/2$
(31)	$\int\limits_0^a x^{\rho-1}(a-x)^{\sigma-1} J_{\mu}(x) J_{\nu}(a-x)\,dx,$ $\int\limits_0^a x^{\rho-1}(a-x)^{\sigma-1} J_{\lambda}(bx) J_{\mu}(cx) J_{\nu}(a-x)\,dx.$ Относительно этих интегралов и их частных случаев см. Bailey W. N., 1930: Proc. London Math. Soc. (2), 30, 422—424 и 31, 200—208; Rutgers J. G., 1931: Nederl. Akad. Wetensch., Proc. 44, 75—85.
(32)	$\int\limits_{-\infty}^{\infty} \frac{J_{\mu}[a(x+y)]}{(x+y)^{\mu}}\,\frac{J_{\nu}[a(x+z)]}{(x+z)^{\nu}}\,dx = \frac{(2\pi/a)^{1/2}\,\Gamma(\mu+\nu)}{\Gamma(\mu+1/2)\,\Gamma(\nu+1/2)}\,\frac{J_{\mu+\nu-1/2}[a(y-z)]}{(y-z)^{\mu+\nu-1/2}},$ $a>0,\ \operatorname{Re}(\mu+\nu)>0$
(33)	$\int\limits_0^{\infty} x^2 J_{2\nu}(2ax) J_{\nu-1/2}(x^2)\,dx = 2^{-1} a J_{\nu+1/2}(a^2),$ $a>0,\ \operatorname{Re}\nu > -1/2$
(34)	$\int\limits_0^{\infty} x^2 J_{2\nu}(2ax) J_{\nu+1/2}(x^2)\,dx = 2^{-1} a J_{\nu-1/2}(a^2),$ $a>0,\ \operatorname{Re}\nu > -1$
(35)	$\int\limits_0^{\infty} x^2 J_{2\nu}(2ax) Y_{\nu+1/2}(x^2)\,dx = -2^{-1} a\, \mathbf{H}_{\nu-1/2}(a^2),$ $a>0,\ \operatorname{Re}\nu > -1$
(36)	$\int\limits_0^{\infty} x J_0(4ax)\,[J_0(x^2)]^2\,dx = -2^{-2} J_0(a^2)\,Y_0(a^2),$ $\quad a>0$
(37)	$\int\limits_0^{\infty} x J_0(4ax)\,J_0(x^2)\,Y_0(x^2)\,dx = -2^{-2}\,[J_0(a^2)]^2,$ $\quad a>0$

(38)	$\displaystyle\int_0^\infty x^{(\nu+2)/3}\sin(x^2)\,J_{(\nu+1/2)/3}(x^2)\,J_\nu(4ax)\,dx=$ $\displaystyle=-\frac{a^{(\nu-1)/3}}{8}\left[J_{(\nu+1/2)/3}(a^2)\sin\left(a^2+\frac{\nu-1}{6}\pi\right)+\right.$ $\displaystyle\left.+Y_{(\nu+1/2)/3}(a^2)\cos\left(a^2+\frac{\nu-1}{4}\pi\right)\right],\qquad -5/2>\operatorname{Re}\nu<-1/2$
(39)	$\displaystyle\int_0^\infty x^{(\nu+2)/3}\cos(x^2)\,J_{(\nu+1/2)/3}(x^2)\,J_\nu(2ax)\,dx=$ $\displaystyle=\frac{a^{(\nu-1)/3}}{8}\left[J_{(\nu+1/2)/3}(a^2)\cos\left(a^2+\frac{\nu-1}{6}\pi\right)+\right.$ $\displaystyle\left.+Y_{(\nu+1/2)/3}(a^2)\sin\left(a^2+\frac{\nu-1}{6}\pi\right)\right],\qquad -1<\operatorname{Re}\nu<11/2$
(40)	$\displaystyle\int_0^\infty x\sin(ax^2)\,J_\nu(bx^2)\,J_{2\nu}(2cx)\,dx=$ $\displaystyle=\begin{cases}2^{-1}h\sin(ac^2h)\,J_\nu(bc^2h), & 0<a<b\\ 2^{-1}k\cos(ac^2k)\,J_\nu(bc^2k), & 0<b<a\end{cases}$ $\operatorname{Re}\nu>-1,\ h=(b^2-a^2)^{-1/2},\ k=(a^2-b^2)^{-1/2}$
(41)	$\displaystyle\int_0^\infty x\cos(ax^2)\,J_\nu(bx^2)\,J_{2\nu}(2cx)\,dx=$ $\displaystyle=\begin{cases}2^{-1}h\cos(ac^2h)\,J_\nu(bc^2h), & 0<a<b\\ 2^{-1}k\sin(ac^2k)\,J_\nu(bc^2k), & 0<b<a\end{cases}$ $\operatorname{Re}\nu>-1/2,\ h=(b^2-a^2)^{-1/2},\ k=(a^2-b^2)^{-1/2}$
(42)	$\displaystyle\int_0^a\frac{x^2}{a^2-x^2}\,J_{1/4}(x)\,J_{-1/4}(x)\,J_{2\nu}(2a^2-2x^2)\,dx=$ $\displaystyle=\frac{a}{4\nu}\,J_{\nu+1/4}(a^2)\,J_{\nu-1/4}(a^2),\qquad \operatorname{Re}\nu>0$
(43)	$\displaystyle\int_0^\infty J_\nu\left(\frac{x}{a}\right)J_{\nu+1}\left(\frac{b}{x}\right)\frac{dx}{x}=\left(\frac{a}{b}\right)^{1/2}J_{2\nu+1}\left(\frac{2b^{1/2}}{a^{1/2}}\right),\qquad a,b>0,\ \operatorname{Re}\nu>-1$
(44)	$\displaystyle\int_0^\infty x^{\rho-1}J_\mu(ax)\,J_\nu(bx^{-1})\,dx=$ $\displaystyle=2^{\rho-1}a^{-\rho}\,G_{04}^{20}\left(\frac{a^2b^2}{16}\,\middle\vert\,\frac{\nu}{2},\,\frac{\rho+\mu}{2},\,\frac{\rho-\mu}{2},\,-\frac{\nu}{2}\right),$ $a,b>0,\ \operatorname{Re}(\rho-\nu)<3/2,\ \operatorname{Re}(\rho+\mu)>-3/2$

(45)	$\int\limits_0^\infty J_\nu\left(\frac{a}{x}\right)Y_\nu\left(\frac{x}{b}\right)dx = b\left[\frac{2}{\pi}K_{2\nu}\left(\frac{2a^{1/2}}{b^{1/2}}\right)+Y_{2\nu}\left(\frac{2a^{1/2}}{b^{1/2}}\right)\right],$ $a,b>0,\ -1/2<\operatorname{Re}\nu<1/2$
(46)	$\int\limits_0^\infty J_\nu\left(\frac{a}{x}\right)Y_\nu\left(\frac{x}{b}\right)\frac{dx}{x^2} = -\frac{1}{a}\left[\frac{2}{\pi}K_{2\nu}\left(\frac{2a^{1/2}}{b^{1/2}}\right)-Y_{2\nu}\left(\frac{2a^{1/2}}{b^{1/2}}\right)\right],$ $a,b>0,\ -1/2<\operatorname{Re}\nu<1/2$
(47)	$\int\limits_0^\infty Y_\nu\left(\frac{a}{x}\right)Y_\nu\left(\frac{x}{b}\right)dx = -b\,J_{2\nu}\left(\frac{2a^{1/2}}{b^{1/2}}\right),$ $a,b>0,\ -1/2<\operatorname{Re}\nu<1/2$

19.4. Функции Бесселя других аргументов

(1)	$\int\limits_0^\infty x^{\nu+1}y^{\mu}J_\nu(ax)J_\mu(by)\,dx = \begin{cases} a^\nu b^\mu(\beta/h)^{\mu+\nu+1}[\sin(\nu\pi)Y_{\mu+\nu+1}(\beta h) - \cos(\nu\pi)J_{\mu+\nu+1}(\beta h)], & 0<a<b \\ -2\pi^{-1}a^\nu b^\mu(\beta/k)^{\mu+\nu+1}\sin(\mu\pi)K_{\mu+\nu+1}(\beta k), & 0<b<a \end{cases}$ $a,b>0,\ \operatorname{Re}\beta>0,\ \operatorname{Re}(\mu+\nu)<0,\ \operatorname{Re}\nu>-1$
(2)	$\int\limits_0^\infty x^{\nu+1}y^{\mu}J_\nu(ax)Y_\mu(by)\,dx = \begin{cases} -a^\nu b^\mu(\beta/h)^{\mu+\nu+1}[\sin(\nu\pi)J_{\mu+\nu+1}(\beta h)+\cos(\nu\pi)Y_{\mu+\nu+1}(\beta h)], & 0<a<b \\ -2\pi^{-1}a^\nu b^\mu(\beta/k)^{\mu+\nu+1}\cos(\mu\pi)K_{\mu+\nu+1}(\beta k), & 0<b<a \end{cases}$ $a,b>0,\ \operatorname{Re}\beta>0,\ \operatorname{Re}(\mu+\nu)<0,\ \operatorname{Re}\nu>-1$
(3)	$\int\limits_0^\infty x^{\nu+1}y^{-\mu}J_\nu(ax)Y_\mu(by)\,dx = \begin{cases} a^\nu b^{-\mu}(\beta/h)^{\nu-\mu+1}Y_{\mu-\nu-1}(\beta h), & 0<a<b \\ -2\pi^{-1}a^\nu b^{-\mu}(\beta/k)^{\nu-\mu+1}K_{\mu-\nu-1}(\beta k), & 0<b<a \end{cases}$ $a,b>0,\ \operatorname{Re}\beta>0,\ \operatorname{Re}\mu>\operatorname{Re}\nu>-1$

$y=(x^2+\beta^2)^{1/2},\ h=(b^2-a^2)^{1/2},\ k=(a^2-b^2)^{1/2}$

(4)	$\int\limits_0^\infty x^{\rho-1} y^{-\mu} (x^2+\lambda^2)^{-1} \left[\cos\left(\frac{\rho-\nu}{2}\pi\right) J_\nu(ax) + \sin\left(\frac{\rho-\nu}{2}\pi\right) Y_\nu(ax)\right] J_\mu(by)\,dx = -\lambda^{\rho-2} \frac{J_\mu\left[b(\beta^2-\lambda^2)^{1/2}\right]}{(\beta^2-\lambda^2)^{\mu/2}} K_\nu(a\lambda),\quad 0<b<a$ $\lvert\operatorname{Re}\nu\rvert<\operatorname{Re}\rho<\operatorname{Re}\mu+4,\ \operatorname{Re}\lambda>0$
(5)	$\int\limits_{-\infty}^\infty \frac{\sin[a(x+\lambda)]}{x+\lambda} \frac{J_\nu(by)}{y^\nu}\,dx = \pi \frac{J_\nu\left[b(\beta^2+\lambda^2)^{1/2}\right]}{(\beta^2+\lambda^2)^{\nu/2}},\quad 0<b<a,\ \operatorname{Re}\nu>-{}^3/_2$
(6)	$\int\limits_{-\infty}^\infty \frac{\sin[a(x-z)]}{x-z} \frac{J_\nu\left[a(x^2-2bx\cos\theta+b^2)^{1/2}\right]}{(x^2-2bx\cos\theta+b^2)^{\nu/2}}\,dx = \pi \frac{J_\nu\left[a(z^2-2bz\cos\theta+b^2)^{1/2}\right]}{(z^2-2bz\cos\theta+b^2)^{\nu/2}},\quad \operatorname{Re}\nu>-{}^1/_2$
(7)	$\int\limits_0^\infty x^{\rho-1}\left(\frac{a+bx}{ax+b}\right)^\nu J_{2\nu}\left[x^{-1/2}(a+bx)^{1/2}(ax+b)^{1/2}\right]dx = -\pi\left[J_{\nu+\rho}(a)Y_{\nu-\rho}(b)+J_{\nu-\rho}(b)Y_{\nu+\rho}(a)\right],$ $a,\ b>0,\ -{}^3/_4<\operatorname{Re}\rho<{}^3/_4$
(8)	$\int\limits_0^\infty x^{\rho-1}\left(\frac{a+bx}{ax+b}\right)^\nu Y_{2\nu}\left[x^{-1/2}(a+bx)^{1/2}(ax+b)^{1/2}\right]dx = \pi\left[J_{\nu+\rho}(a)J_{\nu-\rho}(b)-Y_{\nu+\rho}(a)Y_{\nu-\rho}(b)\right],$ $a,\ b>0,\ -{}^3/_4<\operatorname{Re}\nu<{}^3/_4$
(9)	$\int\limits_0^\infty x^{\rho-1}\left(\frac{a+bx}{ax+b}\right)^\nu H_\nu^{(2)}\left[x^{-1/2}(a+bx)^{1/2}(ax+b)^{1/2}\right]dx = -i\pi H_{\nu+\rho}^{(2)}(a) H_{\nu-\rho}^{(2)}(b),\quad a,\ b>0,\ -{}^3/_4<\operatorname{Re}\nu<{}^3/_4$
(10)	$\int\limits_0^\infty \operatorname{ch} x\cos(2a\operatorname{sh}x) J_\nu(be^x) J_\nu(be^{-x})\,dx = \begin{cases} 2^{-1}(b^2-a^2)^{-1/2} J_{2\nu}\left[2(b^2-a^2)^{1/2}\right], & 0<a<b \\ 0, & 0<b<a \end{cases}$ $\operatorname{Re}\nu>-1$

$y=(x^2+\beta^2)^{1/2}$

(11)	$\int_0^\infty \operatorname{ch} x \cos(2a \operatorname{sh} x)\, Y_\nu(be^x)\, Y_\nu(be^{-x})\, dx =$ $= \begin{cases} -2^{-1}(b^2-a^2)^{-1/2} J_{2\nu}[2(b^2-a^2)^{1/2}], & 0<a<b \\ 2\pi^{-1}\cos(\nu\pi)(a^2-b^2)^{-1/2} K_{2\nu}[2(a^2-b^2)^{1/2}], & 0<b<a \end{cases}$ $-1<\operatorname{Re}\nu<1$
(12)	$\int_0^\infty \operatorname{ch} x \sin(2a \operatorname{sh} x)\,[J_\nu(be^x)\, Y_\nu(be^{-x}) - Y_\nu(be^x)\, J_\nu(be^{-x})]\, dx =$ $= \begin{cases} 0, & 0<a<b \\ -2\pi^{-1}\cos(\nu\pi)(a^2-b^2)^{-1/2} K_{2\nu}[2(a^2-b^2)^{1/2}], & 0<b<a \end{cases}$ $-1/2<\operatorname{Re}\nu<1/2$
(13)	$\int_0^\pi \sin(2\mu x)\, J_{2\nu}(2\alpha \sin x)\, dx = \pi \sin(\mu\pi)\, J_{\nu-\mu}(\alpha)\, J_{\nu+\mu}(\alpha), \quad \operatorname{Re}\nu > -1$
(14)	$\int_0^\pi \cos(2\mu x)\, J_{2\nu}(2\alpha \sin x)\, dx = \pi \cos(\mu\pi)\, J_{\nu-\mu}(\alpha)\, J_{\nu+\mu}(\alpha), \quad \operatorname{Re}\nu > -1/2$
(15)	$\int_0^{\pi/2} \cos(2nx)\, J_0(2\alpha \sin x)\, dx = 2^{-1}\pi\,[J_n(\alpha)]^2, \quad n=0, 1, 2, \ldots$
(16)	$\int_0^{\pi/2} \cos(2nx)\, Y_0(2\alpha \sin x)\, dx = 2^{-1}\pi\, J_n(\alpha)\, Y_n(\alpha), \quad n=0, 1, 2, \ldots$
(17)	$\int_0^\pi [\operatorname{tg}(x/2)]^{-2\varkappa} e^{-\beta\cos x} J_{2\nu}(\alpha \sin x)\, dx =$ $= \frac{\Gamma(1/2+\varkappa+\nu)\,\Gamma(1/2-\varkappa+\nu)}{\alpha\,[\Gamma(2\nu+1)]^2} M_{\varkappa,\nu}\left[\beta+(\beta^2-\alpha^2)^{1/2}\right] M_{\varkappa,\nu}\left[\beta-(\beta^2-\alpha^2)^{1/2}\right],$ $\operatorname{Re}\nu + 1/2 > \lvert \operatorname{Re}\varkappa \rvert$
(18)	$\int_0^{\pi/2} \cos(2\beta \cos x)\, J_{2\nu}(2\alpha \sin x)\, dx =$ $= 2^{-1}\pi\, J_\nu\left[(\beta^2+\alpha^2)^{1/2}+\beta\right] J_\nu\left[(\beta^2+\alpha^2)^{1/2}-\beta\right], \quad \operatorname{Re}\nu > -1/2$
(19)	$\int_0^{\pi/2} (\sin x)^{\nu+1} \cos(\beta\cos x)\, J_\nu(\alpha \sin x)\, dx =$ $= 2^{-1/2}\pi^{1/2}\alpha^\nu(\alpha^2+\beta^2)^{-\nu/2-1/4} J_{\nu+1/2}\left[(\alpha^2+\beta^2)^{1/2}\right], \quad \operatorname{Re}\nu > -1$

(20)	$\int\limits_0^{\pi/2} \sin(2x)\, P_n(\cos 2x)\, J_0(\alpha \sin x)\, dx = 2\alpha^{-1} J_{2n+1}(\alpha), \qquad n = 0, 1, 2, \ldots$
(21)	$\int\limits_0^{\pi/2} (\sin x)^{\nu+1} \cos(\alpha \cos\theta \cos x)\, C_{2n}^{\nu+1/2}(\cos x)\, J_\nu(\alpha \sin\theta \sin x)\, dx =$ $= (-1)^n 2^{-1/2} \pi^{1/2} (\sin\theta)^\nu \alpha^{-1/2} C_{2n}^{\nu+1/2}(\cos\theta)\, J_{\nu+2n+1/2}(\alpha),$ $n = 0, 1, 2, \ldots, \quad \operatorname{Re} \nu > -1$
(22)	$\int\limits_0^{\pi/2} (\sin x)^{\nu+1} \sin(\alpha \cos\theta \cos x)\, C_{2n+1}^{\nu+1/2}(\cos x)\, J_\nu(\alpha \sin\theta \sin x)\, dx =$ $= (-1)^n 2^{-1/2} \pi^{1/2} (\sin\theta)^\nu \alpha^{-1/2} C_{2n+1}^{\nu+1/2}(\cos\theta)\, J_{\nu+2n+3/2}(\alpha),$ $n = 0, 1, 2, \ldots, \quad \operatorname{Re} \nu > -1$
(23)	$\int\limits_0^{\pi/2} \cos(2\mu x)\, J_{2\nu}(2\alpha \cos x)\, dx = 2^{-1}\pi\, J_{\nu+\mu}(\alpha),\ J_{\nu-\mu}(\alpha) \qquad \operatorname{Re} \nu > -1/2$
(24)	$\int\limits_0^{\pi/2} \cos(2\mu x)\, Y_{2\nu}(2\alpha \cos x)\, dx =$ $= 2^{-1}\pi \operatorname{ctg}(2\nu\pi)\, J_{\nu+\mu}(\alpha)\, J_{\nu-\mu}(\alpha) - \frac{\pi J_{\mu-\nu}(\alpha)\, J_{-\mu-\nu}(\alpha)}{2 \sin(2\nu\pi)},$ $-1/2 < \operatorname{Re} \nu < 1/2$
(25)	$\int\limits_0^{\pi/2} (\sin x)^{\mu+1} (\cos x)^{\nu+1} J_\mu(\alpha \sin x)\, J_\nu(\beta \cos x)\, dx =$ $= \alpha^\mu \beta^\nu (\alpha^2+\beta^2)^{-(\mu+\nu+1)/2} J_{\mu+\nu+1}\left[(\alpha^2+\beta^2)^{1/2}\right],$ $\operatorname{Re} \mu > -1, \ \operatorname{Re} \nu > -1$
(26)	$\int\limits_0^{\pi/2} (\sin x)^\rho (\cos x)^\sigma J_\mu(\alpha \sin x)\, J_\nu(\beta \sin x)\, dx,$ $\int\limits_0^{\pi/2} (\sin x)^\rho (\cos x)^\sigma J_\mu(\alpha \sin x)\, J_\nu(\beta \cos x)\, dx.$ См. Bailey W. N., 1938: Quart. J. Math. Oxford Ser. 9, 141—147.
(27)	$\int\limits_0^{\pi} (\sin x)^{2\nu} \frac{J_\nu(\omega)}{\omega^\nu}\, dx = 2^\nu \pi^{1/2}\, \Gamma(\nu + 1/2)\, \frac{J_\nu(\alpha)}{\alpha^\nu}\, \frac{J_\nu(\beta)}{\beta^\nu},$ $\omega = (\alpha^2+\beta^2-2\alpha\beta \cos x)^{1/2}, \ \operatorname{Re} \nu > -1/2$

(28)	$\int\limits_0^\pi (\sin x)^{2\nu} \frac{Y_\nu(\omega)}{\omega^\nu}\,dx = 2^\nu \pi^{1/2}\, \Gamma(\nu+1/2)\, \frac{J_\nu(\alpha)}{\alpha^\nu}\, \frac{Y_\nu(\beta)}{\beta^\nu},$ $\omega = (\alpha^2+\beta^2-2\alpha\beta\cos x)^{1/2},\ \lvert\alpha\rvert<\lvert\beta\rvert,\quad \operatorname{Re}\nu > -1/2$
(29)	$\int\limits_0^\pi (\sin x)^{2\nu} C_n^\nu(\cos x)\, \frac{J_\nu(\omega)}{\omega^\nu}\,dx = \frac{\pi\,\Gamma(2\nu+n)}{2^{\nu-1} n!\,\Gamma(\nu)}\, \frac{J_{\nu+n}(\alpha)}{\alpha^\nu}\, \frac{J_{\nu+n}(\beta)}{\beta^\nu},$ $n=0,\ 1,\ 2,\ \ldots,\ \omega=(\alpha^2+\beta^2-2\alpha\beta\cos x)^{1/2},\ \operatorname{Re}\nu>-1/2$
(30)	$\int\limits_0^\pi (\sin x)^{2\nu} C_n^\nu(\cos x)\, \frac{Y_\nu(\omega)}{\omega^\nu}\,dx = \frac{\pi\,\Gamma(2\nu+n)}{2^{\nu-1} n!\,\Gamma(\nu)}\, \frac{J_{\nu+n}(\alpha)}{\alpha^\nu}\, \frac{Y_{\nu+n}(\beta)}{\beta^\nu},$ $n=0,\ 1,\ 2,\ \ldots,\ \omega=(\alpha^2+\beta^2-2\alpha\beta\cos x)^{1/2},\quad \lvert\alpha\rvert<\lvert\beta\rvert,\ \operatorname{Re}\nu>-1/2$
(31)	$\int\limits_0^\infty e^{-2\mu x}\, Y_{2\nu}(2a\,\operatorname{sh} x)\,dx = \operatorname{ctg}(2\nu\pi)\, I_{\mu+\nu}(a)\, K_{\mu-\nu}(a) - \frac{I_{\mu-\nu}(a)\, K_{\mu+\nu}(a)}{\sin(2\nu\pi)},$ $a>0,\ \operatorname{Re}\mu>-3/2,\quad -1/2<\operatorname{Re}\nu<1/2$
(32)	$\int\limits_0^\infty [\operatorname{cth}(x/2)]^{-2\varkappa} \{\sin[(\mu-\varkappa)\pi]\, J_{2\mu}(a\,\operatorname{sh} x) +$ $+\cos[(\mu-\varkappa)\pi]\, Y_{2\mu}(a\,\operatorname{sh} x)\}\,dx = -a^{-1} W_{\varkappa,\mu}(a)\, W_{-\varkappa,\mu}(a),$ $a>0,\ \operatorname{Re}\varkappa>\lvert\operatorname{Re}\mu\rvert-1/2$
(33)	$\int\limits_0^\infty \operatorname{sh} x\, [\operatorname{th}(x/2)]^\nu e^{-\beta\operatorname{ch} x} J_\nu(\alpha\operatorname{sh} x)\,dx =$ $=(\alpha^2+\beta^2)^{-1/2}\left[\frac{(\alpha^2+\beta^2)^{1/2}+\beta}{(\alpha^2+\beta^2)^{1/2}-\beta}\right]^{-\nu/2} \exp[-(\alpha^2+\beta^2)^{1/2}],$ $\operatorname{Re}\beta>\lvert\operatorname{Re}\alpha\rvert,\ \operatorname{Re}\nu>-1$
(34)	$\int\limits_0^\infty [\operatorname{cth}(x/2)]^{2\varkappa} e^{-\beta\operatorname{ch} x} J_{2\mu}(\alpha\operatorname{sh} x)\,dx =$ $=\frac{\Gamma(1/2-\varkappa+\mu)}{\alpha\,\Gamma(2\mu+1)} M_{-\varkappa,\mu}\left[(\alpha^2+\beta^2)^{1/2}-\beta\right] W_{\varkappa,\mu}\left[(\alpha^2+\beta^2)^{1/2}+\beta\right],$ $\operatorname{Re}\beta>\lvert\operatorname{Re}\alpha\rvert,\ \operatorname{Re}(\mu-\varkappa)>-1/2$
(35)	$\int\limits_0^\infty [\operatorname{cth}(x/2)]^{2\varkappa} e^{-\beta\operatorname{ch} x} Y_{2\mu}(\alpha\operatorname{sh} x)\,dx =$ $=-\frac{W_{\varkappa,\mu}(h)\, W_{-\varkappa,\mu}(k)}{\alpha\cos[(\mu+\varkappa)\pi]} - \frac{\operatorname{tg}[(\mu+\varkappa)\pi]\,\Gamma(1/2-\varkappa+\mu)}{\alpha\,\Gamma(2\mu+1)} W_{\varkappa,\mu}(h)\, M_{-\varkappa,\mu}(k),$ $h=(\alpha^2+\beta^2)^{1/2}+\beta,\quad k=(\alpha^2+\beta^2)^{1/2}-\beta$ $\operatorname{Re}\beta>\lvert\operatorname{Re}\alpha\rvert,\quad \operatorname{Re}\varkappa<1/2-\lvert\operatorname{Re}\mu\rvert$

(36)	$\int_0^\infty (\operatorname{sh} x)^{\mu+1} (\operatorname{ch} x)^{\nu+1} J_\mu (a \operatorname{sh} x) H_\nu^{(2)} (b \operatorname{ch} x)\, dx =$ $= \begin{cases} -e^{-\mu\pi i} a^\mu b^\nu h^{-\mu-\nu-1} H^{(2)}_{\mu+\nu+1}(h), & 0<a<b \\ 2i\pi^{-1} e^{\nu\pi i} a^\mu b^\nu k^{-\mu-\nu-1} K_{\mu+\nu+1}(k), & 0<b<a \end{cases}$ $h=(b^2-a^2)^{1/2}, \quad k=(a^2-b^2)^{1/2}$ $\operatorname{Re}\mu > -1, \ \operatorname{Re}(\mu+\nu)<0$
(37)	$\int_0^\infty (\operatorname{sh} x)^{\mu+1} (\operatorname{ch} x)^{1-\nu} J_\mu (a \operatorname{sh} x) H_\nu^{(2)} (b \operatorname{ch} x)\, dx =$ $= \begin{cases} a^\mu b^{-\nu} h^{\nu-\mu-1} H^{(2)}_{\nu-\mu-1}(h), & 0<a<b \\ 2i\pi^{-1} a^\mu b^{-\nu} k^{\nu-\mu-1} K_{\nu-\mu-1}(k), & 0<b<a \end{cases}$ $h=(b^2-a^2)^{1/2}, \quad k=(a^2-b^2)^{1/2},$ $\operatorname{Re}\nu > \operatorname{Re}\mu > -1$
(38)	$\int_{-\infty}^\infty \frac{\exp(2\varkappa x - \beta \operatorname{th} x)}{\operatorname{ch} x} J_{2\mu}\left(\frac{\alpha}{\operatorname{ch} x}\right) dx =$ $= \frac{\Gamma(1/2+\varkappa+\mu)\,\Gamma(1/2-\varkappa+\mu)}{\alpha\,[\Gamma(2\mu+1)]^2} M_{\varkappa,\mu}(h)\, M_{\varkappa,\mu}(k),$ $h+k=2\beta, \ hk=\alpha^2, \ \operatorname{Re}\mu > \lvert \operatorname{Re}\varkappa \rvert - 1/2$

19.5. Модифицированные функции Бесселя аргумента x

Относительно интегралов, содержащих $\operatorname{ber}_\nu x$, $\operatorname{bei}_\nu x$, $\operatorname{ker}_\nu x$, $\operatorname{kei}_\nu x$ и подобные им функции, см. McLachlan N. W., 1955: Bessel functions for engineers, Oxford, Second edition.

(1)	$\int_0^a I_\nu(x)\, dx = 2 \sum_{n=0}^\infty (-1)^n I_{\nu+2n+1}(a),$	$\operatorname{Re}\nu > -1$
(2)	$\int_0^a x^\nu I_\nu(x)\, dx = 2^{\nu-1}\pi^{1/2}\Gamma(\nu+1/2)\, a\, [I_\nu(a)\mathbf{L}_{\nu-1}(a) - \mathbf{L}_\nu(a) I_{\nu-1}(a)],$	$\operatorname{Re}\nu > -1/2$
(3)	$\int_0^a x^{\nu+1} I_\nu(x)\, dx = a^{\nu+1} I_{\nu+1}(a),$	$\operatorname{Re}\nu > -1$
(4)	$\int_0^a x^{1-\nu} I_\nu(x)\, dx = a^{1-\nu} I_{\nu-1}(a) - \frac{2^{1-\nu}}{\Gamma(\nu)}$	

(5)	$\int_0^a x^{\nu}(a^2-x^2)^{\nu-1/2} I_\nu(x)\,dx = 2^{-\nu-1}\pi^{1/2}a^{2\nu}\,\Gamma(\nu+1/2)\,[I_\nu(a/2)]^2$
(6)	$\int_0^a x^{\nu+1}(a^2-x^2)^{\sigma-1} I_\nu(x)\,dx = 2^{\sigma-1}a^{\nu+\sigma}\,\Gamma(\sigma)\,I_{\nu+\sigma}(a),$ $\operatorname{Re}\nu > -1,\ \operatorname{Re}\sigma > 0$
(7)	$\int_0^a x^{\rho-1}(a^2-x^2)^{\sigma-1} I_\nu(x)\,dx =$ $= \dfrac{\Gamma\left(\frac{\nu+\rho}{2}\right)\Gamma(\sigma)\,a^{\nu+\rho+2\sigma-2}}{2^{\nu+1}\Gamma(\nu+1)\,\Gamma\left(\frac{\nu+\rho}{2}+\sigma\right)}\ {}_1F_2\left(\frac{\nu+\rho}{2};\ \nu+1,\ \frac{\nu+\rho}{2}+\sigma;\ \frac{a^2}{4}\right),$ $\operatorname{Re}(\rho+\nu)>0,\ \operatorname{Re}(\sigma)>0$
(8)	$\int_0^a x^{n+1}\exp(-x^2)\,I_n(2ax)\,dx =$ $= 2^{-2}a^n\left[\exp(a^2)-\exp(-a^2)\sum_{r=-n}^{n} I_r(2a^2)\right],\quad n=0,1,2,\ldots$
(9)	$\int_0^a x^{\nu+1}y^{-1}\cos y\,I_\nu(x)\,dx = \dfrac{\pi^{1/2}a^{2\nu+1}}{2^{\nu+1}\Gamma(\nu+3/2)},\quad y=(a^2-x^2)^{1/2},\ \operatorname{Re}\nu>-1$
(10)	$\int_0^a y^{-1}\operatorname{ch}(y\operatorname{sh}t)\,I_{2\nu}(x)\,dx = 2^{-1}\pi\,I_\nu(2^{-1}ae^{t})\,I_\nu(2^{-1}ae^{-t}),$ $y=(a^2-x^2)^{1/2},\ \operatorname{Re}\nu>-1/2$
(11)	$\int_0^a e^{-x}P_n(1-2xa^{-1})\,I_0(x)\,dx = \dfrac{ae^{-a}}{2n+1}\,[I_n(a)+I_{n+1}(a)]$
(12)	$\int_0^a x^{\mu}e^{-x}P_\nu(1-2x/a)\,I_\mu(x)\,dx.$ См. Bose B. N., 1948: Bull. Calcutta Math. Soc. 40, 8—14.
(13)	$\int_0^\infty x^{-1/3}e^{-x}\sin\left(4ax^{1/2}\right)I_{1/3}(x)\,dx = (2\pi)^{-1/2}a^{1/3}\exp(-a^2)\,K_{1/3}(a^2),\quad a>0$

(14)	$\int\limits_0^\infty x^{-\nu} e^{-x} \sin(4ax^{1/2}) I_\nu(x)\,dx = (2^{3/2}a)^{\nu-1} \exp(-a^2) W_{1/2-3\nu/2,\ 1/2-\nu/2}(2a^2), \quad a>0,\ \operatorname{Re}\nu>0$
(15)	$\int\limits_0^\infty x^{-1/2} e^{-x} \cos(4ax^{1/2}) I_0(x)\,dx = (2\pi)^{-1/2} \exp(-a^2) K_0(a^2), \quad a>0$
(16)	$\int\limits_0^\infty x^{-\nu-1/2} e^{-x} \cos(4ax^{1/2}) I_\nu(x)\,dx = 2^{3\nu/2-1} a^{\nu-1} \exp(-a^2) W_{-3\nu/2,\ \nu/2}(2a^2), \quad a>0,\ \operatorname{Re}\nu>-1/2$
(17)	$\int\limits_a^\infty (x^2-a^2)^{-1/2} T_n(ax^{-1}) K_{2\mu}(x)\,dx = \frac{\pi}{2a} W_{n/2,\ \mu}(a) W_{-n/2,\ \mu}(a), \quad n=0,1,2,\ldots$
(18)	$\int\limits_0^\infty (1+x/\alpha)^{\mu} e^{-x} P_\nu^{-2\mu}(1+2x/\alpha) I_\mu(x)\,dx = 0, \quad -1/2<\operatorname{Re}\mu<0,\ -1/2+\operatorname{Re}\mu<\operatorname{Re}\nu<-1/2-\operatorname{Re}\mu$
(19)	$\int\limits_0^\infty (x+\alpha)^{-\mu} e^{-x} P_\nu^{-2\mu}(1+2x/\alpha) I_\mu(x)\,dx = \frac{2^{\mu-1}\Gamma(\mu+\nu+1/2)\Gamma(\mu-\nu-1/2)}{\pi^{1/2}\Gamma(2\mu+\nu+1)\Gamma(2\mu-\nu)} e^\alpha W_{1/2-\mu,\ 1/2+\nu}(2\alpha), \quad \lvert\arg\alpha\rvert<\pi,\ \operatorname{Re}\mu>\lvert\operatorname{Re}\nu+1/2\rvert$
(20)	$\int\limits_a^\infty x^{1-n} \exp(-x^2) I_n(2ax)\,dx = 2^{-2}a^{-n}\left[\exp(a^2)-\exp(-a^2)\sum_{r=1-n}^{n-1} I_r(2a^2)\right], \quad n=1,2,\ldots$
(21)	$\int\limits_0^a x^\nu K_\nu(x)\,dx = 2^{\nu-1}\pi^{1/2}\Gamma(\nu+1/2)\, a\,[K_\nu(a)\mathbf{L}_{\nu-1}(a)+\mathbf{L}_\nu(a)K_{\nu-1}(a)], \quad \operatorname{Re}\nu>-1/2$
(22)	$\int\limits_0^a x^{\nu+1} K_\nu(x)\,dx = 2^\nu\Gamma(\nu+1)-a^{\nu+1}K_{\nu+1}(a), \quad \operatorname{Re}\nu>-1$

(23)	$\int\limits_0^a x^{1-\nu} K_\nu(x)\,dx = 2^{-\nu}\Gamma(1-\nu) - a^{1-\nu} K_{\nu-1}(a), \qquad \operatorname{Re}\nu < 1$
(24)	$\int\limits_0^a x^{\mu}(a^2-x^2)^{\mu-1/2} K_\mu(x)\,dx = 2^{\mu-1}\pi^{1/2}a^{2\mu}\,\Gamma(\mu+1/2)\, I_\mu(a/2)\, K_\mu(a/2),$ $\operatorname{Re}\mu > -1/2$
(25)	$\int\limits_0^a y^{-1}\operatorname{ch}(y\operatorname{sh}t)\, K_{2\nu}(x)\,dx =$ $= \frac{\pi^2}{4\sin(\nu\pi)}\left[I_{-\nu}(ae^t)\, I_{-\nu}(ae^{-t}) - I_\nu(ae^t)\, I_\nu(ae^{-t})\right],$ $y=(a^2-x^2)^{1/2}, \quad -1/2 < \operatorname{Re}\nu < 1/2$
(26)	$\int\limits_0^a x\, J_\nu(\lambda x)\, K_\nu(\varkappa x)\,dx =$ $= (\varkappa^2+\lambda^2)^{-1}\left[(\lambda/\varkappa)^\nu + \lambda a\, J_{\nu+1}(\lambda a)\, K_\nu(\varkappa a) - \varkappa a\, J_\nu(\lambda a)\, K_{\nu+1}(\varkappa a)\right],$ $\operatorname{Re}\nu > -1$
(27)	$\int\limits_0^a x^{2\nu+1} P_\mu(1-2x^2a^{-2})\, I_\nu(x)\, K_\nu(x)\,dx.$ См. Bose B. N., 1948: Bull. Calcutta Math. Soc. 40, 8—14.
(28)	$\int\limits_0^a x^{2\nu+1} P_n\left[(1-x^2a^{-2})^{1/2}\right] I_\nu(x)\, K_\nu(x)\,dx.$ См. Bose B. N., 1944: Bull. Calcutta Math. Soc. 36, 125—132.
(29)	$\int\limits_0^\infty x^{-1/2}(x+\alpha)^{-1} e^{-x} K_\nu(x)\,dx = \frac{\pi e^\alpha}{\alpha^{1/2}\cos(\nu\pi)} K_\nu(\alpha),$ $\lvert\arg\alpha\rvert < \pi, \quad \lvert\operatorname{Re}\nu\rvert < 1/2$
(30)	$\int\limits_0^\infty x^{-1/4}\exp(-2\alpha x^{1/2})\, K_{1/4}(x)\,dx =$ $= \left(\frac{\alpha\pi}{2}\right)^{1/2} K_{1/4}(\alpha^2) + 2^{-1}\pi^{3/2}\alpha^{1/2}\left[\mathbf{L}_{-1/4}(\alpha^2) - \mathbf{L}_{1/4}(\alpha^2)\right]$
(31)	$\int\limits_0^\infty x^{-1/2} y^{-1} e^{-y} K_\nu(x)\,dx = \frac{\pi^{3/2}\alpha^{-1/2} K_\nu(\alpha)}{\Gamma(3/4+\nu/2)\,\Gamma(3/4-\nu/2)\cos(\nu\pi)},$ $y=(x^2+\alpha^2)^{1/2}, \quad \operatorname{Re}\alpha>0, \quad -1/2<\operatorname{Re}\nu<1/2$

(32)	$\int_0^\infty x^{-1/2} y^{-1} e^{-y} K_\nu(x)\,dx = \dfrac{\pi P_{\nu-1/2}(-\cos\varphi) K_\nu(\alpha)}{\alpha^{1/2}\cos(\nu\pi)}$, $y=(x^2+\alpha^2-2\alpha x\cos\varphi)^{1/2}$, $\lvert\arg\alpha\rvert+\lvert\operatorname{Re}\varphi\rvert<\pi$, $\lvert\operatorname{Re}\nu\rvert<1/2$
(33)	$\int_0^\infty x^{-3/2}(1+\alpha^2/x)^{-1/2}\exp[-(\beta+x)(1+\alpha^2/x)^{1/2}] K_\nu(x)\,dx = 4\alpha^{-1} K_\nu(\beta) K_{2\nu}(2\alpha\beta^{1/2})$, $\operatorname{Re}\alpha>0$, $\operatorname{Re}(\alpha\beta)>0$
(34)	$\int_0^\infty x\sin\left(\frac{a}{2x}\right) K_0(x)\,dx = 2^{-1}\pi a J_1(a^{1/2}) K_1(a^{1/2})$, $a>0$
(35)	$\int_0^\infty x\cos\left(\frac{a}{2x}\right) K_0(x)\,dx = -2^{-1}\pi a Y_1(a^{1/2}) K_1(a^{1/2})$, $a>0$
(36)	$\int_0^\infty x^{-1/3} e^x \sin(4ax^{1/2}) K_{1/3}(x)\,dx = (\pi/2)^{1/2} a^{1/3}\exp(a^2) K_{1/3}(a^2)$, $a>0$
(37)	$\int_0^\infty x^{-1/3} e^{-x}\sin(4ax^{1/2}) K_{1/3}(x)\,dx = 2^{-1/2}\pi^{3/2} a^{1/3}\exp(-a^2) I_{1/3}(a^2)$
(38)	$\int_0^\infty x^{-\nu} e^x \sin(4ax^{1/2}) K_\nu(x)\,dx = (2^{3/2}a)^{\nu-1}\pi\dfrac{\Gamma(3/2-2\nu)}{\Gamma(1/2+\nu)}\exp(a^2) W_{3\nu/2-1/2,\,1/2-\nu/2}(2a^2)$, $a>0$, $0<\operatorname{Re}\nu<3/4$
(39)	$\int_0^\infty x^{\rho-3/2} e^{-x}\sin(4\alpha x^{1/2}) K_\nu(x)\,dx = \dfrac{\pi^{1/2}\alpha\,\Gamma(\rho+\nu)\Gamma(\rho-\nu)}{2^{\rho-2}\Gamma(\rho+1/2)}\,{}_2F_2(\rho+\nu,\ \rho-\nu;\ 3/2,\ \rho+1/2;\ -2\alpha^2)$, $\operatorname{Re}\rho>\lvert\operatorname{Re}\nu\rvert$
(40)	$\int_0^\infty x^{-1/2} e^x \cos(4ax^{1/2}) K_0(x)\,dx = (\pi/2)^{1/2}\exp(a^2) K_0(a^2)$, $a>0$
(41)	$\int_0^\infty x^{-1/2} e^{-x}\cos(4\alpha x^{1/2}) K_0(x)\,dx = 2^{-1/2}\pi^{3/2}\exp(-\alpha^2) I_0(\alpha^2)$

(42)	$\int\limits_0^\infty x^{-\nu-1/2} e^{x} \cos\left(4ax^{1/2}\right) K_\nu(x)\,dx =$ $= 2^{3\nu/2-1}\pi a^{\nu-1}\frac{\Gamma(1/2-2\nu)}{\Gamma(1/2+\nu)}\exp(a^2)\, W_{3\nu/2,\,-\nu/2}(2a^2),$ $a>0,\ -1/2<\operatorname{Re}\nu<1/4$
(43)	$\int\limits_0^\infty x^{\rho-1} e^{-x}\cos\left(4\alpha x^{1/2}\right) K_\nu(x)\,dx =$ $= \frac{\pi^{1/2}\,\Gamma(\rho+\nu)\,\Gamma(\rho-\nu)}{2^{\rho}\,\Gamma(\rho+1/2)}\,{}_2F_2(\rho+\nu,\ \rho-\nu;\ 1/2,\ \rho+1/2;\ -2\alpha^2),\quad \operatorname{Re}\rho>\lvert\operatorname{Re}\nu\rvert$
(44)	$\int\limits_0^\infty x^{n+2\nu-1/2}\exp[-(1+\alpha)x]\, L_n^{2\nu}(\alpha x)\, K_\nu(x)\,dx =$ $= \frac{\pi^{1/2}\,\Gamma(n+\nu+1/2)\,\Gamma(n+3\nu+1/2)}{2^{n+2\nu+1/2}\,n!\,\Gamma(2\nu+1)}\,{}_2F_1(n+\nu+1/2,\ n+3\nu+1/2;\ 2\nu+1;\ -\alpha/2),$ $\operatorname{Re}\alpha>-2,\ \operatorname{Re}(n+\nu)>-1/2,\ \operatorname{Re}(n+3\nu)>-1/2$
(45)	$\int\limits_a^\infty x^{-\nu}(x^2-a^2)^{1/4-\nu/2}\, P_\mu^{\nu-1/2}(2a^2x^{-2}-1)\, K_\nu(x)\,dx =$ $= \pi^{1/2}2^{-\nu}a^{-1/2-\nu}\, W_{\mu+1/2,\,\nu-1/2}(a)\, W_{-\mu-1/2,\,\nu-1/2}(a),\quad \operatorname{Re}\nu<3/2$
(46)	$\int\limits_0^\infty x^{-1}\exp\left(\frac{\alpha^2}{2x}-x\right)\operatorname{Erfc}\left[\frac{\alpha}{(2x)^{1/2}}\right] K_\nu(x)\,dx =$ $= \frac{\pi^{5/2}}{4\cos(\nu\pi)}\{[J_\nu(\alpha)]^2+[Y_\nu(\alpha)]^2\},\quad \operatorname{Re}\alpha>0,\ -1/2<\operatorname{Re}\nu<1/2$
(47)	$\int\limits_0^\infty J_\mu(x)\, K_\nu(x)\, x^{\mu-\nu+1}\,dx = 2^{-1}\,\Gamma(\mu-\nu+1),$ $\operatorname{Re}\mu>-1,\ \operatorname{Re}(\mu-\nu)>-1$
(48)	$\int\limits_0^\infty e^{-2ax} I_0(x)\, K_0(x)\,dx =$ $= \begin{cases} 2^{-1}K\left[(1-a^2)^{1/2}\right], & 0<a<1 \\ (2a)^{-1}K\left[(1-a^{-2})^{1/2}\right], & 1<a<\infty \end{cases}$
(49)	$\int\limits_0^\infty x\exp\left(-\frac{x^2}{2\alpha}\right)[I_\nu(x)+I_{-\nu}(x)]\, K_\nu(x)\,dx = \alpha e^{\alpha} K_\nu(\alpha),$ $\operatorname{Re}\alpha>0,\ -1<\operatorname{Re}\nu<1$

(50)
$$\int_0^\infty x^{\rho-1}\sin(2\alpha x)\,K_\mu(x)\,K_\nu(x)\,dx =$$
$$=\frac{2^{\rho-1}\alpha}{\Gamma(\rho+1)}\Gamma\left(\frac{\rho+\mu+\nu+1}{2}\right)\Gamma\left(\frac{\rho+\mu-\nu+1}{2}\right)\Gamma\left(\frac{\rho-\mu+\nu+1}{2}\right)\Gamma\left(\frac{\rho-\mu-\nu+1}{2}\right)\times$$
$$\times\,{}_4F_3\left(\frac{\rho+\mu+\nu+1}{2},\ \frac{\rho+\mu-\nu+1}{2},\ \frac{\rho-\mu+\nu+1}{2},\ \frac{\rho-\mu-\nu+1}{2};\ \frac{3}{2},\ \frac{\rho+1}{2},\ \frac{\rho}{2}+1;\ -\alpha^2\right),\qquad |\operatorname{Re}\alpha|<1,\ \operatorname{Re}\rho>|\operatorname{Re}\mu|+|\operatorname{Re}\nu|-1$$

(51)
$$\int_0^\infty x^{\rho-1}\cos(2\alpha x)\,K_\mu(x)\,K_\nu(x)\,dx =$$
$$=\frac{2^{\rho-3}}{\Gamma(\rho)}\Gamma\left(\frac{\rho+\mu+\nu}{2}\right)\Gamma\left(\frac{\rho+\mu-\nu}{2}\right)\Gamma\left(\frac{\rho-\mu+\nu}{2}\right)\Gamma\left(\frac{\rho-\mu-\nu}{2}\right)\times$$
$$\times\,{}_4F_3\left(\frac{\rho+\mu+\nu}{2},\ \frac{\rho+\mu-\nu}{2},\ \frac{\rho-\mu+\nu}{2},\ \frac{\rho-\mu-\nu}{2};\ \frac{1}{2},\ \frac{\rho}{2},\ \frac{\rho+1}{2};\ -\alpha^2\right),$$
$$|\operatorname{Re}\alpha|<1,\ \operatorname{Re}\rho>|\operatorname{Re}\mu|+|\operatorname{Re}\nu|$$

(52)
$$\int_0^\infty x^3\cos\left(\frac{x^2}{2a}\right)Y_1(x)\,K_1(x)\,dx = -a^3K_0(a),\qquad a>0$$

19.6. Модифицированные функции Бесселя других аргументов

(1)
$$\int_0^\infty \cos\left(\frac{x^2}{2a}\right)K_{2\nu}\left(x\,e^{i\pi/4}\right)K_{2\nu}\left(x\,e^{-i\pi/4}\right)dx =$$
$$=\frac{\Gamma(1/4+\nu)\,\Gamma(1/4-\nu)}{8a^{1/2}\pi^{-1/2}}\,W_{1/4,\,\nu}\left(a\,e^{i\pi/2}\right)W_{1/4,\,\nu}\left(a\,e^{-i\pi/2}\right),$$
$$a>0,\ -1/4<\operatorname{Re}\nu<1/4$$

(2)
$$\int_0^\infty x^{-1/2}I_\nu(x)\,K_\nu(x)\,K_\mu(2x)\,dx =$$
$$=\frac{\Gamma(1/4+\mu/2)\,\Gamma(1/4-\mu/2)\,\Gamma(1/4+\nu+\mu/2)\,\Gamma(1/4+\nu-\mu/2)}{4\,\Gamma(3/4+\nu+\mu/2)\,\Gamma(3/4+\nu-\mu/2)},$$
$$|\operatorname{Re}\mu|<1/2,\ 2\operatorname{Re}\nu>|\operatorname{Re}\mu|-1/2$$

(3)
$$\int_0^\infty \left[J_0(ax)\,Y_1(bx)+2\pi^{-1}I_0(ax)\,K_1(bx)\right]dx = 0,\qquad 0<a<b$$

(4)
$$\int_0^\infty x^\rho\left[Y_\mu(ax)\pm 2\pi^{-1}K_\mu(ax)\right]\left[Y_\nu(bx)\pm 2\pi^{-1}K_\mu(bx)\right]dx.$$

См. Dixon A. L. and W. L. Ferrar, 1930: Quart. J. Math. Oxford Ser. 1, 122—145.

(5)	$\int\limits_0^\infty \operatorname{sh}(cx) K_1(ax) J_0(bx)\,dx, \quad \int\limits_0^\infty \operatorname{ch}(cx) K_0(ax) J_0(bx)\,dx.$ См. Watson G. N., 1928: J. London Math. Soc. 3, 22—27.
(6)	$\int\limits_0^\infty x J_0(\alpha x) I_0(\beta x) K_0(\gamma x)\,dx = [(\alpha^2+\beta^2+\gamma^2)^2 - 4\beta^2\gamma^2]^{-1/2},$ $\operatorname{Re}\gamma > \lvert \operatorname{Im}\alpha \rvert + \lvert \operatorname{Re}\beta \rvert$
(7)	$\int\limits_0^\infty x J_0(\alpha x) I_1(\beta x) K_1(\gamma x)\,dx =$ $= \frac{1}{2\beta\gamma}\left\{(\alpha^2+\beta^2+\gamma^2)\,[(\alpha^2+\beta^2+\gamma^2)^2 - 4\beta^2\gamma^2]^{1/2} - 1\right\},$ $\operatorname{Re}\gamma > \lvert \operatorname{Im}\alpha \rvert + \lvert \operatorname{Re}\beta \rvert$
(8)	$\int\limits_0^\infty x^{\lambda-1} J_\mu(\alpha x) J_\nu(\beta x) K_\rho(\gamma x)\,dx =$ $= \frac{2^{\lambda-2}\alpha^\mu\beta^\nu\gamma^{-\lambda-\mu-\nu}}{\Gamma(\mu+1)\,\Gamma(\nu+1)}\,\Gamma\left(\frac{\lambda+\mu+\nu-\rho}{2}\right)\Gamma\left(\frac{\lambda+\mu+\nu+\rho}{2}\right)\times$ $\times F_4\left(\frac{\lambda+\mu+\nu-\rho}{2}, \frac{\lambda+\mu+\nu+\rho}{2}; \mu+1, \nu+1; -\frac{\alpha^2}{\gamma^2}, -\frac{\beta^2}{\gamma^2}\right),$ $\operatorname{Re}(\lambda+\mu+\nu) > \lvert \operatorname{Re}\rho \rvert, \ \operatorname{Re}\gamma > \lvert \operatorname{Im}\alpha \rvert + \lvert \operatorname{Im}\beta \rvert$
(9)	$\int\limits_0^\infty x\,[J_0(ax) K_0(bx)]^2\,dx = \frac{\pi}{8ab} - \frac{1}{4ab}\arcsin\left(\frac{b^2-a^2}{b^2+a^2}\right), \qquad a, b > 0$
(10)	$\int\limits_0^\infty J_\nu(ax) J_\nu(bx) K_\nu(ax) K_\nu(bx)\, x^{2\nu+1}\,dx =$ $= \frac{2^{\nu-3}a^{2\nu}\,\Gamma\left(\frac{\nu+1}{2}\right)\Gamma\left(\nu+\frac{1}{2}\right)\Gamma\left(\frac{3\nu+1}{2}\right)}{b^{4\nu+2}\pi^{1/2}\,\Gamma(\nu+1)}\ {}_2F_1\left(\nu+\frac{1}{2}, \frac{3\nu+1}{2}; 2\nu+1; 1-\frac{a^4}{b^4}\right),$ $0 < a < b, \ \operatorname{Re}\nu > -1/3$
Относительно других интегралов такого типа см. п. 6.8.	
(11)	$\int\limits_0^\infty x^{-\mu} e^x P_\nu^{2\mu}(1+2x/\alpha) K_\mu(x+\alpha)\,dx =$ $= \pi^{-1/2}\, 2^{\mu-1}\cos(\mu\pi)\,\Gamma(\mu+\nu+1/2)\,\Gamma(\mu-\nu+1/2)\, W_{1/2-\mu,\,1/2+\nu}(2\alpha),$ $\lvert \arg\alpha \rvert < \pi, \ \operatorname{Re}\mu > \lvert \operatorname{Re}\nu + 1/2 \rvert$

(12)	$\int_0^\infty x^{-\mu/2}(x+a)^{-1/2} e^{-x} \mathrm{P}^{\mu}_{\nu-1/2}\left(\frac{a-x}{a+x}\right) K_\nu(a+x)\,dx = (\pi/2)^{1/2} a^{-\mu/2}\,\Gamma(\mu, 2a), \quad a>0,\ \mathrm{Re}\,\mu<1$
(13)	$\int_0^\infty x^{\mu-1}(x+\beta)^{-\mu} I_\mu(x+\beta) K_\nu(x)\,dx.$ См. MacRobert T. M., 1950: Functions of a complex variable. Macmillan, стр. 379.
(14)	$\int_0^\infty x^{\mu-1}\,\lvert x-b\rvert^{-\mu} K_\mu(\lvert x-b\rvert) K_\nu(x)\,dx = \pi^{-1/2}(2b)^{-\mu}\,\Gamma(1/2-\mu)\,\Gamma(\mu+\nu)\,\Gamma(\mu-\nu) K_\nu(b),$ $b>0,\ \mathrm{Re}\,\mu<1/2,\ \mathrm{Re}\,\mu>\lvert\mathrm{Re}\,\nu\rvert$
(15)	$\int_0^\infty x^{\mu-1}(x+\beta)^{-\mu} K_\mu(x+\beta) K_\nu(x)\,dx = \frac{\pi^{1/2}\,\Gamma(\mu+\nu)\,\Gamma(\mu-\nu)}{2^\mu \beta^\mu\,\Gamma(\mu+1/2)} K_\nu(\beta),$ $\lvert\arg\beta\rvert<\pi,\ \mathrm{Re}\,\mu>\lvert\mathrm{Re}\,\nu\rvert$
(16)	$\int_0^\infty x^{1+2\nu} J_{2\nu-1}(2\alpha x) K_{2\nu-1}(2\alpha x) J_\nu(x^2)\,dx = \pi^{-1/2} 2^{\nu-2} \alpha^{2\nu-1} K_{\nu-1/2}(2\alpha^2),$ $\lvert\arg\alpha\rvert<\pi/4,\ \mathrm{Re}\,\nu>0$
(17)	$\int_0^\infty x^{1-2\nu} J_{2\nu+1}(2\alpha x) K_{2\nu+1}(2\alpha x) J_\nu(x^2)\,dx = \frac{\pi^{1/2} 2^{-\nu-3} \alpha^{-2\nu-1}}{\sin(\nu\pi)}\left[I_{\nu+1/2}(2\alpha^2) - \mathbf{L}_{\nu+1/2}(2\alpha^2)\right],$ $\lvert\arg\alpha\rvert<\pi/4,\ \mathrm{Re}\,\nu>-1$
(18)	$\int_0^\infty x^{1-2\nu} Y_{2\nu+1}(2\alpha x) K_{2\nu+1}(2\alpha x) J_\nu(x^2)\,dx = \pi^{1/2} 2^{-\nu-3} \alpha^{-2\nu-1} \operatorname{ctg}(\nu\pi)\left[I_{\nu+1/2}(2\alpha^2) - \mathbf{L}_{\nu+1/2}(2\alpha^2) + \frac{2 K_{\nu+1/2}(2\alpha^2)}{\pi\cos(\nu\pi)}\right],$ $\lvert\arg\alpha\rvert<\pi/4,\ -1<\mathrm{Re}\,\nu<0$
(19)	$\int_0^\infty x^{3-2\nu} K_{2\nu}(2\alpha x)\left[\cos(\nu\pi) J_{2\nu}(2\alpha x) - \sin(\nu\pi) Y_{2\nu}(2\alpha x)\right] J_\nu(x^2)\,dx = \pi^{-1/2} 2^{-\nu-1} \alpha^{1-2\nu} K_{\nu+1/2}(2\alpha^2),$ $\lvert\arg\alpha\rvert<\pi/4,\ -1<\mathrm{Re}\,\nu<1$

(20)	$\int\limits_0^\infty x J_\nu(x^2)\,[\sin(\nu\pi) J_\nu(x^2) - \cos(\nu\pi) Y_\nu(x^2)]\, J_{4\nu}(4ax)\,dx =$ $= 2^{-2} J_\nu(a^2) J_{-\nu}(a^2), \quad a>0,\ \operatorname{Re}\nu > -1$
(21)	$\int\limits_0^\infty x^{\rho-1} K_\mu(\alpha x) J_\nu(bx^{-1})\,dx = 2^{\rho-2}\alpha^{-\rho} G_{04}^{30}\left(\frac{\alpha^2 b^2}{16}\,\middle\vert\, \frac{\nu}{2}, \frac{\rho+\mu}{2}, \frac{\rho-\mu}{2}, -\frac{\nu}{2}\right),$ $b>0,\ \operatorname{Re}\alpha>0,\ \operatorname{Re}\rho > \vert\operatorname{Re}\mu\vert - {}^3/_2$
(22)	$\int\limits_0^\infty x^{\rho-1} K_\mu(\alpha x) Y_\nu(bx^{-1})\,dx =$ $= (-1)^{m+1} 2^{\rho-2}\alpha^{-\rho} G_{1}^{40}\left(\frac{\alpha^2 b^2}{16}\,\middle\vert\, {\frac{1-\nu}{2} - m \atop \frac{\nu}{2}, -\frac{\nu}{2}, \frac{\rho+\nu}{2}, \frac{\rho-\mu}{2}, \frac{1-\nu}{2} - m}\right),$ m — целое, $\quad b>0,\ \operatorname{Re}\alpha>0,\ \operatorname{Re}\rho > \vert\operatorname{Re}\mu\vert - {}^3/_2$
(23)	$\int\limits_0^\infty K_\nu(\alpha x) K_\nu(\beta x^{-1})\,dx = \pi\alpha^{-1} K_{2\nu}(2\alpha^{1/2}\beta^{1/2}), \qquad \operatorname{Re}\alpha>0,\ \operatorname{Re}\beta>0$
(24)	$\int\limits_0^\infty x^{2\nu-1/2} K_{1/2-\nu}(\alpha x) K_\nu(\beta x^{-1})\,dx =$ $= (2\pi)^{1/2}\alpha^{-\nu-1/2}\beta^\nu K_{2\nu}[(2\alpha\beta)^{1/2} e^{\pi i/4}]\, K_{2\nu}[(2\alpha\beta)^{1/2} e^{-\pi i/4}],$ $\operatorname{Re}\alpha>0,\ \operatorname{Re}\beta>0$
(25)	$\int\limits_0^\infty x^{\rho-1} K_\mu(\alpha x) K_\nu(\beta x^{-1})\,dx =$ $= 2^{\rho-3}\alpha^{-\rho} G_{04}^{40}\left(\frac{\alpha^2\beta^2}{16}\,\middle\vert\, \frac{\nu}{2}, -\frac{\nu}{2}, \frac{\rho+\mu}{2}, \frac{\rho-\mu}{2}\right), \qquad \operatorname{Re}\alpha>0,\ \operatorname{Re}\beta>0$
(26)	$\int\limits_0^\infty x^{-2}[K_\nu(\alpha x)]^2 J_0(bx^{-1})\,dx =$ $= -2\pi b^{-1} K_{2\nu}(2\alpha^{1/2}b^{1/2})\,[\sin(\nu\pi) J_{2\nu}(2\alpha^{1/2}b^{1/2}) + \cos(\nu\pi) Y_{2\nu}(2\alpha^{1/2}b^{1/2})],$ $b>0,\ \operatorname{Re}\alpha>0,\ -{}^1/_4 < \operatorname{Re}\nu < {}^1/_4$
(27)	$\int\limits_0^a x^{\mu+1} y^{-\mu-2} J_\mu(x) I_\nu(y)\,dx = \frac{\Gamma(\nu/2-\mu/2)\,(a/2)^\mu}{\Gamma(\nu/2+\mu/2+1)} J_\nu(a),$ $y=(a^2-x^2)^{1/2},\ \operatorname{Re}\nu > \operatorname{Re}\mu > -1$

(28)	$\int\limits_0^a x^{2\nu} y^{-2\nu-1} J_{2\nu-1}(2x) K_{2\mu}(2y)\,dx = -2^{-2}\Gamma(1/2+\mu-\nu)\Gamma(1/2-\mu-\nu)\,a^{2\nu-1}\times \{\sin[(\mu-\nu)\pi] J_{2\mu}(2a) + \cos[(\mu-\nu)\pi] Y_{2\mu}(2a)\},$ $y=(a^2-x^2)^{1/2},\quad 0<\operatorname{Re}\nu<1/2-\lvert\operatorname{Re}\mu\rvert$
(29)	$\int\limits_0^a x^{1-3\nu} y^{2\nu-1} J_{-3\nu}(2x) I_\nu(y) I_{\nu-1}(y)\,dx = \frac{\Gamma(\nu+1/2)}{2\pi^{1/2}a^{2\nu}} J_\nu(a) J_{-\nu}(a),$ $y=(a^2-x^2)^{1/2},\quad 0<\operatorname{Re}\nu<1/3$
(30)	$\int\limits_0^a x^{1-2\nu} y^{2\nu-3/2} I_{-\nu}(x) K_\nu(x) J_{2\nu-3/2}(2y)\,dx = -2^{-2}\Gamma(1/2-\nu)\,a^{\nu-1} Y_\nu(2a),$ $y=(a^2-x^2)^{1/2},\quad 1/4<\operatorname{Re}\nu<1/2$
(31)	$\int\limits_0^a x J_\lambda(2x) I_\lambda(2x) J_\mu(2y) I_\mu(2y)\,dx = \frac{a^{2\lambda+2\mu+2}}{2\Gamma(\lambda+1)\Gamma(\mu+1)\Gamma(\lambda+\mu+2)}\times {}_1F_4\left(\frac{\lambda+\mu+1}{2};\lambda+1,\ \mu+1,\ \lambda+\mu+1,\ \frac{\lambda+\mu+3}{2};\ -a^4\right),$ $y=(a^2-x^2)^{1/2},\quad \operatorname{Re}\lambda,\ \operatorname{Re}\mu>-1$
(32)	$\int\limits_0^a x^{\varkappa-1/2}(a-x)^{-\varkappa-1/2} e^{-x\operatorname{sh} t} I_{2\mu}[x^{1/2}(a-x)^{1/2}]\,dx = \frac{2\Gamma(1/2+\varkappa+\mu)\Gamma(1/2-\varkappa+\mu)}{a[\Gamma(2\mu+1)]^2} M_{\varkappa,\mu}(2^{-1}ae^{t}) M_{-\varkappa,\mu}(2^{-1}ae^{-t}),$ $\operatorname{Re}\mu>\lvert\operatorname{Re}\varkappa\rvert-1/2$
(33)	$\int\limits_0^\infty x^{\nu-1}(\alpha^2+x^2)^{-\nu/2} K_\nu[(\alpha^2+x^2)^{1/2}] K_\mu(x)\,dx = 2^{\nu-2}\alpha^{-\nu}\Gamma\left(\frac{\nu-\mu}{2}\right)\Gamma\left(\frac{\nu+\mu}{2}\right) K_\mu(\alpha),$ $\operatorname{Re}\nu>\lvert\operatorname{Re}\mu\rvert,\quad \operatorname{Re}\alpha>0$
(34)	$\int\limits_0^\infty x^{\nu-1} y^{-\nu} K_\nu(y) K_\mu(x)\,dx = 2^{-1/2}\pi^{1/2}\beta^{1/2-\nu}c^{-1/2}\Gamma(\nu+\mu)\Gamma(\nu-\mu) P^{1/2-\nu}_{\mu-1/2}(\alpha/c) K_\mu(c),$ $y=[(x+\alpha)^2+\beta^2]^{1/2},\quad c=(\alpha^2+\beta^2)^{1/2}$ $\operatorname{Re}\beta>\lvert\operatorname{Im}\alpha\rvert,\quad \operatorname{Re}\nu>\lvert\operatorname{Re}\mu\rvert$

(35)	$\int_0^\infty x^{-\varkappa-1/2}(\alpha+x)^{\varkappa-1/2} K_{2\mu}[x^{1/2}(\alpha+x)^{1/2}]\,dx =$ $= \alpha^{-1}\Gamma(1/2-\varkappa+\mu)\Gamma(1/2-\varkappa-\mu) W_{\varkappa,\mu}(2^{-1}\alpha e^{i\pi/2}) W_{\varkappa,\mu}(2^{-1}\alpha e^{-i\pi/2})$, $\lvert\arg\alpha\rvert<\pi$, $\operatorname{Re}\varkappa+\lvert\operatorname{Re}\mu\rvert<1/2$
(36)	$\int_0^\infty x^{-1/2}(\alpha+x)^{-1/2} e^{-x\,\mathrm{ch}\,t} K_\nu[x^{1/2}(\alpha+x)^{1/2}]\,dx =$ $= \frac{\exp(2^{-1}\alpha\,\mathrm{ch}\,t)}{2\cos(\nu\pi/2)} K_{\nu/2}(2^{-2}\alpha e^{t}) K_{\nu/2}(2^{-2}\alpha e^{-t})$, $-1<\operatorname{Re}\nu<1$
(37)	$\int_0^\infty x^{-\varkappa-1/2}(\alpha+x)^{\varkappa-1/2}\exp(-\beta x) K_{2\mu}[x^{1/2}(\alpha+x)^{1/2}]\,dx =$ $= \alpha^{-1} e^{\alpha\beta/2}\Gamma(1/2-\varkappa+\mu)\Gamma(1/2-\varkappa-\mu) W_{\varkappa,\mu}(z_1) W_{\varkappa,\mu}(z_2)$, $\lvert\arg\alpha\rvert<\pi$, $\operatorname{Re}\beta>-1$, $\operatorname{Re}\varkappa+\lvert\operatorname{Re}\mu\rvert<1/2$ $z_1,\ z_2 = 2^{-1}\alpha[\beta\pm(\beta^2-1)^{1/2}]$
(38)	$\int_0^\infty x^{\rho-1}\left(\frac{\alpha+\beta x}{\alpha x+\beta}\right)^{\nu} K_{2\nu}[x^{-1/2}(\alpha+\beta x)^{1/2}(\alpha x+\beta)^{1/2}]\,dx = 2K_{\nu+\rho}(\alpha)K_{\nu-\rho}(\beta)$, $\mathrm{R}\,\alpha>0$, $\operatorname{Re}\beta>0$
(39)	$\int_0^{\pi/2}\cos[(\mu-\nu)x]\, I_{\mu+\nu}(2\alpha\cos x)\,dx = 2^{-1}\pi I_\mu(\alpha) I_\nu(\alpha)$, $\operatorname{Re}(\mu+\nu)>-1$
(40)	$\int_0^{\pi/2}\cos[[\mu-\nu)x]\, K_{\mu+\nu}(2\alpha\cos x)\,dx =$ $= \frac{\pi}{2\sin[(\mu+\nu)\pi]}[I_{-\mu}(\alpha) I_{-\nu}(\alpha) - I_\mu(\alpha) I_\nu(\alpha)]$, $-1<\operatorname{Re}(\mu+\nu)<1$
(41)	$\int_0^{\pi/2}\frac{\cos(2\varkappa x)}{\cos x} K_{2\mu}\left(\frac{\alpha}{\cos x}\right)dx = \frac{\pi}{2\alpha} W_{\varkappa,\mu}(\alpha) W_{-\varkappa,\mu}(\alpha)$, $\operatorname{Re}\alpha>0$
(42)	$\int_0^\infty \mathrm{ch}(2\mu x) K_{2\nu}(2\alpha\,\mathrm{ch}\,x)\,dx = 2^{-1} K_{\mu+\nu}(\alpha) K_{\mu-\nu}(\alpha)$, $\operatorname{Re}\alpha>0$

(43)	$\int\limits_0^\infty \frac{\operatorname{ch}(2\varkappa x)}{\operatorname{ch} x} I_{2\mu}\left(\frac{\alpha}{\operatorname{ch} x}\right) dx = \frac{\Gamma(1/2+\varkappa+\mu)\,\Gamma(1/2-\varkappa+\mu)}{2\alpha\,[\Gamma(2\mu+1)]^2} M_{\varkappa,\mu}(\alpha)\, M_{-\varkappa,\mu}(\alpha),$ $\lvert \operatorname{Re}\varkappa \rvert - \operatorname{Re}\mu < 1/2$
(44)	$\int\limits_0^\infty (\operatorname{sh} x)^{\mu+1} (\operatorname{ch} x)^{-2\mu-3/2} P_\nu^{-\mu}[\operatorname{ch}(2x)]\, I_{\mu-1/2}\left(\frac{\alpha}{\operatorname{ch} x}\right) dx =$ $= \frac{2^{\mu-1/2}\,\Gamma(\mu-\nu)\,\Gamma(\mu+\nu+1)}{\pi^{1/2}\alpha^{\mu+3/2}\,[\Gamma(\mu+1)]^2} M_{\nu+1/2,\mu}(\alpha)\, M_{-\nu-1/2,\mu}(\alpha),$ $\operatorname{Re}\mu > \operatorname{Re}\nu,\ \operatorname{Re}\mu > -\operatorname{Re}\nu - 1$
(45)	$\int\limits_{-\infty}^\infty e^{\rho x}\left(\frac{\alpha+\beta e^x}{\alpha e^x+\beta}\right)^\nu K_{2\nu}\left[(\alpha^2+\beta^2+2\alpha\beta\operatorname{ch} x)^{1/2}\right] dx = 2K_{\nu+\rho}(\alpha)\, K_{\nu-\rho}(\beta),$ $\operatorname{Re}\alpha,\ \operatorname{Re}\beta > 0$

19.7. Функции Бесселя и модифицированные функции Бесселя переменного порядка

(1)	$\int\limits_{-\infty}^\infty J_{\nu-x}(\alpha)\, J_{\mu+x}(\alpha)\, dx = J_{\mu+\nu}(2\alpha),$ $\qquad \operatorname{Re}(\mu+\nu) > -1$
(2)	$\int\limits_{-\infty}^\infty a^{-\mu-x} b^{-\nu+x} e^{cxi} J_{\mu+x}(a)\, J_{\nu-x}(b)\, dx =$ $= \begin{cases} \left[\frac{2\cos(c/2)}{a^2 e^{-ci/2}+b^2 e^{ci/2}}\right]^{\mu/2+\nu/2} \exp\left[2^{-1}c(\nu-\mu)i\right] \times \\ \qquad \times J_{\mu+\nu}\left\{\left[2\cos(c/2)\left(a^2 e^{-ci/2}+b^2 e^{ci/2}\right)\right]^{1/2}\right\}, & -\pi < c < \pi \\ 0, & c \geqslant \pi \text{ или } c \leqslant -\pi \end{cases}$ $\operatorname{Re}(\mu+\nu) > 1$
(3)	$\int\limits_{-\infty}^\infty J_{\varkappa+x}(\alpha)\, J_{\lambda-x}(\alpha)\, J_{\mu+x}(\alpha)\, J_{\nu-x}(\alpha)\, dx =$ $= \frac{\Gamma(\varkappa+\lambda+\mu+\nu+1)}{\Gamma(\varkappa+\lambda+1)\,\Gamma(\lambda+\mu+1)\,\Gamma(\mu+\nu+1)\,\Gamma(\nu+\varkappa+1)} \times$ $\times\, {}_4F_5\left(\frac{\varkappa+\lambda+\mu+\nu+1}{2}, \frac{\varkappa+\lambda+\mu+\nu+1}{2}, \frac{\varkappa+\lambda+\mu+\nu}{2}+1, \frac{\varkappa+\lambda+\mu+\nu}{2}+1;\right.$ $\left.\varkappa+\lambda+\mu+\nu+1,\ \varkappa+\lambda+1,\ \lambda+\mu+1,\ \mu+\nu+1,\ \nu+\varkappa+1;\ -4\alpha^2\right),$ $\operatorname{Re}(\varkappa+\lambda+\mu+\nu) > -1$

Относительно аналогичных интегралов см. т. I, стр. 61 и далее и стр. 118.

(4)	$\int\limits_0^\infty J_x(xz)\,J_{-x}(xz)\cos(\pi x)\,dx = 2^{-2}(1-z^2)^{-1/2}$,	$\lvert z\rvert<1$
(5)	$\int\limits_0^\infty [J_x(xz)\,J_{-x}(xz)\cos(\pi x)-1]\,x^{-2}\,dx = -2^{-1}\pi^2$	
(6)	$\int\limits_{-\infty}^\infty \frac{J_{ix}(a)\,dx}{\operatorname{ch}(\pi x/2)} = 2\sin a$,	$a>0$
(7)	$\int\limits_{-\infty}^\infty \frac{J_{ix}(a)\,dx}{\operatorname{sh}(\pi x/2)} = -2i\cos a$,	$a>0$
(8)	$\int\limits_{-\infty}^\infty \frac{e^{\pi x/2}\cos(bx)}{\operatorname{sh}(\pi x)}\,J_{ix}(a)\,dx = -i\exp(ia\operatorname{ch}b)$,	$a,\ b>0$
(9)	$\int\limits_{-\infty}^\infty e^{-cxi}\,[J_{\nu-ix}(a)\,Y_{\nu+ix}(b)+Y_{\nu-ix}(a)J_{\nu+ix}(b)]dx = -2\,(h/k)^{2\nu}\,J_{2\nu}(hk)$, $a,\ b>0$, c — вещественное, $h=\left(ae^{c/2}+be^{-c/2}\right)^{1/2}$, $k=\left(ae^{-c/2}+be^{c/2}\right)^{1/2}$	
(10)	$\int\limits_{-\infty}^\infty e^{-cxi}\,[J_{\nu-ix}(a)\,J_{\nu+ix}(b)-Y_{\nu-ix}(a)\,Y_{\nu+ix}(b)]\,dx = 2\,(h/k)^{2\nu}\,Y_{2\nu}(hk)$, $a,\ b>0$, c — вещественное, $h=\left(ae^{c/2}+be^{-c/2}\right)^{1/2}$, $k=\left(ae^{-c/2}+be^{c/2}\right)^{1/2}$	
(11)	$\int\limits_{-\infty}^\infty e^{-cxi}\,H^{(2)}_{\nu-ix}(a)\,H^{(2)}_{\nu+ix}(b)\,dx = 2i\,(h/k)^{2\nu}\,H^{(2)}_{2\nu}(hk)$, $a,\ b>0$, c — вещественное, $h=\left(ae^{c/2}+be^{-c/2}\right)^{1/2}$, $k=\left(ae^{-c/2}+be^{c/2}\right)^{1/2}$	
(12)	$\int\limits_0^\infty \frac{\{[J_{ix}(a)]^2+[Y_{ix}(a)]^2\}\,dx}{\operatorname{ch}(\pi x)} = -Y_0(2a)+\mathbf{H}_0(2a)$,	$a>0$
(13)	$\int\limits_0^\infty x\,e^{\pi x}\operatorname{th}(\pi x)\,H^{(2)}_{ix}(a)\,H^{(2)}_{ix}(b)\,dx = -\frac{2\,(ab)^{1/2}}{\pi\,(a+b)}\exp[-ik(a+b)]$,	$a,\ b>0$

(14)	$\int\limits_0^\infty x\, e^{\pi x} \operatorname{sh}(\pi x)\, \Gamma(\nu + ix)\, \Gamma(\nu - ix)\, H_{ix}^{(2)}(a)\, H_{ix}^{(2)}(b)\, dx =$ $= i 2^\nu \pi^{1/2}\, \Gamma(1/2 + \nu)\, (ab)^\nu (a+b)^{-\nu} K_\nu(a+b), \quad a,\ b > 0,\ \operatorname{Re}\nu > 0$
(15)	$\int\limits_0^\infty x\, e^{\pi x} \operatorname{sh}(\pi x) \operatorname{ch}(\pi x)\, \Gamma(\nu + ix)\, \Gamma(\nu - ix)\, H_{ix}^{(2)}(a)\, H_{ix}^{(2)}(b)\, dx =$ $= \frac{i\pi^{3/2} 2^\nu}{\Gamma(1/2 - \nu)} (b-a)^{-\nu} H_\nu^{(2)}(b-a), \quad 0 < a < b,\ 0 < \operatorname{Re}\nu < 1/2$
(16)	$\int\limits_0^\infty x\, e^{\pi x} \operatorname{sh}(\pi x)\, \Gamma\left(\frac{\nu + ix}{2}\right) \Gamma\left(\frac{\nu - ix}{2}\right) H_{ix}^{(2)}(a)\, H_{ix}^{(2)}(b)\, dx =$ $= i\pi 2^{2-\nu} (ab)^\nu (a^2 + b^2)^{-\nu/2} H_\nu^{(2)}[(a^2+b^2)^{1/2}], \quad a,\ b > 0,\ \operatorname{Re}\nu > 0$
(17)	$\int\limits_0^\infty x\, e^{\pi x} \operatorname{th}(\pi x)\, \mathrm{P}_{-1/2+ix}(-\cos\varphi)\, H_{ix}^{(2)}(a)\, H_{ix}^{(2)}(b)\, dx = -\frac{2(ab)^{1/2}}{\pi R} e^{-iR},$ $a,\ b > 0,\ 0 < \varphi < \pi,\ R = (a^2 + b^2 - 2ab\cos\varphi)^{1/2}$
(18)	$\int\limits_0^\infty x\, e^{\pi x} \operatorname{sh}(\pi x)\, \Gamma(\nu + ix)\, \Gamma(\nu - ix)\, \mathrm{P}_{-1/2+ix}^{1/2-\nu}(-\cos\varphi) \times$ $\times H_{ix}^{(2)}(a)\, H_{ix}^{(2)}(b)\, dx = i\,(2\pi)^{1/2} (\sin\varphi)^{\nu - 1/2} (ab)^\nu R^{-\nu} H_\nu^{(2)}(R),$ $a,\ b > 0,\ 0 < \varphi < \pi,\ R = (a^2 + b^2 - 2\,ab\cos\varphi)^{1/2},\ \operatorname{Re}\nu > 0$
(19)	$\int\limits_0^\infty \operatorname{ch}(\pi x/2)\, K_{ix}(a)\, dx = \pi/2, \qquad a > 0$
(20)	$\int\limits_0^\infty x \operatorname{sh}(\pi x/2)\, K_{ix}(a)\, dx = \pi a/2, \qquad a > 0$
(21)	$\int\limits_{-\infty}^\infty K_{ix+iy}(\alpha)\, K_{ix+iz}(\beta)\, dx = \pi K_{iy-iz}(\alpha + \beta), \qquad \vert\arg\alpha\vert + \vert\arg\beta\vert < \pi$
(22)	$\int\limits_{-\infty}^\infty e^{\pi x} K_{ix+iy}(a)\, K_{ix+iz}(b)\, dx = \pi e^{-\pi z} K_{iy-iz}(a-b), \qquad a > b > 0$
(23)	$\int\limits_{-\infty}^\infty e^{-i\rho x} K_{\nu+ix}(\alpha)\, K_{\nu-ix}(\beta)\, dx = \pi \left(\frac{\alpha + \beta e^\rho}{\alpha e^\rho + \beta}\right)^\nu K_{2\nu}(w),$ $\vert\arg\alpha\vert + \vert\arg\beta\vert + \vert\operatorname{Im}\rho\vert < \pi, \quad w = (\alpha^2 + \beta^2 + 2\alpha\beta\operatorname{ch}\rho)^{1/2}$

(24)	$\int\limits_{-\infty}^{\infty} \exp[(\pi-\gamma)x]\, K_{ix+iy}(a)\, K_{ix+iz}(b)\, dx = \pi e^{-\beta y-\alpha z} K_{iy-iz}(c),$ где $0<\gamma<\pi$, $a, b, c>0$ и α, β, γ — углы треугольника со сторонами a, b, c
(25)	$\int\limits_{-\infty}^{\infty} (n+\nu+ix)^{-1} \sin[(\nu+ix)\pi]\, K_{\nu+ix}(a)\, K_{\nu-ix}(b)\, dx = \begin{cases} \pi^2 I_n(a)\, K_{n+2\nu}(b), & 0<a<b \\ \pi^2 K_{n+2\nu}(a)\, I_n(b), & 0<b<a \end{cases}$ $n=0, 1, 2, \ldots$
(26)	$\int\limits_{0}^{\infty} \sin(bx)\,\mathrm{sh}(\pi x)\,[K_{ix}(a)]^2\, dx = 2^{-2}\pi^2 J_0[2a\,\mathrm{sh}(b/2)],$ $a, b>0$
(27)	$\int\limits_{0}^{\infty} \cos(bx)\,\mathrm{ch}(\pi x)\,[K_{ix}(a)]^2\, dx = -2^{-2}\pi^2 Y_0[2a\,\mathrm{sh}(b/2)],$ $a, b>0$
(28)	$\int\limits_{0}^{\infty} \mathrm{ch}(\rho x)\, K_{\nu+ix}(\alpha)\, K_{\nu-ix}(\alpha)\, dx = 2^{-1}\pi K_{2\nu}[2\alpha\cos(\rho/2)],$ $2\lvert\arg\alpha\rvert+\lvert\mathrm{Re}\,\rho\rvert<\pi$
(29)	$\int\limits_{-\infty}^{\infty} (\nu-{}^1/_2+ix)\,\Gamma({}^1/_2-ix)\,\Gamma(2\nu-{}^1/_2+ix)\, P^{1/2-\nu}_{\nu+ix-1}(\cos\varphi)\times I_{\nu-1/2+ix}(a)\, K_{\nu-1/2+ix}(b)\, dx = (2\pi)^{1/2}(\sin\varphi)^{\nu-1/2}(ab/\omega)^{\nu} K_{\nu}(\omega),$ $\omega=(a^2+b^2+2ab\cos\varphi)^{1/2}$

Относительно подобных интегралов см. также главу XII.

19.8. Функции, родственные функциям Бесселя

(1)	$\int\limits_{0}^{\infty} x J_{\nu}(ax)\,[\mathbf{J}_{\nu}(x)-J_{\nu}(x)]\, dx = \frac{\sin(\nu\pi)}{\pi a(a+1)},$ $a>0$, $\mathrm{Re}\,\nu>-1$
(2)	$\int\limits_{0}^{\infty} x^{-\nu-1}\,\mathbf{H}_{\nu}(x)\, dx = \frac{2^{-\nu-1}\pi}{\Gamma(\nu+1)},$ $\mathrm{Re}\,\nu>-{}^3/_2$

(3)	$\int\limits_{-\infty}^{\infty} x^{-\nu-1} \frac{\sin[a(x+\lambda)]}{x+\lambda} \mathbf{H}_\nu(x)\,dx = \pi\lambda^{-\nu-1}\mathbf{H}_\nu(\lambda), \qquad a \geqslant 1, \quad \operatorname{Re}\nu > {}^5/_2$
(4)	$\int\limits_0^{\infty} x^{1-\mu-\nu} J_\nu(x)\,\mathbf{H}_\mu(x)\,dx = \frac{(2\nu-1)\,2^{-\mu-\nu}}{(\mu+\nu-1)\,\Gamma(\mu+{}^1/_2)\,\Gamma(\nu+{}^1/_2)},$ $\operatorname{Re}\nu > {}^1/_2, \quad \operatorname{Re}(\mu+\nu) > 1$
(5)	$\int\limits_0^{\infty} [\cos(\nu\pi/2)\,J_\nu(x) + \sin(\nu\pi/2)\,\mathbf{H}_\nu(x)]\frac{dx}{x^2+\alpha^2} = \frac{\pi}{2\alpha}[I_\nu(\alpha) - \mathbf{L}_\nu(\alpha)],$ $\operatorname{Re}\alpha > 0, \quad -{}^1/_2 < \operatorname{Re}\nu < 2$
(6)	$\int\limits_a^{\infty} x^{1/2}(x^2-a^2)^{-1/4-\nu/2} P_\mu^{\nu+1/2}(2x^2a^{-2}-1)\,[\mathbf{H}_\nu(x) - Y_\nu(x)]\,dx =$ $= \frac{\pi^{1/2}a\cos(\nu\pi)}{2^{\nu+2}\sin(\mu\pi)}\{[Y_\nu(a/2)]^2 - [J_\nu(a/2)]^2\}, \qquad -1 < \operatorname{Re}\mu < 0, \quad \operatorname{Re}\nu < {}^1/_2$
(7)	$\int\limits_0^{\infty} x^{\rho-1} K_\mu(\alpha x)\,K_\nu(\alpha x)\,\mathbf{H}_\lambda(x)\,dx$ См. Mohan B., 1942: Bull. Calcutta Math. Soc. 34, 55—59.

Относительно других интегралов, содержащих функции Бесселя и функции Струве, см. McLachlan N. W. and A. L. Meyers, 1936: Philos. Mag. 21, 425—448.

(8)	$\int\limits_0^{\infty} x^{-\mu-\nu}\,\mathbf{H}_\mu(x)\,\mathbf{H}_\nu(x)\,dx = \frac{2^{-\mu-\nu}\pi^{1/2}\,\Gamma(\mu+\nu)}{\Gamma(\mu+{}^1/_2)\,\Gamma(\nu+{}^1/_2)\,\Gamma(\mu+\nu+{}^1/_2)},$ $\operatorname{Re}(\mu+\nu) > 0$
(9)	$\int\limits_0^{\pi/2} \cos[(\nu+1)x]\,\mathbf{H}_\nu(\alpha\cos x)\,dx = \pi^{1/2}\alpha^{-1/2}\sin(\alpha/2)\,J_{\nu+1/2}(\alpha/2),$ $\operatorname{Re}\nu > -2$
(10)	$\int\limits_0^{\pi/2} \frac{\cos(2\nu x)}{\cos x}\left[\mathbf{H}_{2\nu}\left(\frac{\alpha}{\cos x}\right) - Y_{2\nu}\left(\frac{\alpha}{\cos x}\right)\right] dx =$ $= \frac{\pi^{1/2}\alpha^{-1}}{\Gamma(2\nu+{}^1/_2)} W_{\nu,\nu}(\alpha e^{i\pi/2})\,W_{\nu,\nu}(\alpha e^{-i\pi/2}), \qquad \operatorname{Re}\nu < {}^1/_2$

(11)	$\int\limits_0^\infty \exp[(\nu+1)x]\, \mathbf{H}_\nu(\alpha \operatorname{sh} x)\, dx =$ $= \frac{\pi^{1/2}\alpha^{-1/2}}{\sin(\nu\pi)}\left[\operatorname{sh}(\alpha/2)\, I_{\nu+1/2}(\alpha/2) - \operatorname{ch}(\alpha/2)\, I_{-\nu-1/2}(\alpha/2)\right],$ $\operatorname{Re}\alpha > 0, \quad -2 < \operatorname{Re}\nu < 0$
(12)	$\int\limits_0^\infty x^{-1}\cos(2ax^{-1})\,[I_0(x) - \mathbf{L}_0(x)]\, dx = 2\, J_0(2a^{1/2})\, K_0(2a^{1/2}), \qquad a > 0$
(13)	$\int\limits_0^\infty x^{\nu-1/2}\exp[-(1+\alpha)x]\, K_0(\alpha x)\, \mathbf{L}_\nu(x)\, dx =$ $= \frac{\pi^{1/2}[\Gamma(\nu+1/2)]^2}{(2\alpha)^{\nu+1/2}\,\Gamma(\nu+1)} P_{\nu-1/2}(1+\alpha^{-1}), \quad \operatorname{Re}\alpha > 0, \quad \operatorname{Re}\nu > -1/2$
(14)	$\int\limits_0^a x\, P_n(1-2x^2a^{-2})\,[I_0(x) - \mathbf{L}_0(x)]\, dx = (-1)^n a\,[I_{2n+1}(a) - \mathbf{L}_{2n+1}(a)],$ $n = 0, 1, 2, \ldots$
(15)	$\int\limits_a^\infty x^{1/2}(x^2-a^2)^{-1/4-\nu/2} P_\mu^{\nu+1/2}(2x^2a^{-2}-1)\,[I_{-\nu}(x) - \mathbf{L}_\nu(x)]\, dx =$ $= \frac{\pi^{1/2}\, a\cos(\nu\pi)}{2^{\nu+1}\sin(2\mu\pi)}\{[I_\nu(a/2)]^2 - [I_{-\nu}(a/2)]^2\}, \quad -1 < \operatorname{Re}\mu < 0, \ \operatorname{Re}\nu < 1/2$
(16)	$\int\limits_0^{\pi/2} \frac{\cos(2\nu x)}{\cos x}\left[I_{2\nu}\left(\frac{\alpha}{\cos x}\right) - \mathbf{L}_{2\nu}\left(\frac{\alpha}{\cos x}\right)\right] dx =$ $= \frac{\pi^{1/2}\alpha^{-1}}{\Gamma(2\nu+1)} W_{\nu,\nu}(\alpha)\, M_{-\nu,\nu}(\alpha), \quad \operatorname{Re}\nu < 1/2$
(17)	$\int\limits_0^\infty x^{\lambda-1} s_{\mu,\nu}(x)\, dx = \frac{\Gamma\left(\frac{1+\lambda+\mu}{2}\right)\Gamma\left(\frac{1-\lambda-\mu}{2}\right)\Gamma\left(\frac{1+\mu+\nu}{2}\right)\Gamma\left(\frac{1+\mu-\nu}{2}\right)}{2^{2-\lambda-\mu}\,\Gamma\left(1-\frac{\lambda+\nu}{2}\right)\Gamma\left(1-\frac{\lambda-\nu}{2}\right)},$ $-\operatorname{Re}\mu < \operatorname{Re}\lambda + 1 < 5/2$
(18)	$\int\limits_0^\infty x^{-\mu-1}\cos(ax)\, s_{\mu,\nu}(x)\, dx =$ $= \begin{cases} 0, & a > 1 \\ 2^{\mu-1/2}\pi^{1/2}\Gamma\left(\frac{\mu+\nu+1}{2}\right)\Gamma\left(\frac{\mu-\nu+1}{2}\right)(1-a^2)^{\mu/2+1/4} \mathrm{P}_{\nu-1/2}^{-\mu-1/2}(a), & 0 < a < 1 \end{cases}$

(19)	$\int\limits_0^\infty x^{-\mu} J_\nu(ax)\, s_{\mu+\nu,\, \mu-\nu+1}(x)\, dx = \begin{cases} 0, & a>1 \\ 2^{\nu-1}\Gamma(\nu)\, a^{-\nu}(1-a^2)^\mu, & 0<a<1 \end{cases}$ $\operatorname{Re}\nu > -1$
(20)	$\int\limits_0^\infty x^\nu K_{1-2\nu}(\alpha x^{1/2} e^{i\pi/4}) K_{1-2\nu}(\alpha x^{1/2} e^{-i\pi/4})\, s_{\mu,\nu}(x)\, dx =$ $= 2^{2\mu+2\nu-2}\pi^{-1/2}\Gamma\left(\frac{1+\mu+\nu}{2}\right)\Gamma\left(\frac{1+\mu-\nu}{2}\right)\times$ $\times\Gamma\left(\frac{\mu+3\nu+1}{2}\right)\Gamma\left(\frac{\mu+\nu}{2}+1\right) S_{-\mu-2\nu-1/2,\, 1/2-\nu}(\alpha),$ $\lvert\arg\alpha\rvert<\pi/4, \quad \operatorname{Re}(\mu-\nu)>-3, \quad \operatorname{Re}(\mu+3\nu)>-1$
(21)	$\int\limits_0^{\pi/2} \cos[(\mu+1)x]\, s_{\mu,\nu}(\alpha\cos x)\, dx =$ $= 2^{\mu-2}\pi\,\Gamma\left(\frac{\mu+\nu+1}{2}\right)\Gamma\left(\frac{\mu-\nu+1}{2}\right) J_{(\mu+\nu+1)/2}(\alpha/2)\, J_{(\mu-\nu+1)/2}(\alpha/2),$ $\operatorname{Re}\mu>-2$
(22)	$\int\limits_0^\infty \exp[(\mu+1)x]\, s_{\mu,\nu}(a\operatorname{sh}x)\, dx =$ $= \frac{2^{\mu-2}\pi\,\Gamma(\rho)\,\Gamma(\sigma)}{\sin(\mu\pi)}[I_\rho(a/2)\, I_\sigma(a/2) - I_{-\rho}(a/2)\, I_{-\sigma}(a/2)],$ $2\rho=\mu+\nu+1,\ 2\sigma=\mu-\nu+1,\ a>0,\ -2<\operatorname{Re}\mu<0$
(23)	$\int\limits_0^\infty x^{-\mu}\sin(ax)\, S_{\mu,\nu}(x)\, dx =$ $= 2^{-\mu-1/2}\pi^{1/2}\Gamma\left(1-\frac{\mu+\nu}{2}\right)\Gamma\left(1-\frac{\mu-\nu}{2}\right)(a^2-1)^{\mu/2-1/4} P_{\nu-1/2}^{\mu-1/2}(a),$ $a>1,\ \operatorname{Re}\mu<1-\lvert\operatorname{Re}\nu\rvert$
(24)	$\int\limits_0^a x^{(\nu-\mu-1)/2}(a^2-x^2)^{(\nu-\mu-2)/4}\mathrm{P}_{\nu-1/2}^{(\mu-\nu+2)/2}(x/a)\, S_{\mu,\nu}(x)\, dx =$ $= 2^{\mu-3/2}\pi^{1/2}a^{(\nu-\mu)/2}\Gamma\left(\frac{\mu+\nu+3}{4}\right)\Gamma\left(\frac{\mu-3\nu+3}{4}\right)\cos\left[2^{-1}(\mu-\nu)\pi\right]\times$ $\times[J_\nu(a/2)\, Y_{-(\mu-\nu+1)/2}(a/2) - Y_\nu(a/2)\, J_{-(\mu-\nu+1)/2}(a/2)],$ $\operatorname{Re}(\mu-\nu)<0,\ -1<\operatorname{Re}(\mu+\nu)<1,\ \operatorname{Re}(\mu-3\nu)<1$

(25) $$\int_a^\infty x^{1/2}(x^2-a^2)^{-\beta/2}P_\nu^\beta(x/a)\,S_{\mu,\,1/2}(x)\,dx=\frac{a^{1/2}\,\Gamma\left(\frac{\beta-\mu+\nu}{2}+\frac{1}{4}\right)\Gamma\left(\frac{\beta-\mu-\nu}{2}-\frac{1}{4}\right)}{\pi^{1/2}2^{3/2-\beta+\mu}\,\Gamma\left(1/2-\mu\right)}\,S_{\mu-\beta+1,\ \nu+1/2}(a),$$

$\operatorname{Re}\beta<1,\ \operatorname{Re}(\mu+\nu-\beta)<-1/2,\ \operatorname{Re}(\mu-\nu-\beta)<1/2$

(26) $$\int_a^\infty x(x^2-a^2)^{-\nu/2}P_\lambda^\nu(2x^2a^{-2}-1)\,S_{\mu,\,\nu}(x)\,dx=\frac{a\,\Gamma\left(\frac{\nu-\mu-1}{2}-\lambda\right)\Gamma\left(\frac{\nu-\mu+1}{2}+\lambda\right)}{2\,\Gamma\left(\frac{1-\mu-\nu}{2}\right)\Gamma\left(\frac{1-\mu+\nu}{2}\right)}\,S_{\mu-\nu+1,\,2\lambda+1}(a),$$

$\operatorname{Re}\nu<1,\ \operatorname{Re}(\mu-\nu+\lambda)<-1,\ \operatorname{Re}(\mu-\nu-\lambda)<0$

(27) $$\int_a^\infty x^{-\nu}(x^2-a^2)^{1/4-\nu/2}\,\mathrm{P}^{\nu-1/2}_{\mu/2-\nu/2}(2a^2x^{-2}-1)\,S_{\mu,\,\nu}(x)\,dx=\frac{\pi^{1/2}2^{\mu-\nu}\,\Gamma\left(\frac{3\nu-\mu-1}{2}\right)}{a^{\nu+1/2}\,\Gamma\left(\frac{1+\nu-\mu}{2}\right)}\,W_{\rho,\,\sigma}\left(ae^{i\pi/2}\right)W_{\rho,\,\sigma}\left(ae^{-i\pi/2}\right),$$

$\rho=2^{-1}(\mu+1-\nu),\quad \sigma=\nu-1/2$

$\operatorname{Re}(\mu-\nu)<0,\ \operatorname{Re}\nu<3/2,\ \operatorname{Re}(3\nu-\mu)>1$

(28) $$\int_0^\infty x^{1-\mu-\nu}J_\nu(ax)\,S_{\mu,\,-\mu-2\nu}(x)\,dx=\frac{\pi^{1/2}a^{\nu-1}\,\Gamma(1-\mu-\nu)}{2^{\mu+2\nu}\,\Gamma(\nu+1/2)}\,(a^2-1)^{(\mu+\nu-1)/2}\,P^{\mu+\nu-1}_{\mu+\nu}(a),$$

$a>1,\ \operatorname{Re}\nu>-1/2,\ \operatorname{Re}(\mu+\nu)<1$

(29) $$\int_0^{\pi/2}\cos(2\mu x)\,S_{2\mu-1,\,2\nu}(\alpha\cos x)\,dx=\frac{\pi 2^{2\mu-3}\alpha^{2\mu}}{\Gamma(1-\mu-\nu)\,\Gamma(1-\mu+\nu)\sin(2\nu\pi)}\times$$

$$\times\left[J_{\mu+\nu}(\alpha/2)\,Y_{\mu-\nu}(\alpha/2)-J_{\mu-\nu}(\alpha/2)\,Y_{\mu+\nu}(\alpha/2)\right],$$

$\operatorname{Re}\mu>-2,\ -1<\operatorname{Re}\nu<1$

(30) $$\int_0^{\pi/2}\frac{\cos(2\mu x)}{\cos x}\,S_{2\mu,\,2\nu}\left(\frac{\alpha}{\cos x}\right)dx=\frac{\pi 2^{2\mu-1}}{\alpha}\,W_{\mu,\,\nu}\left(\alpha e^{i\pi/2}\right)W_{\mu,\,\nu}\left(\alpha e^{-i\pi/2}\right),$$

$|\arg\alpha|<\pi,\ \operatorname{Re}\mu<1$

(31)	$\int_0^\infty (\operatorname{sh} x)^{1/2} \operatorname{ch}(\nu x)\, S_{\mu,\,1/2}(\alpha \operatorname{ch} x)\, dx = \dfrac{\Gamma\left(\frac{1}{4}-\frac{\mu+\nu}{2}\right)\Gamma\left(\frac{1}{4}-\frac{\mu-\nu}{2}\right)}{2^{\mu+3/2}\alpha^{1/2}\,\Gamma(1/2-\mu)}\, S_{\mu+1/2,\,\nu}(\alpha),$	$\lvert\arg\alpha\rvert<\pi,\ \operatorname{Re}\mu+\lvert\operatorname{Re}\nu\rvert<1/2$
(32)	$\int_0^\infty x^{2\nu-1}\, U_\nu(w,\, x)\, dx = 2^{\nu-1}\,\Gamma(\nu)\, w^\nu \cos(w/2),$	$\operatorname{Re}\nu>0$
(33)	$\int_0^\infty x^{2\nu-3}\, U_\nu(w,\, x)\, dx = 2^{\nu-2}\,\Gamma(\nu-1)\, w^{\nu-1} \sin(w/2),$	$\operatorname{Re}\nu>1$
(34)	$\int_0^\infty x^{1-\nu} \sin(ax/2)\, U_\nu(x,\, z)\, dx = \begin{cases} 0, & a>1 \\ 2^{-1}\pi\,(1-a)^{\nu/2-1} z^{2-\nu} J_{\nu-2}\left[z(1-a)^{1/2}\right], & 0<a<1 \end{cases}$	
(35)	$\int_0^\infty x^{-\nu} \cos(ax/2)\, U_\nu(x,\, z)\, dx = \begin{cases} 0, & a>1 \\ 2^{-1}\pi\,(1-a)^{\nu/2-1/2} z^{1-\nu} J_{\nu-1}\left[z(1-a)^{1/2}\right], & 0<a<1 \end{cases}$	
(36)	$\int_0^\infty x^{\nu-1} J_{\nu/2-1}\left(\frac{x^2}{2w}\right) U_\nu(w,\, x)\, dx = \dfrac{w^\nu}{2(\nu-1)}\, J_{\nu/2}\left(\frac{w}{2}\right),$	$\operatorname{Re}\nu>1$
(37)	$\int_{-\infty}^\infty \dfrac{\sin[a(x+z)]}{x+z}\, U_\nu(w,\, x)\, dx = \pi\, U_\nu(w,\, z),$	$a>1$

ГЛАВА XX

ГИПЕРГЕОМЕТРИЧЕСКИЕ ФУНКЦИИ

Большинство высших трансцендентных функций, рассмотренных в главах XVI—XIX, являются частными случаями гипергеометрических функций. В этой главе мы дадим список гипергеометрических функций, не вошедших в главы XVI—XIX, а также обобщенных гипергеометрических функций. Интегралы, приведенные в пп. 20.4 и 20.5, являются ключевыми, и из них можно вывести громадное число интегралов, содержащих частные случаи гипергеометрических функций. Относительно частных случаев E-функции и G-функции см. Приложение. Мы не приводим в этой главе интегралов, содержащих обобщенные гипергеометрические ряды ${}_pF_q$, так как

$$
\begin{aligned}
{}_pF_q(a_1, \ldots, a_p; b_1, \ldots, b_q; x) &= \\
&= \frac{\Gamma(b_1)\ldots\Gamma(b_q)}{\Gamma(a_1)\ldots\Gamma(a_p)} E(a_1, \ldots, a_p; b_1, \ldots, b_q; -x^{-1}) = \\
&= \frac{\Gamma(b_1)\ldots\Gamma(b_q)}{\Gamma(a_1)\ldots\Gamma(a_p)} G^{p,1}_{q+1,p}\left(-\frac{1}{x}\,\middle|\,\begin{matrix}1, b_1, \ldots, b_q\\ a_1, \ldots, a_p\end{matrix}\right).
\end{aligned}
$$

Такие интегралы могут быть выведены из указанных в пп. 20.4 и 20.5. Ввиду большой важности интегралов, содержащих E-функции и G-функции, мы повторяем здесь интегралы от этих функций, приведенные в предыдущих главах, и в некоторых случаях указываем уточненные условия справедливости формул.

Функции параболического цилиндра. Относительно теории этих функций см. ВТФ, т. II, главу VIII и указанную там литературу. См. также книгу Buchholz Herbert, 1953: Die konfluente hypergeometrische Funktion. Springer-Verlag.

$$
\begin{aligned}
D_\nu(z) &= 2^{\nu/2+1/4} z^{-1/2} W_{\nu/2+1/4,\,\pm 1/4}\left(\frac{z^2}{2}\right) = \\
&= \frac{2^{-\nu-1}z^\nu}{\pi^{1/2}\,\Gamma(-\nu)} \exp\left(-\frac{z^2}{4}\right) E\left(-\frac{\nu}{2}, \frac{1-\nu}{2} : : \frac{z^2}{2}\right) = \\
&= 2^{\nu/2} \exp\left(\frac{z^2}{4}\right) G^{20}_{12}\left(\frac{z^2}{2}\,\middle|\,\begin{matrix}1/2 - \nu/2\\ 0,\ 1/2\end{matrix}\right) = \\
&= \frac{2^{-\nu/2-1}}{\pi^{1/2}\,\Gamma(-\nu)} \exp\left(-\frac{z^2}{4}\right) G^{21}_{12}\left(\frac{z^2}{2}\,\middle|\,\begin{matrix}1 + \nu/2\\ 0,\ 1/2\end{matrix}\right).
\end{aligned}
$$

Другие выражения через G-функцию и выражения для произведений функций параболического цилиндра могут быть выведены с помощью формул, приведенных в Приложении.

Гипергеометрические ряды Гаусса. Относительно теории этих рядов см. ВТФ, т. I, главу II и указанную там литературу в особенности монографии Гурса, Кампе де Ферье (Kampé de Feriét), Клейна (Klein), Сноу (Snow) и главу XIV книги Уиттекера и Ватсона.

$$ {}_2F_1(a, b; c; x) = \frac{\Gamma(c)}{\Gamma(a)\Gamma(b)} E(a, b: c: -x^{-1}) = \frac{\Gamma(c)}{\Gamma(a)\Gamma(b)} G_{22}^{21}\left(-\frac{1}{x}\,\middle|\,\begin{matrix}1, c\\ a, b\end{matrix}\right). $$

Вычисление интегралов, содержащих гипергеометрические ряды Гаусса, часто облегчается благодаря применению формул преобразования; относительно этих формул см. ВТФ, т. I, пп. 2.9 и 2.11.

Вырожденные гипергеометрические функции. Относительно теории этих функций см. ВТФ, т. I, главу VI и указанную там литературу, в особенности главу XVI книги Уиттекера и Ватсона, а также Трикоми (Tricomi F. G., 1952: Lezioni sulle funzioni ipergeometriche confluenti. Torino, Gheroni) и Бухгольца (Buchholz Herbert, 1953: Die konfluente hypergeometrische Funktion, Springer-Verlag).

$$ \begin{aligned} M_{\varkappa,\mu}(z) &= z^{\mu+1/2} e^{-z/2} {}_1F_1(1/2 - \varkappa + \mu;\ 2\mu + 1;\ z) = \\ &= z^{\mu+1/2} e^{z/2} {}_1F_1(1/2 + \varkappa + \mu;\ 2\mu + 1;\ -z) = \\ &= \frac{\Gamma(2\mu+1)}{\Gamma(1/2 - \varkappa + \mu)} z^{\mu+1/2} e^{-z/2} E(1/2 - \varkappa + \mu:\ 2\mu + 1:\ -z^{-1}) = \\ &= \frac{\Gamma(2\mu+1)}{\Gamma(1/2 + \varkappa + \mu)} z^{\mu+1/2} e^{z/2} E(1/2 + \varkappa + \mu: 2\mu + 1: z^{-1}) = \\ &= \frac{\Gamma(2\mu+1)}{\Gamma(1/2 + \varkappa + \mu)} e^{z/2} G_{12}^{11}\left(z\,\middle|\,\begin{matrix}1-\varkappa\\ 1/2 + \mu,\ 1/2 - \mu\end{matrix}\right), \end{aligned} $$

$$ \begin{aligned} W_{\varkappa,\mu}(z) &= \frac{z^{\varkappa} e^{-z/2}}{\Gamma(1/2 - \varkappa + \mu)\,\Gamma(1/2 - \varkappa - \mu)} E(1/2 - \varkappa + \mu,\ 1/2 - \varkappa - \mu : : z) = \\ &= \frac{e^{-z/2}}{\Gamma(1/2 - \varkappa + \mu)\,\Gamma(1/2 - \varkappa - \mu)} G_{12}^{21}\left(z\,\middle|\,\begin{matrix}1+\varkappa\\ 1/2 + \mu,\ 1/2 - \mu\end{matrix}\right) = \\ &= e^{z/2} G_{12}^{20}\left(z\,\middle|\,\begin{matrix}1-\varkappa\\ 1/2 + \mu,\ 1/2 - \mu\end{matrix}\right). \end{aligned} $$

Относительно других выражений через G-функцию и выражений для произведений вырожденных гипергеометрических функций см. Приложение.

***E*-функция Мак-Роберта.** Краткое введение в теорию этой функции содержится в ВТФ, т. I, пп. 5.2—5.2.2, а более детальное изложение теории можно найти в книге MacRobert T.M., 1950: Functions of a complex variable. Macmillan, Приложение V и Разные примеры III. См. также работы Мак-Роберта, указанные на стр. 286 ВТФ, т. I, и дальнейшие работы Мак-Роберта и его учеников в Proc. Glasgow Math. Ass. т. I, 1953.

$$ E(a_1, \ldots, a_p: b_1, \ldots, b_q: x) = G_{q+1,\,p}^{p,\,1}\left(z\,\middle|\,\begin{matrix}1,\ b_1, \ldots, b_q\\ a_1, \ldots, a_p\end{matrix}\right). $$

Многие высшие трансцендентные функции и некоторые их комбинации являются частными случаями E-функции; часть соответствующих формул приведена в Приложении.

***G*-функция Мейера.** Относительно теории этой функции см. ВТФ, т. I, пп. 5.3—5.6 и работы Мейера, перечисленные на стр. 286 ВТФ, т. I, а также дальнейшие работы Мейера, помещенные в Proc. Nederl Akad. Wetensch.

Следует отметить, что очень многие интегралы, содержащие специальные функции, могут быть сведены к интегралам от G-функции. Примеры этого и необходимые формулы приведены в работах Мейсра. Часть соответствующих формул приведена в Приложении.

20.1. Функции параболического цилиндра

См. также вырожденные гипергеометрические функции, E-функцию, G-функцию.

(1)	$\int\limits_0^\infty x^{\nu-1}\exp\left(-\frac{3x^2}{4}\right)D_\nu(x)\,dx=2^{-\nu/2}\,\Gamma(\nu)\cos(2^{-2}\nu\pi),$ $\operatorname{Re}\nu>0$
(2)	$\int\limits_0^\infty x^{\nu}\exp\left(-\frac{3x^2}{4}\right)D_{\nu-1}(x)\,dx=2^{-\nu/2-1}\,\Gamma(\nu)\sin(2^{-2}\nu\pi),$ $\operatorname{Re}\nu>-1$
(3)	$\int\limits_0^a x^{2\nu-1}(a^2-x^2)^{\lambda-1}\exp\left(\frac{x^2}{4}\right)D_{-2\lambda-2\nu}(x)\,dx=$ $=\frac{\Gamma(\lambda)\,\Gamma(2\nu)}{\Gamma(2\lambda+2\nu)}\,2^{\lambda-1}\,a^{2\lambda+2\nu-2}\exp\left(\frac{a^2}{4}\right)D_{-2\nu}(a),$ $\operatorname{Re}\lambda>0,\ \operatorname{Re}\nu>0$
(4)	$\int\limits_a^\infty x^{\nu}(x-a)^{\mu/2-\nu/2-1}\exp\left[-2^{-2}(x-a)^2\right]D_\mu(x)\,dx=$ $=2^{\mu-\nu-2}\,a^{\mu-1}\,\Gamma\left(\frac{\mu-\nu}{2}\right)D_\nu(a),$ $\operatorname{Re}(\mu-\nu)>0$
(5)	$\int\limits_{-\infty}^\infty (x-i\alpha)^{-1}\exp\left(-\frac{x^2}{4}\right)D_n(x)\,dx=$ $=-(2\pi)^{1/2}(-i)^n\,n!\exp\left(-\frac{\alpha^2}{4}\right)D_{-n-1}(\alpha),$ $n=0,\ 1,\ 2,\ \ldots,\ \operatorname{Re}\alpha>0$
(6)	$\int\limits_0^\infty x^{\nu}(x^2+\alpha^2)^{-1}\exp\left(-\frac{x^2}{4}\right)D_\nu(x)\,dx=$ $=(\pi/2)^{1/2}\alpha^{\nu-1}\,\Gamma(\nu+1)\exp\left(\frac{\alpha^2}{4}\right)D_{-\nu-1}(\alpha),$ $\operatorname{Re}\alpha>0,\ 0<\operatorname{Re}\nu<1$

(7)	$\int\limits_0^\infty x^{\nu-1}(x^2+\alpha^2)^{-1/2}\exp\left(-\frac{x^2}{4}\right)D_\nu(x)\,dx = \alpha^{\nu-1}\Gamma(\nu)\exp\left(\frac{\alpha^2}{4}\right)D_{-\nu}(\alpha),\quad \operatorname{Re}\alpha>0,\ \operatorname{Re}\nu>0$
(8)	$\int\limits_0^\infty x^{2\rho-1}\sin(\alpha x)\exp\left(-\frac{x^2}{4}\right)D_{2\nu}(x)\,dx = 2^{\nu-\rho-1/2}\pi^{1/2}\alpha\frac{\Gamma(2\rho+1)}{\Gamma(\rho-\nu+1)}\,{}_2F_2\left(\rho+\frac{1}{2},\ \rho+1;\ \frac{3}{2},\ \rho-\nu+1;\ -\frac{\alpha^2}{2}\right),\quad \operatorname{Re}\rho>-1/2$
(9)	$\int\limits_0^\infty x^{2\rho-1}\sin(ax)\exp\left(\frac{x^2}{4}\right)D_{2\nu}(x)\,dx = \frac{2^{\rho-\nu-2}}{\Gamma(-2\nu)}G_{23}^{22}\left(\frac{a^2}{2}\,\middle\vert\,\begin{matrix}1/2-\rho,\ 1-\rho\\ -\rho-\nu,\ 1/2,\ 0\end{matrix}\right),\quad a>0,\ \operatorname{Re}\rho>-1/2,\ \operatorname{Re}(\rho+\nu)<1/2$
(10)	$\int\limits_0^\infty x^{2\rho-1}\cos(\alpha x)\exp\left(-\frac{x^2}{4}\right)D_{2\nu}(x)\,dx = \frac{2^{\nu-\rho}\Gamma(2\rho)\pi^{1/2}}{\Gamma(\rho-\nu+1/2)}\,{}_2F_2\left(\rho,\ \rho+1/2;\ 1/2,\ \rho-\nu+1/2;\ -\frac{\alpha^2}{2}\right),\quad \operatorname{Re}\rho>0$
(11)	$\int\limits_0^\infty x^{2\rho-1}\cos(ax)\exp\left(\frac{x^2}{4}\right)D_{2\nu}(x)\,dx = \frac{2^{\rho-\nu-2}}{\Gamma(-2\nu)}G_{23}^{22}\left(\frac{a^2}{2}\,\middle\vert\,\begin{matrix}1/2-\rho,\ 1-\rho\\ -\rho-\nu,\ 0,\ 1/2\end{matrix}\right),\quad a>0,\ \operatorname{Re}\rho>0,\ \operatorname{Re}(\rho+\nu)<1/2$
(12)	$\int\limits_0^\infty x^{\nu-1/2}\exp[-(x+\alpha)^2]\,I_{\nu-1/2}(2\alpha x)\,D_\nu(2x)\,dx = 2^{-1}\pi^{-1/2}\Gamma(\nu)\,\alpha^{\nu-1/2}D_{-\nu}(2\alpha),\quad \operatorname{Re}\alpha>0,\ \operatorname{Re}\nu>0$
(13)	$\int\limits_0^\infty x^{\nu-3/2}\exp[-(x+\alpha)^2]\,I_{\nu-3/2}(2\alpha x)\,D_\nu(2x)\,dx = 2^{-1}\pi^{-1/2}\Gamma(\nu)\,\alpha^{\nu-3/2}D_{-\nu}(2\alpha),\quad \operatorname{Re}\alpha>0,\ \operatorname{Re}\nu>1$
(14)	$\int\limits_0^\infty [D_\nu(x)]^2\,dx = (\pi/2)^{1/2}\Gamma(\nu+1)+\frac{\pi^{1/2}}{2^{3/2}\Gamma(-\nu)}\left[\psi\left(\frac{\nu+1}{2}\right)-\psi\left(\frac{\nu}{2}+1\right)\right]$

(15)	$\int\limits_0^\infty D_\nu(x)\, D_\mu(x)\, dx = \frac{\pi 2^{(\mu+\nu+1)/2}}{\mu-\nu}\left[\frac{1}{\Gamma\left(-\frac{\nu}{2}\right)\Gamma\left(\frac{1-\mu}{2}\right)} - \frac{1}{\Gamma\left(-\frac{\mu}{2}\right)\Gamma\left(\frac{1-\nu}{2}\right)}\right]$
(16)	$\int\limits_0^\infty J_0(xy)\, D_\nu(x)\, D_{\nu-1}(-x)\, dx = y^{-1}[D_\nu(y)\, D_{\nu-1}(y) +$ $+ 2^{-1} D_\nu(y)\, D_{\nu-1}(-y) + 2^{-1} D_\nu(-y)\, D_{\nu-1}(y)],\quad y>0$
(17)	$\int\limits_0^\infty J_0(xy)\, D_\nu(x)\, D_{\nu-1}(x)\, dx = \frac{1}{2y}[D_\nu(y)\, D_{\nu-1}(-y) - D_\nu(-y)\, D_{\nu-1}(y)]$
(18)	$\int\limits_0^\infty J_0(xy)\, D_\nu(-x)\, D_{\nu-1}(x)\, dx =$ $= y^{-1}[2^{-1} D_\nu(y)\, D_{\nu-1}(-y) + 2^{-1} D_\nu(-y)\, D_{\nu-1}(y) - D_\nu(y)\, D_{\nu-1}(y)]$
(19)	$\int\limits_0^{\pi/2} (\sin x)^{-\nu}(\cos x)^{-\mu-2} D_\nu(\alpha\sin x)\, D_\mu(\alpha\cos x)\, dx =$ $= -(\pi/2)^{1/2}(\mu+1)^{-1} D_{\mu+\nu+1}(\alpha),\quad \operatorname{Re}\nu<1,\ \operatorname{Re}\mu<-1$
(20)	$\int\limits_0^\infty \operatorname{ch}(2\mu x)\exp[-(\alpha\operatorname{sh}x)^2]\, D_{2\varkappa}(2\alpha\operatorname{ch}x)\, dx = 2^{\varkappa-3/2}\pi^{1/2}\alpha^{-1} W_{\varkappa,\mu}(2\alpha^2),$ $\operatorname{Re}\alpha^2>0$
(21)	$\int\limits_0^\infty \operatorname{ch}(2\mu x)\exp[(\alpha\operatorname{sh}x)^2]\, D_{2\varkappa}(2\alpha\operatorname{ch}x)\, dx =$ $= \frac{\Gamma(\mu-\varkappa)\,\Gamma(-\mu-\varkappa)}{2^{\varkappa+5/2}\alpha\,\Gamma(-2\varkappa)} W_{\varkappa+1/2,\mu}(2\alpha^2),$ $\lvert\arg\alpha\rvert<3\pi/4,\ \operatorname{Re}\varkappa+\lvert\operatorname{Re}\mu\rvert<0$
(22)	$\int\limits_0^\infty \cos(ax)\, D_{x-1/2}(\beta)\, D_{-x-1/2}(\beta)\, dx =$ $=\begin{cases}\frac{1}{2}\left(\frac{\pi}{\cos a}\right)^{1/2}\exp\left(-\frac{\beta^2\cos a}{2}\right), & -\pi/2<a<\pi/2\\ 0, & a<-\pi/2 \text{ или } a>\pi/2\end{cases}$

20.2. Гипергеометрические ряды Гаусса

См. также функции Лежандра, E-функцию и G-функцию.

(1)	$\int\limits_0^1 x^{\alpha-\gamma}(1-x)^{\gamma-\beta-1}\,{}_2F_1(\alpha, \beta; \gamma; x)\,dx = \dfrac{\Gamma(1+\alpha/2)\,\Gamma(\gamma)\,\Gamma(\alpha-\gamma+1)\,\Gamma(\gamma-\alpha/2-\beta)}{\Gamma(1+\alpha)\,\Gamma(1+\alpha/2-\beta)\,\Gamma(\gamma-\alpha/2)}$, $\operatorname{Re}\alpha+1>\operatorname{Re}\gamma>\operatorname{Re}\beta,\ \operatorname{Re}(\gamma-\alpha/2-\beta)>0$
(2)	$\int\limits_0^1 x^{\rho-1}(1-x)^{\beta-\gamma-n}\,{}_2F_1(-n, \beta; \gamma; x)\,dx = \dfrac{\Gamma(\gamma)\,\Gamma(\rho)\,\Gamma(\beta-\gamma+1)\,\Gamma(\gamma-\rho+n)}{\Gamma(\gamma+n)\,\Gamma(\gamma-\rho)\,\Gamma(\beta-\gamma+\rho+1)}$, $n=0, 1, 2, \ldots,\ \operatorname{Re}\rho>0,\ \operatorname{Re}(\beta-\gamma)>n-1$
(3)	$\int\limits_0^1 x^{\rho-1}(1-x)^{\beta-\rho-1}\,{}_2F_1(\alpha, \beta; \gamma; x)\,dx = \dfrac{\Gamma(\gamma)\,\Gamma(\rho)\,\Gamma(\beta-\rho)\,\Gamma(\gamma-\alpha-\rho)}{\Gamma(\beta)\,\Gamma(\gamma-\alpha)\,\Gamma(\gamma-\rho)}$, $\operatorname{Re}\rho>0,\ \operatorname{Re}(\beta-\rho)>0,\ \operatorname{Re}(\gamma-\alpha-\rho)>0$
(4)	$\int\limits_0^1 x^{\gamma-1}(1-x)^{\rho-1}\,{}_2F_1(\alpha, \beta; \gamma; x)\,dx = \dfrac{\Gamma(\gamma)\,\Gamma(\rho)\,\Gamma(\gamma+\rho-\alpha-\beta)}{\Gamma(\gamma+\rho-\alpha)\,\Gamma(\gamma+\rho-\beta)}$, $\operatorname{Re}\gamma>0,\ \operatorname{Re}\rho>0,\ \operatorname{Re}(\gamma+\rho-\alpha-\beta)>0$
(5)	$\int\limits_0^1 x^{\rho-1}(1-x)^{\sigma-1}\,{}_2F_1(\alpha, \beta; \gamma; x)\,dx = \dfrac{\Gamma(\rho)\,\Gamma(\sigma)}{\Gamma(\rho+\sigma)}\,{}_3F_2(\alpha, \beta, \rho; \gamma, \rho+\sigma; 1)$, $\operatorname{Re}\rho>0,\ \operatorname{Re}\sigma>0,\ \operatorname{Re}(\gamma+\sigma-\alpha-\beta)>0$
(6)	$\int\limits_0^1 x^{\gamma-1}(1-x)^{\rho-1}(1-zx)^{-\sigma}\,{}_2F_1(\alpha, \beta; \gamma; x)\,dx = \dfrac{\Gamma(\gamma)\,\Gamma(\rho)\,\Gamma(\gamma+\rho-\alpha-\beta)}{\Gamma(\gamma+\rho-\alpha)\,\Gamma(\gamma+\rho-\beta)}(1-z)^{-\sigma}\times$ $\times\,{}_3F_2\left(\rho, \sigma, \gamma+\rho-\alpha-\beta; \gamma+\rho-\alpha, \gamma+\rho-\beta; \dfrac{z}{z-1}\right)$, $\operatorname{Re}\gamma>0,\ \operatorname{Re}\rho>0,\ \operatorname{Re}(\gamma+\rho-\alpha-\beta)>0,\ \lvert\arg(1-z)\rvert<\pi$
(7)	$\int\limits_0^1 x^{\rho-1}(1-x)^{\sigma-1}\,{}_2F_1(\alpha, \beta; \gamma; xz)\,dx = \dfrac{\Gamma(\rho)\,\Gamma(\sigma)}{\Gamma(\rho+\sigma)}\,{}_3F_2(\alpha, \beta, \rho; \gamma, \rho+\sigma; z)$, $\operatorname{Re}\rho>0,\ \operatorname{Re}\sigma>0,\ \lvert\arg(1-z)\rvert<\pi$

(8)	$\int_0^1 x^{\gamma-1}(1-x)^{\rho-1}e^{-xz}\,{}_2F_1(\alpha,\ \beta;\ \gamma;\ x)\,dx = \frac{\Gamma(\gamma)\,\Gamma(\rho)\,\Gamma(\gamma+\rho-\alpha-\beta)}{\Gamma(\gamma+\rho-\alpha)\,\Gamma(\gamma+\rho-\beta)}\,e^{-z}\,{}_2F_2(\rho,\gamma+\rho-\alpha-\beta;\ \gamma+\rho-\alpha,\ \gamma+\rho-\beta;\ z),$ $\operatorname{Re}\gamma>0,\ \operatorname{Re}\rho>0,\ \operatorname{Re}(\gamma+\rho-\alpha-\beta)>0$
(9)	$\int_0^\infty x^{\gamma-1}(1+x)^{-\sigma}\,{}_2F_1(\alpha,\ \beta;\ \gamma;\ -x)\,dx = \frac{\Gamma(\gamma)\,\Gamma(\alpha-\gamma+\sigma)\,\Gamma(\beta-\gamma+\sigma)}{\Gamma(\sigma)\,\Gamma(\alpha+\beta-\gamma+\sigma)},$ $\operatorname{Re}\gamma>0,\ \operatorname{Re}(\alpha-\gamma+\sigma)>0,\ \operatorname{Re}(\beta-\gamma+\sigma)>0$
(10)	$\int_0^\infty x^{\gamma-1}(x+z)^{-\sigma}\,{}_2F_1(\alpha,\ \beta;\ \gamma;\ -x)\,dx = \frac{\Gamma(\alpha-\gamma+\sigma)\,\Gamma(\beta-\gamma+\sigma)\,\Gamma(\gamma)}{\Gamma(\alpha+\beta-\gamma+\sigma)\,\Gamma(\sigma)}\,{}_2F_1(\alpha-\gamma+\sigma,\ \beta-\gamma+\sigma;\ \alpha+\beta-\gamma+\sigma;\ 1-z),$ $\operatorname{Re}\gamma>0,\ \operatorname{Re}(\alpha-\gamma+\sigma)>0,\ \operatorname{Re}(\beta-\gamma+\sigma)>0,\ \lvert\arg z\rvert<\pi$
(11)	$\int_0^1 x^{\gamma-1}(1-x)^{\delta-\gamma-1}\,{}_2F_1(\alpha,\ \beta;\ \gamma;\ xz)\,{}_2F_1[\delta-\alpha,\ \delta-\beta;\ \delta-\gamma;\ (1-x)\zeta]\,dx = \frac{\Gamma(\gamma)\,\Gamma(\delta-\gamma)}{\Gamma(\delta)}(1-\zeta)^{2\alpha-\delta}\,{}_2F_1(\alpha,\ \beta;\ \delta;\ z+\zeta-z\zeta),$ $0<\operatorname{Re}\gamma<\operatorname{Re}\delta,\ \lvert\arg(1-z)\rvert<\pi,\ \lvert\arg(1-\zeta)\rvert<\pi$
(12)	$\int_0^1 x^{\gamma-1}(1-x)^{\varepsilon-1}(1-xz)^{-\delta}\,{}_2F_1(\alpha,\ \beta;\ \gamma;\ xz)\times{}_2F_1\left[\delta,\ \beta-\gamma;\ \varepsilon;\ \frac{(1-x)z}{1-xz}\right]dx = \frac{\Gamma(\gamma)\,\Gamma(\varepsilon)}{\Gamma(\gamma+\varepsilon)}\,{}_2F_1(\alpha+\delta,\ \beta;\ \gamma+\varepsilon;\ z),$ $\operatorname{Re}\gamma>0,\ \operatorname{Re}\varepsilon>0,\ \lvert\arg(z-1)\rvert<\pi$
(13)	$\int_0^\infty e^{-\lambda x}\,{}_2F_1(\alpha,\ \beta;\ {}^1\!/_2;\ -x^2)\,dx = \lambda^{\alpha+\beta-1}S_{1-\alpha-\beta,\ \alpha-\beta}(\lambda),\qquad \operatorname{Re}\lambda>0$
(14)	$\int_0^\infty x\,e^{-\lambda x}\,{}_2F_1(\alpha,\ \beta;\ {}^3\!/_2;\ -x^2)\,dx = \lambda^{\alpha+\beta-2}S_{1-\alpha-\beta,\ \alpha-\beta}(\lambda),\qquad \operatorname{Re}\lambda>0$
(15)	$\int_0^\infty x^{\gamma-1}(x+y)^{-\alpha}(x+z)^{-\beta}e^{-x}\,{}_2F_1\left[\alpha,\ \beta;\ \gamma;\ \frac{x(x+y+z)}{(x+y)(x+z)}\right]dx = \Gamma(\gamma)\,(xy)^{-1/2-\mu}\,e^{y/2+z/2}\,W_{\varkappa,\mu}(y)\,W_{\lambda,\mu}(z),$ $2\varkappa=1-\alpha+\beta-\gamma,\ 2\lambda=1+\alpha-\beta-\gamma,\ 2\mu=\alpha+\beta-\gamma,$ $\operatorname{Re}\gamma>0,\ \lvert\arg y\rvert<\pi,\ \lvert\arg z\rvert<\pi$

(16)	$\int\limits_0^\infty x^{\alpha+\beta-2\nu-1}(x+1)^{-\nu}e^{xz}K_\nu[(x+1)z]\,{}_2F_1(\alpha,\ \beta;\ \alpha+\beta-2\nu;\ -x)\,dx =$ $=\pi^{-1/2}\cos(\nu\pi)\,\Gamma(1/2-\alpha+\nu)\,\Gamma(1/2-\beta+\nu)\,\Gamma(\gamma)\,(2z)^{-1/2-\gamma/2}W_{\gamma/2,\ (\beta-\alpha)/2}(2z),$ $\operatorname{Re}(\alpha+\beta-2\nu)>0,\ \operatorname{Re}(1/2-\alpha+\nu)>0,\ \operatorname{Re}(1/2-\beta+\nu)>0,$ $\lvert\arg z\rvert<3\pi/2,\ \gamma=\alpha+\beta-2\nu$

20.3. Вырожденные гипергеометрические функции

См. также E-функция, G-функция.

Относительно частных случаев вырожденных гипергеометрических функций см. также пп. 16.5, 16.6, 17.3, 20.1 и главу XIX.

(1)	$\int\limits_0^a x^{\beta-1}(a-x)^{\gamma-1}\,{}_1F_1(a;\ \beta;\ x)\,dx=\frac{\Gamma(\beta)\,\Gamma(\gamma)}{\Gamma(\beta+\gamma)}a^{\beta+\gamma-1}\,{}_1F_1(a;\ \beta+\gamma;\ a),$ $\operatorname{Re}\beta>0,\ \operatorname{Re}\gamma>0$
(2)	$\int\limits_0^a x^{\beta-1}(a-x)^{\delta-1}\,{}_1F_1(a;\ \beta;\ x)\,{}_1F_1(\gamma;\ \delta;\ a-x)\,dx=$ $=\frac{\Gamma(\beta)\,\Gamma(\delta)}{\Gamma(\beta+\delta)}a^{\beta+\delta-1}\,{}_1F_1(a+\gamma;\ \beta+\delta;\ a),\quad \operatorname{Re}\beta>0,\ \operatorname{Re}\delta>0$
(3)	$\int\limits_0^1 x^{\beta-1}(1-x)^{\sigma-\beta-1}\,{}_1F_1(\alpha;\ \beta;\ \lambda x)\,{}_1F_1[\sigma-\alpha;\ \sigma-\beta;\ \mu(1-x)]\,dx=$ $=\frac{\Gamma(\beta)\,\Gamma(\sigma-\beta)}{\Gamma(\sigma)}e^{\lambda}\,{}_1F_1(\alpha;\ \sigma;\ \mu-\lambda)\quad 0<\operatorname{Re}\beta<\operatorname{Re}\sigma$
(4)	$\int\limits_0^\infty \cos(ax)\,{}_1F_1(\nu+1;\ 1;\ ix)\,{}_1F_1(\nu+1;\ 1;\ -ix)\,dx=$ $=\begin{cases}-a^{-1}\sin(\nu\pi)\,P_\nu(2a^{-2}-1), & 0<a<1\\ 0, & 1<a<\infty\end{cases}$ $-1<\operatorname{Re}\nu<0$
(5)	$\int\limits_0^a x^{-1}(a-x)^{\varkappa-1}e^{(a-x)/2}M_{\varkappa,\ \mu}(x)\,dx=\frac{\Gamma(\varkappa)\,\Gamma(2\mu+1)}{\Gamma(\varkappa+\mu+1/2)}\pi^{1/2}a^{\varkappa-1/2}I_\mu(a/2),$ $\operatorname{Re}\varkappa>0,\ \operatorname{Re}\mu>-1/2$
(6)	$\int\limits_0^a x^{\varkappa-1}(a-x)^{\lambda-1}e^{(a-x)/2}M_{\varkappa+\lambda,\ \mu}(x)\,dx=$ $=\frac{\Gamma(\lambda)\,\Gamma(\varkappa+\mu+1/2)}{\Gamma(\varkappa+\lambda+\mu+1/2)}a^{\varkappa+\lambda-1}M_{\varkappa,\ \mu}(a),$ $\operatorname{Re}(\varkappa+\mu)>-1/2,\ \operatorname{Re}\lambda>0$

(7)	$\int\limits_0^a x^{\mu-1/2}(a-x)^{\nu-1/2} M_{\varkappa,\mu}(x) M_{\lambda,\nu}(a-x)\,dx = \frac{\Gamma(2\mu+1)\,\Gamma(2\nu+1)}{\Gamma(2\mu+2\nu+2)}\, a^{\mu+\nu} M_{\varkappa+\lambda,\,\mu+\nu+1/2}(a),$ $\operatorname{Re}\mu > -1/2,\quad \operatorname{Re}\nu > -1/2$
(8)	$\int\limits_0^\infty x^{\rho-1}\left[x^{1/2}+(\alpha+x)^{1/2}\right]^{2\sigma} e^{-x/2} M_{\varkappa,\mu}(x)\,dx = -\frac{\sigma\,\Gamma(2\mu+1)\,\alpha^{\sigma}}{\pi^{1/2}\,\Gamma(1/2+\varkappa+\mu)}\, G^{23}_{34}\left(\alpha \,\middle|\, \begin{matrix} 1/2,\ 1,\ 1-\varkappa+\rho \\ 1/2+\mu+\rho,\ -\sigma,\ \sigma,\ 1/2-\mu+\rho \end{matrix}\right),$ $\lvert\arg\alpha\rvert < \pi,\quad \operatorname{Re}(\mu+\rho) > -1/2,\quad \operatorname{Re}(\varkappa-\rho-\sigma) > 0$
(9)	$\int\limits_0^\infty x^{\rho-1}(\alpha+x)^{-1/2}\left[x^{1/2}+(\alpha+x)^{1/2}\right]^{2\sigma} e^{-x/2} M_{\varkappa,\mu}(x)\,dx = \frac{\Gamma(2\mu+1)\,\alpha^{\sigma}}{\pi^{1/2}\,\Gamma(1/2+\varkappa+\mu)}\, G^{23}_{34}\left(\alpha \,\middle|\, \begin{matrix} 0,\ 1/2,\ 1/2-\varkappa+\rho \\ -\sigma,\ \rho+\mu,\ \rho-\mu,\ \sigma \end{matrix}\right),$ $\lvert\arg\alpha\rvert < \pi,\quad \operatorname{Re}(\rho+\mu) > -1/2,\quad \operatorname{Re}(\varkappa-\rho-\sigma) > -1/2$
(10)	$\int\limits_0^\infty x^{-\mu-1/2} e^{-x/2} \sin\left(2ax^{1/2}\right) M_{\varkappa,\mu}(x)\,dx = \pi^{1/2} a^{\varkappa+\mu-1} \frac{\Gamma(3-2\mu)}{\Gamma(1/2+\varkappa+\mu)} \exp\left(-\frac{a^2}{2}\right) W_{\rho,\sigma}(a^2),$ $a > 0,\quad \operatorname{Re}(\varkappa+\mu) > 0,\quad 2\rho = \varkappa-3\mu+1,\quad 2\sigma = \varkappa+\mu-1$
(11)	$\int\limits_0^\infty x^{-1/2}(\alpha+x)^{\mu} e^{-x/2} P_\nu^{-2\mu}(1+2x/\alpha) M_{\varkappa,\mu}(x)\,dx = -\frac{\sin(\nu\pi)}{\pi\,\Gamma(\varkappa)}\,\Gamma(2\mu+1)\,\Gamma(\varkappa-\mu+\nu+1/2)\,\Gamma(\varkappa-\mu-\nu-1/2)\, e^{\alpha/2} W_{\rho,\sigma}(\alpha),$ $\rho = 1/2-\varkappa+\mu,\quad \sigma = 1/2+\nu$ $\lvert\arg\alpha\rvert < \pi,\quad \operatorname{Re}\mu > -1/2,\quad \operatorname{Re}(\varkappa-\mu) > \lvert\operatorname{Re}\nu+1/2\rvert$
(12)	$\int\limits_0^\infty x^{-1/2}(\alpha+x)^{-\mu} e^{-x/2} P_\nu^{-2\mu}(1+2x/\alpha) M_{\varkappa,\mu}(x)\,dx = \frac{\Gamma(2\mu+1)\,\Gamma(\varkappa+\mu+\nu+1/2)\,\Gamma(\varkappa+\mu-\nu-1/2)}{\Gamma(\varkappa+\mu+1/2)\,\Gamma(2\mu+\nu+1)\,\Gamma(2\mu-\nu)}\, e^{\alpha/2} W_{1/2-\varkappa-\mu,\,1/2+\nu}(\alpha),$ $\lvert\arg\alpha\rvert < \pi,\quad \operatorname{Re}\mu > -1/2,\quad \operatorname{Re}(\varkappa+\mu) > \lvert\operatorname{Re}\nu+1/2\rvert$
(13)	$\int\limits_0^\infty x^{-1/2} e^{-x/2} P_\nu^{-2\mu}\left[(1+x/\alpha)^{1/2}\right] M_{\varkappa,\mu}(x)\,dx = \frac{\Gamma(2\mu+1)\,\Gamma(\varkappa+\nu/2)\,\Gamma(\varkappa-\nu/2-1/2)}{2^{2\mu}\alpha^{1/4}\,\Gamma(\varkappa+\mu+1/2)\,\Gamma(\mu+\nu/2+1/2)\,\Gamma(\mu-\nu/2)}\, e^{\alpha/2} W_{3/4-\varkappa,\,1/4+\nu/2}(\alpha),$ $\lvert\arg\alpha\rvert < \pi,\quad \operatorname{Re}\varkappa > 2^{-1}\operatorname{Re}\nu-1/2,\quad \operatorname{Re}\varkappa > -2^{-1}\operatorname{Re}\nu$

(14)	$\int_0^\infty x^{\mu-\varkappa/4-\nu/2-1/2}(\alpha+x)^{\nu/2}e^{-x/2}Q^{\nu}_{\mu-\varkappa+3/2}(1+2x/\alpha)M_{\varkappa,\mu}(x)\,dx=$ $=\dfrac{e^{\nu\pi i}\,\Gamma(1+2\mu-\nu)\,\Gamma(1+2\mu)\,\Gamma(5/2-\varkappa+\mu+\nu)}{2\,\Gamma(1/2+\varkappa+\mu)}\alpha^{(\varkappa+2\mu-2\nu+5)/4}e^{\alpha/2}W_{\rho,\sigma}(\alpha),$ $\lvert\arg\alpha\rvert<\pi,\ \operatorname{Re}\mu>-1/2,\ \operatorname{Re}(2\mu-\nu)>-1$ $2\rho=1/2-\varkappa-\mu+2\nu,\ 2\sigma=\varkappa-3\mu-3/2$
(15)	$\int_0^\infty x^{(\varkappa+\mu+\nu)/2-1}(\alpha+x)^{-1/2}e^{-x/2}Q^{1-\varkappa+\mu-\nu}_{\varkappa-\mu-\nu-1}\left[(1+x/\alpha)^{1/2}\right]M_{\varkappa,\mu}(x)\,dx=$ $=e^{(1-\varkappa+\mu-\nu)\pi i}2^{\mu-\varkappa-\nu}\alpha^{(\varkappa+\mu-1)/2}\dfrac{\Gamma(1/2-\nu)\,\Gamma(1+2\mu)\,\Gamma(\varkappa+\mu+\nu)}{\Gamma(\varkappa+\mu+1/2)}e^{\alpha/2}W_{\rho,\sigma}(\alpha),$ $\operatorname{Re}\mu>-1/2,\ \operatorname{Re}(\varkappa+\mu+\nu)>0$ $\lvert\arg\alpha\rvert<\pi,\ \rho=1/2-\varkappa-\nu/2,\ \sigma=\mu+\nu/2$
(16)	$\int_0^\infty x^{\nu-1/2}e^{-x/2}Q^{2\mu-2\nu}_{2\varkappa-2\nu-3}\left[(1+x/\alpha)^{1/2}\right]M_{\varkappa,\mu}(x)\,dx=$ $=e^{2(\mu-\nu)\pi i}2^{2\mu-2\nu-1}\alpha^{(\varkappa+\mu-1)/2}e^{\alpha/2}\dfrac{\Gamma(2\mu+1)\,\Gamma(\nu+1)\,\Gamma(\varkappa+\mu-2\nu-1/2)}{\Gamma(\varkappa+\mu+1/2)}W_{\rho,\sigma}(\alpha),$ $\lvert\arg\alpha\rvert<\pi,\ \operatorname{Re}\mu>-1/2,\ \operatorname{Re}\nu>-1,\ \operatorname{Re}(\varkappa+\mu-2\nu)>1/2$ $2\rho=1-\varkappa+\mu-2\nu,\ 2\sigma=\varkappa-\mu-2\nu-2$
(17)	$\int_0^\infty x^{\varkappa-3/2}\exp\left[-2^{-1}(\alpha+1)\,x\right]K_\nu(\alpha x/2)\,M_{\varkappa,\nu}(x)\,dx=$ $=\dfrac{\pi^{1/2}\,\Gamma(\varkappa)\,\Gamma(\varkappa+2\nu)}{\alpha^{\varkappa+\nu}\,\Gamma(\varkappa+\nu+1/2)}\,{}_2F_1(\varkappa,\varkappa+2\nu;\ 2\nu+1;\ -\alpha^{-1}),$ $\operatorname{Re}\alpha>0,\ \operatorname{Re}\varkappa>0,\ \operatorname{Re}(\varkappa+2\nu)>0$
(18)	$\int_0^\infty x^{-1/2}J_\nu(ax^{1/2})\,K_{\nu/2-\mu}(x/2)\,M_{\varkappa,\mu}(x)\,dx=$ $=\dfrac{\Gamma(2\mu+1)}{a\Gamma(\varkappa+\nu/2+1)}W_{(\varkappa-\mu)/2,\,\varkappa/2-\nu/4}\left(\dfrac{a^2}{2}\right)M_{(\varkappa+\mu)/2,\,\varkappa/2+\nu/4}\left(\dfrac{a^2}{2}\right),$ $a>0,\ \operatorname{Re}\varkappa>-1/4,\ \operatorname{Re}\mu>-1/2,\ \operatorname{Re}\nu>-1$
(19)	$\int_0^\infty x^{\rho-1}e^{-x/2}M_{\gamma+\rho,\,\beta+\rho+1/2}(x)\,{}_2F_1(\alpha,\beta;\ \gamma;\ -\lambda/x)\,dx=$ $=\dfrac{\Gamma(\alpha+\beta+2\rho)\,\Gamma(2\beta+2\rho)\,\Gamma(\gamma)}{\Gamma(\beta)\,\Gamma(\beta+\gamma+2\rho)}\lambda^{\beta/2+\rho-1/2}e^{\lambda/2}W_{\varkappa,\mu}(\lambda),$ $\lvert\arg\lambda\rvert<\pi,\ \operatorname{Re}(\beta+\rho)>0,\ \operatorname{Re}(\alpha+\beta+2\rho)>0,\ \operatorname{Re}\gamma>0$ $\varkappa=1/2-\alpha-\beta/2-\rho,\ \mu=\beta/2+\rho$

(20)
$$\int_0^\infty x^{2\lambda-1}(\alpha+x)^{-\mu-1/2}e^{-x/2}M_{\varkappa,\mu}(\alpha+x)\,dx=\frac{\alpha^{\lambda-\mu-1/2}\Gamma(2\lambda)\Gamma(2\mu+1)\Gamma(\varkappa+\mu-2\lambda+1/2)}{\Gamma(\varkappa+\mu+1/2)\Gamma(1-2\lambda+2\mu)}M_{\varkappa-\lambda,\,\mu-\lambda}(\alpha),$$
$$\operatorname{Re}\lambda>0,\quad \operatorname{Re}(\varkappa+\mu-2\lambda)>-1/2$$

(21)
$$\int_0^a x^{-\varkappa-\lambda-1}(a-x)^{\lambda-1}e^{x/2}W_{\varkappa,\mu}(x)\,dx=\frac{\Gamma(\lambda)\Gamma(1/2-\varkappa-\lambda+\mu)\Gamma(1/2-\varkappa-\lambda-\mu)}{a^{\varkappa+1}\Gamma(1/2-\varkappa+\mu)\Gamma(1/2-\varkappa-\mu)}W_{\varkappa+\lambda,\,\mu}(a),$$
$$\operatorname{Re}\lambda>0,\quad \operatorname{Re}(\varkappa+\lambda)<1/2-|\operatorname{Re}\mu|$$

(22)
$$\int_0^\infty W_{\varkappa,\mu}(x)\frac{dx}{x}=\frac{\pi^{3/2}2^{\varkappa}}{\Gamma(3/4-\varkappa/2+\mu/2)\Gamma(3/4-\varkappa/2-\mu/2)\cos(\mu\pi)},$$
$$-1/2<\operatorname{Re}\mu<1/2$$

(23)
$$\int_0^\infty x^{\varkappa+2\mu-1}e^{-3x/2}W_{\varkappa,\mu}(x)\,dx=\frac{\Gamma(\varkappa+\mu+1/2)\Gamma\left[2^{-2}(2\varkappa+6\mu+5)\right]}{(\varkappa+3\mu+1/2)\Gamma\left[2^{-2}(2\mu-2\varkappa+3)\right]},$$
$$\operatorname{Re}(\varkappa+\mu)>-1/2,\quad \operatorname{Re}(\varkappa+3\mu)>-1/2$$

(24)
$$\int_0^\infty x^{\rho-1}\left[x^{1/2}+(\alpha+x)^{1/2}\right]^{2\sigma}e^{-x/2}W_{\varkappa,\mu}(x)\,dx=-\pi^{-1/2}\sigma\alpha^{\sigma}G^{32}_{34}\left(\alpha\,\middle|\,\begin{matrix}1/2,\ 1,\ 1-\varkappa+\rho\\ 1/2+\mu+\rho,\ 1/2-\mu+\rho,\ -\sigma,\ \sigma\end{matrix}\right),$$
$$|\arg\alpha|<\pi,\quad \operatorname{Re}\rho>|\operatorname{Re}\mu|-1/2$$

(25)
$$\int_0^\infty x^{\rho-1}\left[x^{1/2}+(\alpha+x)^{1/2}\right]^{2\sigma}e^{x/2}W_{\varkappa,\mu}(x)\,dx=-\frac{\sigma\pi^{-1/2}\alpha^{\sigma}}{\Gamma(1/2-\varkappa+\mu)\Gamma(1/2-\varkappa-\mu)}G^{33}_{34}\left(\alpha\,\middle|\,\begin{matrix}1/2,\ 1,\ 1+\varkappa+\rho\\ 1/2+\mu+\rho,\ 1/2-\mu+\rho,\ -\sigma,\ \sigma\end{matrix}\right),$$
$$|\arg\alpha|<\pi,\quad \operatorname{Re}\rho>|\operatorname{Re}\mu|-1/2,\quad \operatorname{Re}(\varkappa+\rho+\sigma)<0$$

(26)
$$\int_0^\infty x^{\rho-1}(\alpha+x)^{-1/2}\left[x^{1/2}+(\alpha+x)^{1/2}\right]^{2\sigma}e^{x/2}W_{\varkappa,\mu}(x)\,dx=\frac{\pi^{-1/2}\alpha^{\sigma}}{\Gamma(1/2-\varkappa+\mu)\Gamma(1/2-\varkappa-\mu)}G^{33}_{34}\left(\alpha\,\middle|\,\begin{matrix}0,\ 1/2,\ 1/2+\varkappa+\rho\\ -\sigma,\ \rho+\mu,\ \rho-\mu,\ \sigma\end{matrix}\right),$$
$$|\arg\alpha|<\pi,\quad \operatorname{Re}\rho>|\operatorname{Re}\mu|-1/2,\quad \operatorname{Re}(\varkappa+\rho+\sigma)<1/2$$

(27)	$\int\limits_0^\infty x^{\rho-1}(\alpha+x)^{-1/2}\left[x^{1/2}+(\alpha+x)^{1/2}\right]^{2\sigma}e^{-x/2}W_{\varkappa,\mu}(x)\,dx = \pi^{-1/2}\alpha^{\sigma}G_{34}^{32}\left(\alpha\left\vert\begin{matrix}0,\ 1/2,\ 1/2-\varkappa+\rho\\ -\sigma,\ \rho+\mu,\ \rho-\mu,\ \sigma\end{matrix}\right.\right)$, $\vert\arg\alpha\vert<\pi$, $\operatorname{Re}\rho>\vert\operatorname{Re}\mu\vert-1/2$
(28)	$\int\limits_0^\infty x^{\rho-1}\sin\left(cx^{1/2}\right)e^{-x/2}W_{\varkappa,\mu}(x)\,dx = \frac{\Gamma(1+\mu+\rho)\Gamma(1-\mu+\rho)c}{\Gamma(3/2-\varkappa+\rho)}\,{}_2F_2\left(1+\mu+\rho,\ 1-\mu+\rho;\ \frac{3}{2},\ \frac{3}{2}-\varkappa+\rho;\ -\frac{c^2}{4}\right)$, $\operatorname{Re}\rho>\vert\operatorname{Re}\mu\vert-1$
(29)	$\int\limits_0^\infty x^{\rho-1}\sin\left(cx^{1/2}\right)e^{x/2}W_{\varkappa,\mu}(x)\,dx = \frac{\pi^{1/2}}{\Gamma(1/2-\varkappa+\mu)\Gamma(1/2-\varkappa-\mu)}G_{23}^{22}\left(\frac{c^2}{4}\left\vert\begin{matrix}1/2+\mu-\rho,\ 1/2-\mu-\rho\\ 1/2,\ -\varkappa-\rho,\ 0\end{matrix}\right.\right)$, $c>0$, $\operatorname{Re}\rho>\vert\operatorname{Re}\mu\vert-1$, $\operatorname{Re}(\varkappa+\rho)<1/2$
(30)	$\int\limits_0^\infty x^{\rho-1}\cos\left(cx^{1/2}\right)e^{-x/2}W_{\varkappa,\mu}(x)\,dx = \frac{\Gamma(1/2+\mu+\rho)\Gamma(1/2-\mu+\rho)}{\Gamma(1-\varkappa+\rho)}\,{}_2F_2\left(\frac{1}{2}+\mu+\rho,\ \frac{1}{2}-\mu+\rho;\ \frac{1}{2},\ 1-\varkappa+\rho;\ -\frac{c^2}{4}\right)$, $\operatorname{Re}\rho>\vert\operatorname{Re}\mu\vert-1/2$
(31)	$\int\limits_0^\infty x^{\rho-1}\cos\left(cx^{1/2}\right)e^{x/2}W_{\varkappa,\mu}(x)\,dx = \frac{\pi^{1/2}}{\Gamma(1/2-\varkappa+\mu)\Gamma(1/2-\varkappa-\mu)}G_{23}^{22}\left(\frac{c^2}{4}\left\vert\begin{matrix}1/2+\mu-\rho,\ 1/2-\mu-\rho\\ 0,\ -\varkappa-\rho,\ 1/2\end{matrix}\right.\right)$, $c>0$, $\operatorname{Re}\rho>\vert\operatorname{Re}\mu\vert-1/2$, $\operatorname{Re}(\varkappa+\rho)<1/2$
(32)	$\int\limits_0^\infty x^{-1/2-\mu/2-\nu}(\alpha+x)^{\mu/2}e^{-x/2}P_{\varkappa+\nu-3/2}^{\mu}\left(1+2x\alpha^{-1}\right)W_{\varkappa,\nu}(x)\,dx = \frac{\Gamma(1-\mu-2\nu)}{\Gamma(3/2-\varkappa-\mu-\nu)}\alpha^{-1/4+\varkappa/2-\nu/2}e^{\alpha/2}W_{\rho,\sigma}(\alpha)$, $2\rho=1/2+2\mu+\nu-\varkappa$, $2\sigma=\varkappa+3\nu-3/2$, $\vert\arg\alpha\vert<\pi$, $\operatorname{Re}\mu<1$, $\operatorname{Re}(\mu+2\nu)<1$
(33)	$\int\limits_0^\infty x^{-1/2-\mu/2-\nu}(\alpha+x)^{-\mu/2}e^{-x/2}P_{\varkappa+\mu+\nu-3/2}^{\mu}\left(1+2x\alpha^{-1}\right)W_{\varkappa,\nu}(x)\,dx = \frac{\Gamma(1-\mu-2\nu)}{\Gamma(3/2-\varkappa-\mu-\nu)}\alpha^{-1/4+\varkappa/2-\nu/2}e^{\alpha/2}W_{\rho,\sigma}(\alpha)$, $2\rho=1/2-\varkappa+\nu$, $2\sigma=\varkappa+2\mu+3\nu-3/2$, $\vert\arg\alpha\vert<\pi$, $\operatorname{Re}\mu<1$, $\operatorname{Re}(\mu+2\nu)<1$

(34)	$\int_0^\infty x^{-1/2-\mu/2-\nu} e^{-x/2} P^{\mu}_{2\varkappa+\mu+2\nu-3}\left[(1+x/\alpha)^{1/2}\right] W_{\varkappa,\nu}(x)\,dx = \frac{2^{\mu}\,\Gamma(1-\mu-2\nu)}{\Gamma(3/2-\varkappa-\mu-\nu)}\,\alpha^{-1/2+\varkappa/2-\nu/2} e^{\alpha/2}\, W_{\rho,\sigma}(\alpha)$, $2\rho=1-\varkappa+\mu+\nu, \quad 2\sigma=\varkappa+\mu+3\nu-2$, $\lvert\arg\alpha\rvert<\pi, \quad \mathrm{Re}\,\mu<1, \quad \mathrm{Re}\,(\mu+2\nu)<1$
(35)	$\int_0^\infty x^{-1/2-\mu/2-\nu} (\alpha+x)^{-1/2} e^{-x/2} P^{\mu}_{2\varkappa+\mu+2\nu-2}\left[(1+x/\alpha)^{1/2}\right] W_{\varkappa,\nu}(x)\,dx = \frac{2^{\mu}\,\Gamma(1-\mu-2\nu)}{\Gamma(3/2-\varkappa-\mu-\nu)}\,\alpha^{-1/2+\varkappa/2-\nu/2} e^{\alpha/2}\, W_{\rho,\sigma}(\alpha)$, $2\rho=\mu+\nu-\varkappa, \quad 2\sigma=\varkappa+\mu+3\nu-1, \quad \lvert\arg\alpha\rvert<\pi, \quad \mathrm{Re}\,\mu>0, \quad \mathrm{Re}\,\nu>0$
(36)	$\int_0^\infty x^{-\varkappa-3/2} \exp\left[-2^{-1}(\alpha-1)x\right] K_{\mu}(\alpha x/2)\, W_{\varkappa,\mu}(x)\,dx = \frac{\pi\,\Gamma(-\varkappa)\,\Gamma(2\mu-\varkappa)\,\Gamma(-2\mu-\varkappa)}{\Gamma(1/2-\varkappa)\,\Gamma(1/2+\mu-\varkappa)\,\Gamma(1/2-\mu-\varkappa)}\, 2^{2\varkappa+1}\alpha^{\varkappa-\nu} \times$ $\times\, {}_2F_1(-\varkappa, 2\mu-\varkappa; -2\varkappa; 1-\alpha^{-1}), \quad \mathrm{Re}\,\alpha>0, \quad \mathrm{Re}\,\varkappa<2\,\mathrm{Re}\,\mu<-\mathrm{Re}\,\varkappa$
(37)	$\int_0^\infty x^{\rho-1} e^{-x/2} J_{\lambda+\nu}(\alpha x^{1/2})\, J_{\lambda-\nu}(\alpha x^{1/2})\, W_{\varkappa,\mu}(x)\,dx = \frac{(\alpha/2)^{2\lambda}\,\Gamma(1/2+\lambda+\mu+\rho)\,\Gamma(1/2+\lambda-\mu+\rho)}{\Gamma(1+\lambda+\nu)\,\Gamma(1+\lambda-\nu)\,\Gamma(1+\lambda-\varkappa+\rho)} \times$ $\times\, {}_4F_4(1+\lambda, 1/2+\lambda, 1/2+\lambda+\mu+\rho, 1/2+\lambda-\mu+\rho;$ $1+\lambda+\nu, 1+\lambda-\nu, 1+2\lambda, 1+\lambda-\varkappa+\rho; -\alpha^2)$, $\lvert\mathrm{Re}\,\mu\rvert<\mathrm{Re}\,(\lambda+\rho)+1/2$
(38)	$\int_0^\infty x^{\rho-1} e^{-x/2} I_{\lambda+\nu}(\alpha x^{1/2})\, K_{\lambda-\nu}(\alpha x^{1/2})\, W_{\varkappa,\mu}(x)\,dx = \frac{1}{2\pi^{1/2}}\, G^{24}_{45}\left(\alpha^2 \left\lvert \begin{matrix} 0, 1/2, 1/2+\mu-\rho, 1/2-\mu-\rho \\ \lambda, \nu, -\lambda, -\nu, \varkappa-\rho \end{matrix}\right.\right)$, $\lvert\mathrm{Re}\,\mu\rvert<\mathrm{Re}\,(\lambda+\rho)+1/2, \quad \lvert\mathrm{Re}\,\mu\rvert<\mathrm{Re}\,(\nu+\rho)+1/2$
(39)	$\int_0^\infty x^{-1} M_{\varkappa,\mu}(x)\, W_{\lambda,\mu}(x)\,dx = \frac{\Gamma(2\mu+1)}{(\varkappa-\lambda)\,\Gamma(1/2+\mu-\lambda)}$, $\mathrm{Re}\,\mu>-1/2, \quad \mathrm{Re}\,(\varkappa-\lambda)>0$

(40)	$\int_0^\infty x^{-1} W_{\varkappa,\mu}(x)\, W_{\lambda,\mu}(x)\,dx =$ $= \frac{1}{(\varkappa-\lambda)\sin(2\mu\pi)}\left[\frac{1}{\Gamma(1/2-\varkappa+\mu)\,\Gamma(1/2-\lambda-\mu)} - \frac{1}{\Gamma(1/2-\varkappa-\mu)\,\Gamma(1/2-\lambda+\mu)}\right],$ $-1/2 < \operatorname{Re}\mu < 1/2$
(41)	$\int_0^\infty x^{\rho-1} W_{\varkappa,\mu}(x)\, W_{-\varkappa,\mu}(x)\,dx = \frac{\Gamma(\rho+1)\,\Gamma\left(\frac{\rho+1}{2}+\mu\right)\Gamma\left(\frac{\rho+1}{2}-\mu\right)}{2\,\Gamma(1+\rho/2+\varkappa)\,\Gamma(1+\rho/2-\varkappa)},$ $\operatorname{Re}\rho > 2\,\lvert\operatorname{Re}\mu\rvert - 1$
(42)	$\int_0^\infty x^{\rho-1} W_{\varkappa,\mu}(x)\, W_{\lambda,\nu}(x)\,dx = \frac{\Gamma(1+\mu+\nu+\rho)\,\Gamma(1-\mu+\nu+\rho)\,\Gamma(-2\nu)}{\Gamma(1/2-\lambda-\nu)\,\Gamma(3/2-\varkappa+\nu+\rho)} \times$ $\times\, {}_3F_2(1+\mu+\nu+\rho,\ 1-\mu+\nu+\rho,\ 1/2-\lambda+\nu;\ 1+2\nu,\ 3/2-\varkappa+\nu+\rho;\ 1) +$ $+ \frac{\Gamma(1+\mu-\nu+\rho)\,\Gamma(1-\mu-\nu+\rho)\,\Gamma(2\nu)}{\Gamma(1/2-\lambda+\nu)\,\Gamma(3/2-\varkappa-\nu+\rho)} \times$ $\times\, {}_3F_2(1+\mu-\nu+\rho,\ 1-\mu-\nu+\rho,\ 1/2-\lambda-\nu;\ 1-2\nu,\ 3/2-\varkappa-\nu+\rho;\ 1),$ $\lvert\operatorname{Re}\mu\rvert + \lvert\operatorname{Re}\nu\rvert < \operatorname{Re}\rho + 1$
(43)	$\int_0^\infty x^{\rho-1} \exp[-2^{-1}(\alpha+\beta)x]\, M_{\varkappa,\mu}(\alpha x)\, W_{\lambda,\nu}(\beta x)\,dx =$ $= \frac{\Gamma(1+\mu+\nu+\rho)\,\Gamma(1+\mu-\nu+\rho)}{\Gamma(3/2-\lambda+\mu+\rho)}\,\alpha^{\mu+1/2}\beta^{-\mu-\rho-1/2} \times$ $\times\, {}_3F_2(1/2+\varkappa+\mu,\ 1+\mu+\nu+\rho,\ 1+\mu-\nu+\rho;$ $2\mu+1,\ 3/2-\lambda+\mu+\rho;\ -\alpha/\beta),$ $\operatorname{Re}\alpha > 0,\ \operatorname{Re}\beta > 0,\ \operatorname{Re}(\rho+\mu) > \lvert\operatorname{Re}\nu\rvert - 1$
(44)	$\int_0^\infty x^{\rho-1} \exp[2^{-1}(\alpha+\beta)x]\, W_{\varkappa,\mu}(\alpha x)\, W_{\lambda,\nu}(\beta x)\,dx =$ $= \beta^{-\rho}\,[\Gamma(1/2-\varkappa+\mu)\,\Gamma(1/2-\varkappa-\mu)\,\Gamma(1/2-\lambda+\nu)\,\Gamma(1/2-\lambda-\nu)]^{-1} \times$ $\times\, G_{33}^{33}\left(\frac{\beta}{\alpha}\,\middle\vert\,\begin{matrix} 1/2+\mu,\ 1/2-\mu,\ 1+\lambda+\rho \\ 1/2+\nu+\rho,\ 1/2-\nu+\rho,\ -\varkappa \end{matrix}\right),$ $\lvert\operatorname{Re}\mu\rvert + \lvert\operatorname{Re}\nu\rvert < \operatorname{Re}\rho + 1,\quad \operatorname{Re}(\varkappa+\lambda+\rho) < 0$
(45)	$\int_0^\infty x^{\rho-1} \exp[-2^{-1}(\alpha-\beta)x]\, W_{\varkappa,\mu}(\alpha x)\, W_{\lambda,\nu}(\beta x)\,dx =$ $= \frac{\beta^{-\rho}}{\Gamma(1/2-\lambda+\nu)\,\Gamma(1/2-\lambda-\nu)}\, G_{33}^{23}\left(\frac{\beta}{\alpha}\,\middle\vert\,\begin{matrix} 1/2+\mu,\ 1/2-\mu,\ 1+\lambda+\rho \\ 1/2+\nu+\rho,\ 1/2-\nu+\rho,\ \varkappa \end{matrix}\right),$ $\operatorname{Re}\alpha > 0,\quad \lvert\operatorname{Re}\mu\rvert + \lvert\operatorname{Re}\nu\rvert < \operatorname{Re}\rho + 1$

(46)	$\int_0^\infty x^{\rho-1} \exp\left[-2^{-1}(\alpha+\beta)x\right] W_{\varkappa,\mu}(\alpha x)\, W_{\lambda,\nu}(\beta x)\,dx =$ $= \beta^{-\rho} G^{22}_{33}\left(\frac{\beta}{\alpha}\,\middle\vert\, \begin{matrix} 1/2+\mu,\ 1/2-\nu,\ 1-\lambda+\rho \\ 1/2+\nu+\rho,\ 1/2-\nu+\rho,\ \varkappa \end{matrix}\right),$ $\operatorname{Re}(\alpha+\beta)>0, \quad \lvert\operatorname{Re}\mu\rvert+\lvert\operatorname{Re}\nu\rvert<\operatorname{Re}\rho+1$
(47)	$\int_0^\infty x^{2\lambda-1}(\alpha+x)^{-\mu-1/2} e^{-x/2} W_{\varkappa,\mu}(\alpha+x)\,dx =$ $= \Gamma(2\lambda)\,\alpha^{\lambda-\mu-1/2} W_{\varkappa-\lambda,\,\mu-\lambda}(\alpha), \quad \lvert\arg\alpha\rvert<\pi, \ \operatorname{Re}\lambda>0$
(48)	$\int_0^\infty x^{\lambda-1}(\alpha+x)^{\varkappa-\lambda-1} e^{-x/2} W_{\varkappa,\mu}(\alpha+x)\,dx = \Gamma(\lambda)\,\alpha^{\varkappa-1} W_{\varkappa-\lambda,\,\mu}(\alpha),$ $\lvert\arg\alpha\rvert<\pi, \quad \operatorname{Re}\lambda>0$
(49)	$\int_0^\infty x^{\rho-1}(\alpha+x)^{-\sigma} e^{-x/2} W_{\varkappa,\mu}(\alpha+x)\,dx =$ $= \Gamma(\rho)\,\alpha^{\rho} e^{\alpha/2} G^{30}_{23}\left(\alpha\,\middle\vert\, \begin{matrix} 0,\ 1-\varkappa-\sigma \\ -\rho,\ 1/2+\mu-\sigma,\ 1/2-\mu-\sigma \end{matrix}\right), \quad \lvert\arg\alpha\rvert<\pi, \ \operatorname{Re}\rho>0$
(50)	$\int_0^\infty x^{2\lambda-1}(\alpha+x)^{-\mu-1/2} e^{x/2} W_{\varkappa,\mu}(\alpha+x)\,dx =$ $= \frac{\Gamma(2\lambda)\,\Gamma(1/2-\varkappa+\mu-2\lambda)}{\Gamma(1/2-\varkappa+\mu)}\,\alpha^{\lambda-\mu-1/2} W_{\varkappa+\lambda,\,\mu-\lambda}(\alpha),$ $\lvert\arg\alpha\rvert<\pi, \quad 0<2\operatorname{Re}\lambda<1/2-\operatorname{Re}(\varkappa+\mu)$
(51)	$\int_0^\infty x^{\rho-1}(\alpha+x)^{-\sigma} e^{x/2} W_{\varkappa,\mu}(\alpha+x)\,dx =$ $= \frac{\Gamma(\rho)\,\alpha^{\rho} e^{-\alpha/2}}{\Gamma(1/2-\varkappa+\mu)\,\Gamma(1/2-\varkappa-\mu)}\, G^{31}_{23}\left(\alpha\,\middle\vert\, \begin{matrix} \varkappa-\sigma+1,\ 0 \\ -\rho,\ 1/2+\mu-\sigma,\ 1/2-\mu-\sigma \end{matrix}\right),$ $\lvert\arg\alpha\rvert<\pi, \quad 0<\operatorname{Re}\rho<\operatorname{Re}(\sigma-\varkappa)$
(52)	$\int_0^\infty x^{\tau-1}(\alpha+x)^{-\lambda}\,{}_2F_1(\rho,\sigma;\tau;-x/\alpha)\, e^{\pm x/2} W_{\varkappa,\mu}(\alpha+x)\,dx.$ Выразить $W_{\varkappa,\mu}$ через G-функцию и найти соответствующий интеграл в п. 20.5.

Интегралы, содержащие произведения $e^{\pm x/2} W_{\varkappa,\mu}(\alpha+x)$ и функции Лежандра.

Выразить $W_{\varkappa,\mu}$ через G-функцию, а функцию Лежандра — через гипергеометрическую функцию и найти соответствующий интеграл в п. 20.5.

(53)	$\int\limits_0^\infty x^{-1/2}\exp\left[-\frac{1}{2}\left(\frac{x}{\alpha}+\frac{\beta}{x}\right)\right]W_{\varkappa,\mu}\left(\frac{x}{\alpha}\right)W_{\varkappa,\mu}\left(\frac{\beta}{x}\right)dx=$ $=\pi^{1/2}2^{1/2-2\varkappa}(\alpha\beta)^{1/4}\exp\left[-\left(\frac{\beta}{\alpha}\right)^{1/2}\right]W_{2\varkappa-1/2,\,2\mu}\left[2\left(\frac{\beta}{\alpha}\right)^{1/2}\right],$ $\operatorname{Re}\alpha>0,\ \operatorname{Re}\beta>0$
(54)	$\int\limits_0^\infty x^{\rho-1}\exp\left[-\frac{1}{2}\left(\frac{x}{\alpha}+\frac{\beta}{x}\right)\right]W_{\varkappa,\mu}\left(\frac{x}{\alpha}\right)W_{\lambda,\nu}\left(\frac{\beta}{x}\right)dx=$ $=\beta^\rho G^{40}_{24}\left(\frac{\beta}{\alpha}\,\middle\vert\,\begin{matrix}1-\varkappa,\ 1-\lambda-\rho\\ 1/2+\mu,\ 1/2-\mu,\ 1/2+\nu-\rho,\ 1/2-\nu-\rho\end{matrix}\right),\quad \operatorname{Re}\alpha>0,\ \operatorname{Re}\beta>0$
(55)	$\int\limits_0^\infty x^{\rho-1}\exp\left[\frac{1}{2}\left(\frac{x}{\alpha}-\frac{\beta}{x}\right)\right]W_{\varkappa,\mu}\left(\frac{x}{\alpha}\right)W_{\lambda,\nu}\left(\frac{\beta}{x}\right)dx=$ $=\frac{\beta^\rho}{\Gamma(1/2-\varkappa+\mu)\,\Gamma(1/2-\varkappa-\mu)}G^{41}_{24}\left(\frac{\beta}{\alpha}\,\middle\vert\,\begin{matrix}1+\varkappa,\ 1-\lambda-\rho\\ 1/2+\mu,\ 1/2-\mu,\ 1/2+\nu-\rho,\ 1/2-\nu-\rho\end{matrix}\right),$ $\lvert\arg\alpha\rvert<3\pi/2,\ \operatorname{Re}\beta>0,\ \operatorname{Re}(\varkappa+\rho)<-\lvert\operatorname{Re}\nu\rvert-1/2$
(56)	$\int\limits_0^\infty x^{-1/2}\exp\left[\frac{1}{2}\left(\frac{x}{\alpha}+\frac{\beta}{x}\right)\right]W_{\varkappa,\mu}\left(\frac{x}{\alpha}\right)W_{\varkappa,\mu}\left(\frac{\beta}{x}\right)dx=$ $=\frac{\pi^{1/2}\,\Gamma(-\varkappa-\mu)\,\Gamma(-\varkappa+\mu)\,(\alpha\beta)^{1/4}}{2^{1/2+2\varkappa}\,\Gamma(1/2-\varkappa+\mu)\,\Gamma(1/2-\varkappa-\mu)}\exp\left[\left(\frac{\beta}{\alpha}\right)^{1/2}\right]W_{2\varkappa+1/2,\,2\mu}\left[2\left(\frac{\beta}{\alpha}\right)^{1/2}\right],$ $\lvert\arg\alpha\rvert<3\pi/2,\ \lvert\arg\beta\rvert<3\pi/2,\ \operatorname{Re}\varkappa<-\lvert\operatorname{Re}\mu\rvert$
(57)	$\int\limits_0^\infty x^{\rho-1}\exp\left[\frac{1}{2}\left(\frac{x}{\alpha}+\frac{\beta}{x}\right)\right]W_{\varkappa,\mu}\left(\frac{x}{\alpha}\right)W_{\lambda,\nu}\left(\frac{\beta}{x}\right)dx=$ $=\frac{\beta^\rho}{\Gamma(1/2-\varkappa+\mu)\,\Gamma(1/2-\varkappa-\mu)\,\Gamma(1/2-\lambda+\nu)\,\Gamma(1/2-\lambda-\nu)}\times$ $\times G^{42}_{24}\left(\frac{\beta}{\alpha}\,\middle\vert\,\begin{matrix}1+\varkappa,\ 1+\lambda-\rho\\ 1/2+\mu,\ 1/2-\mu,\ 1/2+\nu-\rho,\ 1/2-\nu-\rho\end{matrix}\right),$ $\lvert\arg\alpha\rvert<3\pi/2,\ \lvert\arg\beta\rvert<3\pi/2$ $\operatorname{Re}(\lambda-\rho)<1/2-\lvert\operatorname{Re}\mu\rvert,\ \operatorname{Re}(\varkappa+\rho)<1/2-\lvert\operatorname{Re}\nu\rvert$
(58)	$\int\limits_{-\infty}^\infty e^{-2\rho xi}\,\Gamma(1/2+\nu+ix)\,\Gamma(1/2+\nu-ix)\,M_{ix,\,\nu}(2\alpha)\,dx=$ $=2^{1/2-\nu}\pi\beta^{\nu+1/2}(\operatorname{ch}\rho)^{-2\nu-1}\exp(-\alpha\operatorname{th}\rho)\,\Gamma(2\nu+1),$ $\lvert\operatorname{Im}\rho\rvert<\pi/2,\ \operatorname{Re}\nu>-1/2$

(59)	$\int\limits_{-i\infty}^{i\infty} \Gamma(1/2+\nu+\mu+x)\,\Gamma(1/2+\nu+\mu-x)\,\Gamma(1/2+\nu-\mu+x)\,\Gamma(1/2+\nu-\mu-x)\times$ $\times M_{\mu+ix,\,\nu}(\alpha)\,M_{\mu-ix,\,\nu}(\beta)\,dx =$ $= \frac{2\pi(\alpha\beta)^{\nu+1/2}[\Gamma(2\nu+1)]^2\,\Gamma(2\nu+2\mu+1)\,\Gamma(2\nu-2\mu+1)}{(\alpha+\beta)^{2\nu+1}\,\Gamma(4\nu+2)}\,M_{2\mu,\,2\nu+1/2}(\alpha+\beta),$ $\operatorname{Re}\nu > \lvert\operatorname{Re}\mu\rvert - 1/2$
(60)	$\int\limits_{-\infty}^{\infty} e^{-2\rho x i}\,\Gamma(1/2+\nu+ix)\,\Gamma(1/2+\nu-ix)\,M_{ix,\,\nu}(\alpha)\,M_{ix,\,\nu}(\beta)\,dx =$ $= \frac{2\pi(\alpha\beta)^{1/2}}{\operatorname{ch}\rho}\exp[-(\alpha+\beta)\operatorname{th}\rho]\,J_{2\nu}\left(\frac{2\alpha^{1/2}\beta^{1/2}}{\operatorname{ch}\rho}\right),\quad \lvert\operatorname{Im}\rho\rvert < \pi/2,\ \operatorname{Re}\nu > -1/2$
(61)	$\int\limits_{-\infty}^{\infty} \frac{W_{ix,\,0}(\alpha)\,W_{-ix,\,0}(\beta)\,dx}{\operatorname{ch}(\pi x)} = 2\,\frac{(\alpha\beta)^{1/2}}{\alpha+\beta}\exp\left[-2^{-1}(\alpha+\beta)\right]$
(62)	$\int\limits_{-\infty}^{\infty} \Gamma(ix)\,\Gamma(2\varkappa+ix)\,W_{\varkappa+ix,\,\varkappa-1/2}(\alpha)\,W_{-\varkappa-ix,\,\varkappa-1/2}(\beta)\,dx =$ $= 2\pi^{1/2}\,\Gamma(2\varkappa)\,(\alpha\beta)^{\varkappa}\,(\alpha+\beta)^{1/2-2\varkappa}\,K_{2\varkappa-1/2}\left(\frac{\alpha+\beta}{2}\right)$

Относительно многих интегралов по параметрам см. Buchholz Herbert, 1953: Die konfluente hypergeometrische Funktions. Springer-Verlag, глава VI.

20.4. E-функция Мак-Роберта

См. также п. 20.5.

(1)	$\int\limits_0^1 x^{\beta-1}(1-x)^{\gamma-\beta-1}\,E(\alpha_1,\ \ldots,\ \alpha_p : \rho_1,\ \ldots,\ \rho_q : xz)\,dx.$ См. MacRobert T. M., 1953: Proc. Glasgow Math. Assoc. 1, 118.
(2)	$\int\limits_0^1 x^{\beta-1}(1-x)^{\gamma-\beta-1}\,E(\alpha_1,\ \ldots,\ \alpha_p : \rho_1,\ \ldots,\ \rho_q : x^{-m}z)\,dx =$ $= \Gamma(\gamma-\beta)\,m^{\beta-\gamma}\,E(\alpha_1,\ \ldots,\ \alpha_{p+m} : \rho_1,\ \ldots,\ \rho_{q+m} : z),$ $\alpha_{p+k} = \frac{\beta+k-1}{m},\quad \rho_{q+k} = \frac{\gamma+k-1}{m},\quad k = 1,\ \ldots,\ m$ $\operatorname{Re}\gamma > \operatorname{Re}\beta > 0,\ m = 1,\ 2,\ \ldots$

(3)	$\int_0^\infty x^{\rho-1}(1+x)^{-\sigma} E[\alpha_1, \ldots, \alpha_p : \rho_1, \ldots, \rho_q : (1+x) z]\, dx =$ $= \Gamma(\rho) E(\alpha_1, \ldots, \alpha_p, \sigma-\rho : \rho_1, \ldots, \rho_q, \sigma : z), \quad \operatorname{Re}\sigma > \operatorname{Re}\rho > 0$
(4)	$\int_0^\infty x^{\beta-1} e^{-x} E(\alpha_1, \ldots, \alpha_p : \rho_1, \ldots, \rho_q : xz)\, dx =$ $= \frac{\pi}{\sin(\beta\pi)} \left[E\left(\alpha_1, \ldots, \alpha_p : 1-\beta, \rho_1, \ldots, \rho_q : e^{\pm i\pi} z\right) -\right.$ $\left. - z^{-\beta} E\left(\alpha_1+\beta, \ldots, \alpha_p+\beta : 1+\beta, \rho_1+\beta, \ldots, \rho_q+\beta : e^{\pm i\pi} z\right)\right],$ $p \geqslant q+1, \operatorname{Re}(\alpha_r+\beta) > 0, r = 1, \ldots, p, \lvert\arg z\rvert < \pi.$ При $p \leqslant q$ результат справедлив, если интеграл сходится.
(5)	$\int_0^\infty x^{\beta-1} e^{-x} E\left(\alpha_1, \ldots, \alpha_p : \rho_1, \ldots, \rho_q : x^{-m} z\right) dx =$ $= (2\pi)^{1/2-m/2} m^{\beta-1/2} E(\alpha_1, \ldots, \alpha_{p+m} : \rho_1, \ldots, \rho_q : m^{-m} z),$ $\operatorname{Re}\beta > 0, m = 1, 2, \ldots, \alpha_{p+k} = (\beta+k-1)/m, k = 1, \ldots, m$
(6)	$\int_{-1}^1 (1-x)^{-\alpha_p} (1-x^2)^{-\mu/2} \mathrm{P}_{n-\mu}^{\mu}(x) E\left[\alpha_1, \ldots, \alpha_p : \rho_1, \ldots, \rho_q : (1-x) z\right] dx.$ См. MacRobert T. M., 1953: Proc. Glasgow Math. Assoc. 1, 111–114.
(7)	$\int_0^\infty x^{\beta-1} J_\nu(x) E\left(\alpha_1, \ldots, \alpha_p : \rho_1, \ldots, \rho_q : x^{-2m} z\right) dx =$ $= (2\pi)^{-m} (2m)^{\beta-1} \{\exp[2^{-1}\pi(\beta-\nu-1) i] \times$ $\times E[\alpha_1, \ldots, \alpha_{p+2m} : \rho_1, \ldots, \rho_q : (2m)^{-2m} z e^{-m\pi i}] +$ $+ \exp[-2^{-1}\pi(\beta-\nu-1) i] E[\alpha_1, \ldots, \alpha_{p+2m} : \rho_1, \ldots, \rho_q : (2m)^{-2m} z e^{m\pi i}]\},$ $\operatorname{Re}(\beta+\nu) > 0, \operatorname{Re}(2\alpha_r m - \beta) > -3/2, \quad r = 1, \ldots, p$ $\alpha_{p+k} = \frac{\beta+\nu+2k-2}{2m}, \quad \alpha_{p+m+k} = \frac{\beta-\nu+2k-2}{2m}$ $m = 1, 2, \ldots, k = 1, \ldots, m$
(8)	$\int_0^\infty x^{\beta-1} K_\nu(x) E\left(\alpha_1, \ldots, \alpha_p : \rho_1, \ldots, \rho_q : x^{-2m} z\right) dx =$ $= (2\pi)^{1-m} 2^{\beta-2} m^{\beta-1} E[\alpha_1, \ldots, \alpha_{p+2m} : \rho_1, \ldots, \rho_q : (2m)^{-2m} z],$ $\alpha_{p+k} = \frac{\beta+\nu+2k-2}{2m}, \quad \alpha_{p+m+k} = \frac{\beta-\nu+2k-2}{2m}$ $\operatorname{Re}\beta > \lvert\operatorname{Re}\nu\rvert, m = 1, 2, \ldots, k = 1, \ldots, m$

(9)	$\int\limits_0^\infty x^{\beta-1} e^x K_\nu(x) E(\alpha_1, \ldots, \alpha_p : \rho_1, \ldots, \rho_q : z/x)\, dx.$ См. Ragab F. M., 1953: Proc. Glasgow Math. Assoc. 1, 192—195.
(10)	$\int\limits_0^\infty x^{\beta-1} e^{-x/2} W_{\varkappa, \mu}(x) E(\alpha_1, \ldots, \alpha_p : \rho_1, \ldots, \rho_q : x^{-m} z)\, dx =$ $= (2\pi)^{1/2 - m/2} m^{\beta+\varkappa-1/2} E(\alpha_1, \ldots, \alpha_{p+2m} : \rho_1, \ldots, \rho_{q+m} : m^{-m} z),$ $\operatorname{Re}\beta > \lvert \operatorname{Re}\mu \rvert - 1/2, \quad m = 1, 2, \ldots$ $\alpha_{p+k} = (\beta + k + \mu - 1/2)/m, \quad \alpha_{p+m+k} = (\beta - \mu + k - 1/2)/m$ $\rho_{q+k} = (\beta - \varkappa + k)/m, \quad k = 1, \ldots, m$
(11)	$\int\limits_0^\infty x^{\lambda-1} E(\alpha_1, \ldots, \alpha_p : \rho_1, \ldots, \rho_q : xy) E(\beta_1, \ldots, \beta_r : \sigma_1, \ldots, \sigma_s : xz)\, dx,$ $\int\limits_0^\infty x^{\lambda-1} E(\alpha_1, \ldots, \alpha_p : \rho_1, \ldots, \rho_q : xy) E(\beta_1, \ldots, \beta_r : \sigma_1, \ldots, \sigma_s : z/x)\, dx.$ См. Ragab F. M., 1953: Proc. Glasgow Math. Assoc. 1, 192—195.

20.5. *G*-функция Мейера

(1)	$\int\limits_0^1 x^{\rho-1} (1-x)^{\sigma-1} G_{pq}^{mn}\left(\alpha x \,\middle\vert\, \begin{matrix} a_1, \ldots, a_p \\ b_1, \ldots, b_q \end{matrix}\right) dx =$ $= \Gamma(\sigma)\, G_{p+1, q+1}^{m, n+1}\left(\alpha \,\middle\vert\, \begin{matrix} 1-\rho, a_1, \ldots, a_p \\ b_1, \ldots, b_q, 1-\rho-\sigma \end{matrix}\right).$ Первое множество условий справедливости формулы: $p + q < 2(m+n), \quad \lvert \arg\alpha \rvert < (m + n - p/2 - q/2)\pi,$ $\operatorname{Re}(\rho + b_j) > 0, \quad j = 1, \ldots, m, \quad \operatorname{Re}\sigma > 0.$ Второе множество условий справедливости формулы: $p + q \leqslant 2(m+n), \quad \lvert \arg\alpha \rvert \leqslant (m + n - p/2 - q/2)\pi,$ $\operatorname{Re}(\rho + b_j) > 0, \quad j = 1, \ldots, m, \quad \operatorname{Re}\sigma > 0,$ $\operatorname{Re}\left[\sum_{j=1}^p a_j - \sum_{j=1}^q b_j + (p-q)(\rho - 1/2)\right] > -1/2.$ Третье множество условий справедливости формулы: $p < q$ (или $p \leqslant q$ и $\lvert\alpha\rvert < 1$), $\operatorname{Re}(\rho + b_j) > 0, \quad j = 1, \ldots, m, \quad \operatorname{Re}\sigma > 0$

(2)	$\int\limits_1^\infty x^{-\rho}(x-1)^{\sigma-1}G_{pq}^{mn}\left(\alpha x\,\middle\vert\,\begin{matrix}a_1,\ldots,a_p\\b_1,\ldots,b_q\end{matrix}\right)dx=$ $=\Gamma(\sigma)\,G_{p+1,\,q+1}^{m+1,\,n}\left(\alpha\,\middle\vert\,\begin{matrix}a_1,\ldots,a_p,\,\rho\\\rho-\sigma,\,b_1,\ldots,b_q\end{matrix}\right).$ Первое множество условий справедливости формулы: $p+q<2(m+n),\quad \vert\arg\alpha\vert<(m+n-p/2-q/2)\pi,$ $\operatorname{Re}(\rho-\sigma-a_j)>-1,\quad j=1,\ldots,n,\quad \operatorname{Re}\sigma>0.$ Второе множество условий справедливости формулы: $p+q\leqslant 2(m+n),\quad \vert\arg\alpha\vert\leqslant(m+n-p/2-q/2)\pi,$ $\operatorname{Re}(\rho-\sigma-a_j)>-1,\quad j=1,\ldots,n,\quad \operatorname{Re}\sigma>0,$ $\operatorname{Re}\left[\sum_{j=1}^{p}a_j-\sum_{j=1}^{q}b_j+(q-p)(\rho-\sigma+{}^1/_2)\right]>-{}^1/_2.$ Третье множество условий справедливости формулы: $q<p\quad$ (или $q\leqslant p$ и $\vert\alpha\vert>1$), $\operatorname{Re}(\rho-\sigma-a_j)>-1,\quad j=1,\ldots,n,\quad \operatorname{Re}\sigma>0$
(3)	$\int\limits_0^\infty x^{\rho-1}G_{pq}^{mn}\left(\alpha x\,\middle\vert\,\begin{matrix}a_1,\ldots,a_p\\b_1,\ldots,b_q\end{matrix}\right)dx=\dfrac{\prod\limits_{j=1}^{m}\Gamma(b_j+\rho)\prod\limits_{j=1}^{n}\Gamma(1-a_j-\rho)}{\prod\limits_{j=m+1}^{q}\Gamma(1-b_j-\rho)\prod\limits_{j=n+1}^{p}\Gamma(a_j+\rho)}\,\alpha^{\rho},$ $p+q<2(m+n),\quad \vert\arg\alpha\vert<(m+n-p/2-q/2)\pi$ $-\min\limits_{1\leqslant j\leqslant m}\operatorname{Re}b_j<\operatorname{Re}\rho<1-\max\limits_{1\leqslant j\leqslant n}\operatorname{Re}a_j$
(4)	$\int\limits_0^\infty x^{\rho-1}(x+\beta)^{-\sigma}G_{pq}^{mn}\left(\alpha x\,\middle\vert\,\begin{matrix}a_1,\ldots,a_p\\b_1,\ldots,b_q\end{matrix}\right)dx=$ $=\dfrac{\beta^{\rho-\sigma}}{\Gamma(\sigma)}\,G_{p+1,\,q+1}^{m+1,\,n+1}\left(\alpha\beta\,\middle\vert\,\begin{matrix}1-\rho,\,a_1,\ldots,a_p\\\sigma-\rho,\,b_1,\ldots,b_q\end{matrix}\right).$ Первое множество условий справедливости формулы: $p+q<2(m+n),\quad \vert\arg\alpha\vert<(m+n-p/2-q/2)\pi,\quad \vert\arg\beta\vert<\pi,$ $\operatorname{Re}(\rho+b_j)>0,\quad j=1,\ldots,m,\quad \operatorname{Re}(\rho-\sigma+a_j)<1,\quad j=1,\ldots,n.$ Второе множество условий справедливости формулы: $p\leqslant q,\ p+q\leqslant 2(m+n),\quad \vert\arg\alpha\vert\leqslant(m+n-p/2-q/2)\pi,\quad \vert\arg\beta\vert<\pi,$ $\operatorname{Re}(\rho+b_j)>0,\quad j=1,\ldots,m,\quad \operatorname{Re}(\rho-\sigma+a_j)<1,\quad j=1,\ldots,n,$ $\operatorname{Re}\left[\sum_{j=1}^{p}a_j-\sum_{j=1}^{q}b_j-(q-p)(\rho-\sigma-{}^1/_2)\right]>1.$ Третье множество условий справедливости формулы: $p\geqslant q,\ p+q\leqslant 2(m+n),\quad \vert\arg\alpha\vert\leqslant(m+n-p/2-q/2)\pi,\quad \vert\arg\beta\vert<\pi,$ $\operatorname{Re}(\rho+b_j)>0,\quad j=1,\ldots,m,\quad \operatorname{Re}(\rho-\sigma+a_j)<1,\quad j=1,\ldots,n,$ $\operatorname{Re}\left[\sum_{j=1}^{p}a_j-\sum_{j=1}^{q}b_j+(p-q)(\rho-{}^1/_2)\right]>1$

(5) $$\int_0^\infty x^{-\rho} e^{-\beta x} G_{pq}^{mn}\left(\alpha x \middle| \begin{matrix} a_1, \ldots, a_p \\ b_1, \ldots, b_q \end{matrix}\right) dx = \beta^{\rho-1} G_{p+1,q}^{m,n+1}\left(\frac{\alpha}{\beta} \middle| \begin{matrix} \rho, a_1, \ldots, a_p \\ b_1, \ldots, b_q \end{matrix}\right),$$

$$p+q<2(m+n), \quad |\arg\alpha|<(m+n-p/2-q/2)\pi, \quad |\arg\beta|<\pi/2$$
$$\operatorname{Re}(b_j-\rho)>-1, \quad j=1, \ldots, m$$

(6) $$\int_0^\infty e^{-\beta x} G_{pq}^{mn}\left(\alpha x^2 \middle| \begin{matrix} a_1, \ldots, a_p \\ b_1, \ldots, b_q \end{matrix}\right) dx = \pi^{-1/2}\beta^{-1} G_{p+2,q}^{m,n+2}\left(\frac{4\alpha}{\beta^2} \middle| \begin{matrix} 0, 1/2, a_1, \ldots, a_p \\ b_1, \ldots, b_q \end{matrix}\right),$$

$$p+q<2(m+n), \quad |\arg\alpha|<(m+n-p/2-q/2)\pi, \quad |\arg\beta|<\pi/2$$
$$\operatorname{Re} b_j>-1/2, \quad j=1, \ldots, m$$

(7) $$\int_0^\infty \sin(cx)\, G_{pq}^{mn}\left(\alpha x^2 \middle| \begin{matrix} a_1, \ldots, a_p \\ b_1, \ldots, b_q \end{matrix}\right) dx = \pi^{1/2} c^{-1} G_{p+2,q}^{m,n+1}\left(\frac{4\alpha}{c^2} \middle| \begin{matrix} 0, a_1, \ldots, a_p, 1/2 \\ b_1, \ldots, b_q \end{matrix}\right),$$

$$p+q<2(m+n), \quad |\arg\alpha|<(m+n-p/2-q/2)\pi, \quad c>0$$
$$\operatorname{Re} b_j>-1, \quad j=1, \ldots, m, \quad \operatorname{Re} a_j<1/2, \quad j=1, \ldots, n$$

(8) $$\int_0^\infty \cos(cx)\, G_{pq}^{mn}\left(\alpha x^2 \middle| \begin{matrix} a_1, \ldots, a_p \\ b_1, \ldots, b_q \end{matrix}\right) dx = \pi^{1/2} c^{-1} G_{p+2,q}^{m,n+1}\left(\frac{4\alpha}{c^2} \middle| \begin{matrix} 1/2, a_1, \ldots, a_p, 0 \\ b_1, \ldots, b_q \end{matrix}\right),$$

$$p+q<2(m+n), \quad |\arg\alpha|<(m+n-p/2-q/2)\pi, \quad c>0$$
$$\operatorname{Re} b_j>-1/2, \quad j=1, \ldots, m, \quad \operatorname{Re} a_j<1/2, \quad j=1, \ldots, n$$

(9) $$\int_0^\infty x^{-\rho} J_\nu(2x^{1/2})\, G_{pq}^{mn}\left(\alpha x \middle| \begin{matrix} a_1, \ldots, a_p \\ b_1, \ldots, b_q \end{matrix}\right) dx =$$
$$= G_{p+2,q}^{m,n+1}\left(\alpha \middle| \begin{matrix} \rho-\nu/2, a_1, \ldots, a_p, \rho+\nu/2 \\ b_1, \ldots, b_q \end{matrix}\right),$$

$$p+q<2(m+n), \quad |\arg\alpha|<(m+n-p/2-q/2)\pi$$
$$-3/4+\max_{1\leqslant j\leqslant n} \operatorname{Re} a_j<\operatorname{Re}\rho<1+2^{-1}\operatorname{Re}\nu+\min_{1\leqslant j\leqslant m} \operatorname{Re} b_j$$

(10) $$\int_0^\infty x^{-\rho} Y_\nu(2x^{1/2})\, G_{pq}^{mn}\left(\alpha x \middle| \begin{matrix} a_1, \ldots, a_p \\ b_1, \ldots, b_q \end{matrix}\right) dx =$$
$$= G_{p+3,q+1}^{m,n+2}\left(\alpha \middle| \begin{matrix} \rho-\nu/2, \rho+\nu/2, a_1, \ldots, a_p, \rho+1/2+\nu/2 \\ b_1, \ldots, b_q, \rho+1/2+\nu/2 \end{matrix}\right),$$

$$p+q<2(m+n), \quad |\arg\alpha|<(m+n-p/2-q/2)\pi$$
$$-3/4+\max_{1\leqslant j\leqslant n} \operatorname{Re} a_j<\operatorname{Re}\rho<\min_{1\leqslant j\leqslant m} \operatorname{Re} b_j+2^{-1}|\operatorname{Re}\nu|+1$$

(11)	$\int\limits_0^\infty x^{-\rho} K_\nu(2x^{1/2})\, G_{pq}^{mn}\left(\alpha x \middle| \begin{matrix} a_1, \ldots, a_p \\ b_1, \ldots, b_q \end{matrix}\right) dx =$ $= 2^{-1} G_{p+2,\, q}^{m,\, n+2}\left(\alpha \middle| \begin{matrix} \rho-\nu/2,\ \rho+\nu/2,\ a_1, \ldots, a_p \\ b_1, \ldots, b_q \end{matrix}\right),$ $p+q<2(m+n),\ \lvert\arg\alpha\rvert<(m+n-p/2-q/2)\pi$ $\operatorname{Re}\rho<1-2^{-1}\lvert\operatorname{Re}\nu\rvert+\min\limits_{1\leqslant j\leqslant m}\operatorname{Re} b_j$
(12)	$\int\limits_0^\infty x^{-\rho}\mathbf{H}_\nu(2x^{1/2})\, G_{pq}^{mn}\left(\alpha x \middle| \begin{matrix} a_1, \ldots, a_p \\ b_1, \ldots, b_q \end{matrix}\right) dx =$ $= G_{p+3,\, q+1}^{m+1,\, n+1}\left(\alpha \middle| \begin{matrix} \rho-1/2-\nu/2,\ a_1, \ldots, a_p,\ \rho+\nu/2,\ \rho-\nu/2 \\ \rho-1/2-\nu/2,\ b_1, \ldots, b_q \end{matrix}\right),$ $p+q<2(m+n),\ \lvert\arg\alpha\rvert<(m+n-p/2-q/2)\pi$ $\max\left(-\frac{3}{4},\ \operatorname{Re}\frac{\nu-1}{2}\right)+\max\limits_{1\leqslant j\leqslant n}\operatorname{Re} a_j<\operatorname{Re}\rho<\min\limits_{1\leqslant j\leqslant m}\operatorname{Re} b_j+\frac{1}{2}\operatorname{Re}\nu+\frac{3}{2}$
(13)	$\int\limits_1^\infty x^{-\rho}(x-1)^{\sigma-1}\, {}_2F_1(\varkappa+\sigma-\rho,\ \lambda+\sigma-\rho;\ \sigma;\ 1-x)\times$ $\times G_{pq}^{mn}\left(\alpha x \middle| \begin{matrix} a_1, \ldots, a_p \\ b_1, \ldots, b_q \end{matrix}\right) dx =$ $= \Gamma(\sigma)\, G_{p+2,\, q+2}^{m+2,\, n}\left(\alpha \middle| \begin{matrix} a_1, \ldots, a_p,\ \varkappa+\lambda+\sigma-\rho,\ \rho \\ \varkappa,\ \lambda,\ b_1, \ldots, b_q \end{matrix}\right).$ Первое множество условий справедливости формулы: $p+q<2(m+n),\ \lvert\arg\alpha\rvert<(m+n-p/2-q/2)\pi,$ $\operatorname{Re}\sigma>0,\ \operatorname{Re}\varkappa\geqslant\operatorname{Re}\lambda>\operatorname{Re} a_j-1,\ j=1, \ldots, n.$ Второе множество условий справедливости формулы: $p+q\leqslant 2(m+n),\ \lvert\arg\alpha\rvert\leqslant(m+n-p/2-q/2)\pi,$ $\operatorname{Re}\sigma>0,\ \operatorname{Re}\varkappa\geqslant\operatorname{Re}\lambda>\operatorname{Re} a_j-1,\ j=1, \ldots, n,$ $\operatorname{Re}\left[\sum\limits_{j=1}^p a_j-\sum\limits_{j=1}^q b_j+(q-p)(\varkappa+1/2)\right]>-1/2,$ $\operatorname{Re}\left[\sum\limits_{j=1}^p a_j-\sum\limits_{j=1}^q b_j+(q-p)(\lambda+1/2)\right]>-1/2$
(14)	$\int\limits_0^\infty G_{pq}^{mn}\left(\alpha x \middle| \begin{matrix} a_1, \ldots, a_p \\ b_1, \ldots, b_q \end{matrix}\right) G_{rs}^{kl}\left(\beta x \middle| \begin{matrix} c_1, \ldots, c_r \\ d_1, \ldots, d_s \end{matrix}\right) dx =$ $= \alpha^{-1} G_{q+r,\, p+s}^{k+n,\, l+m}\left(\frac{\beta}{\alpha} \middle| \begin{matrix} -b_1, \ldots, -b_m,\ c_1, \ldots, c_r,\ -b_{m+1}, \ldots, -b_q \\ -a_1, \ldots, -a_n,\ d_1, \ldots, d_s,\ -a_{n+1}, \ldots, -a_p \end{matrix}\right).$ Относительно (пяти) множеств условий справедливости формулы см. Meijer C. S., 1941: Nederl. Akad. Wetensch., Proc. 44, 82—92.

ПРИЛОЖЕНИЕ

ОБОЗНАЧЕНИЯ И ОПРЕДЕЛЕНИЯ ВЫСШИХ ТРАНСЦЕНДЕНТНЫХ ФУНКЦИЙ

Через ВТФ I обозначаются ссылки иа первый том книги «Высшие трансцендентные функции» тех же авторов, а через ВТФ II — на второй том этой книги.

Общие замечания. Большинство обозначений объяснено там, где они встречаются. Обозначения, встречающиеся несколько раз на одной страиице, объясняются внизу страницы.

Как правило, вещественные переменные и параметры обозначаются латинскими буквами, а комплексные переменные и параметры — греческими буквами. Исключения делаются для традиционных обозначений (таких, например, как y в главе XIV). Буквы m, n обычно обозначают целые числа.

Через $\operatorname{Re} z$, $\operatorname{Im} z$ обозначены соответственно вещественная и мнимая части комплексной величины z.

Через $|z|$, $\arg z$ — соответственно модуль и аргумент (фаза) комплексной величины.

Главное значение в смысле Коши. Если подынтегральная функция имеет особенность в точке c, $a < c < b$, то главным значением в смысле Коши интеграла

$$\int_a^b f(x)\,dx$$

называют

$$\fint_a^b f(x)\,dx = \lim \left[\int_a^{c-\varepsilon} f(x)\,dx + \int_{c+\varepsilon}^b f(x)\,dx \right], \qquad \varepsilon > 0,\ \varepsilon \to 0.$$

Пустая сумма интерпретируется как нуль, а пустое произведеие — как единица. $\sum\limits_{n=a}^{b}$, $\prod\limits_{n=a}^{b}$ пусты, если $b < a$.

Через $[x]$ обозначено наибольшее целое число, не превосходящее x.

$$(a)_\nu = \Gamma(a+\nu)/\Gamma(a),$$
$$(a)_0 = 1,$$
$$(a)_n = a(a+1)\ldots(a+n-1), \qquad n = 1, 2, \ldots,$$
$$(a)_n = (-1)^n (1-a-n)_n, \qquad n \text{ — целое},$$
$$(a)_{-n} = (-1)^n/(1-a)_n, \qquad n \text{ — целое}.$$

Биномиальные коэффициенты

$$\binom{\alpha}{\beta} = \frac{\Gamma(\alpha+1)}{\Gamma(\beta+1)\,\Gamma(\alpha-\beta+1)};$$

$$\operatorname{sgn} x = \begin{cases} -1, & x<0, \\ 0, & x=0, \\ 1, & x>0. \end{cases}$$

Постоянная Эйлера — Маскерони

$$C = \lim_{m\to\infty}\left(\sum_{n=1}^{m} 1/n - \ln m\right) = 0{,}5772156649\ \ldots,$$

$$\gamma = e^{C}.$$

Заметим, что в ВТФ и многих других книгах вместо C пишут γ.

Ортогональные многочлены. См. также ВТФ II, гл. 10 и стр. 187—190 астоящего тома.

Многочлены Лежандра

$$P_n(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2-1)^n.$$

Многочлены Гегенбауэра

$$C_n^{\nu}(x) = \frac{(-2)^n(\nu)_n}{n!\,(n+2\nu)_n}(1-x^2)^{1/2-\nu}\frac{d^n}{dx^n}(1-x^2)^{n+\nu-1/2}.$$

Многочлены Чебышева

$$T_n(x) = \cos(n\arccos x),$$

$$U_n(x) = \frac{\sin[(n+1)\arccos x]}{\sin(\arccos x)}.$$

Многочлены Якоби

$$P_n^{(\alpha,\beta)}(x) = \frac{(-1)^n}{2^n n!}(1-x)^{-\alpha}(1+x)^{-\beta}\frac{d^n}{dx^n}\left[(1-x)^{n+\alpha}(1+x)^{n+\beta}\right].$$

Многочлены Лагерра

$$L_n^{\alpha}(z) = \frac{e^{z}z^{-\alpha}}{n!}\frac{d^n}{dz^n}\left(e^{-z}z^{n+\alpha}\right),$$

$$L_n^{0}(z) = L_n(z).$$

Многочлены Эрмита

$$\mathrm{He}_n(x) = (-1)^n\exp\left(2^{-1}x^2\right)\frac{d^n}{dx^n}\left[\exp\left(-2^{-1}x^2\right)\right],$$

$$H_n(x) = (-1)^n\exp(x^2)\frac{d^n}{dx^n}\left[\exp(-x^2)\right].$$

Многочлены Шарлье

$$p_n(x;\ a) = n!\,a^{-n}L_n^{x-n}(a).$$

Гамма-функция и родственные ей функции. См. также ВТФ I, гл. 1
Гамма-функция

$$\Gamma(z)=\int_0^\infty e^{-t}t^{z-1}\,dt,\qquad \operatorname{Re} z>0.$$

Логарифмическая производная от гамма-функции

$$\psi(z)=\frac{\Gamma'(z)}{\Gamma(z)},\quad \psi'(z)=\frac{d\psi}{dz}\qquad \text{и т. д.}$$

Бета-функция

$$B(x,\ y)=\frac{\Gamma(x)\,\Gamma(y)}{\Gamma(x+y)}.$$

Дилогарифм Эйлера

$$L_2(z)=\sum_{n=1}^\infty \frac{z^n}{n^2}=-\int_0^z \frac{\ln(1-z)}{z}\,dz.$$

Неполные гамма-функции. См. «Вырожденные гипергеометрические функции».

Неполная бета-функция. См. «Гипергеометрические функции».

Дзета-функция Римана и родственные функции

$$\zeta(s)=\sum_{n=1}^\infty n^{-s},$$

$$\xi(t)=-2^{-1}(t^2+{}^1/_4)\,\pi^{-it/2-{}^1/_4}\,\Gamma(it/2+{}^1/_4)\,\zeta(it+{}^1/_2),$$

$$\zeta(z,\ a)=\sum_{n=0}^\infty (n+a)^{-z},\qquad \Phi(z,\ s,\ v)=\sum_{n=0}^\infty \frac{z^n}{(v+n)^s}.$$

Функции Лежандра. См. также ВТФ I, гл. 3. Относительно выражений для произведений функций Лежандра через гипергеометрические функции см. Meijer C. S. 1936: Math. Ann. 112, 469—489 и Proc. Nederl. Akad. Wetensch. 39, 394—403 и 519—527: 1938: Nieuw Arch. Wiskunde (2), 19, 207—234.

$$P_\nu^\mu(z)=\frac{1}{\Gamma(1-\mu)}\left(\frac{z+1}{z-1}\right)^{\mu/2}{}_2F_1(-\nu,\ \nu+1;\ 1-\mu;\ {}^1/_2-z/2),$$

$$Q_\nu^\mu(z)=\frac{e^{\mu\pi i}\pi^{1/2}\,\Gamma(\mu+\nu+1)}{2^{\nu+1}\Gamma(\nu+{}^3/_2)}\,z^{-\mu-\nu-1}(z^2-1)^{\mu/2}\,{}_2F_1\left(\frac{\mu+\nu+1}{2},\ \frac{\mu+\nu+2}{2};\ \nu+\frac{3}{2};\ \frac{1}{z^2}\right),$$

z изменяется в комплексной плоскости, разрезанной вдоль отрезка $[-1,\ 1]$ вещественной оси.

$$\mathrm{P}_\nu^\mu(x)=\frac{1}{\Gamma(1-\mu)}\left(\frac{1+x}{1-x}\right)^{\mu/2}{}_2F_1(-\nu,\ \nu+1;\ 1-\mu;\ {}^1/_2-x/2),\qquad -1<x<1,$$

$$\mathrm{Q}_\nu^\mu(x)=2^{-1}e^{-i\mu\pi}\left[\exp(-2^{-1}\mu\pi i)\,Q_\nu^\mu(x+i0)+\exp(2^{-1}\mu\pi i)Q_\nu^\mu(x-i0)\right],\qquad -1<x<1,$$

$$P_\nu(z)=P_\nu^0(z),\ Q_\nu(z)=Q_\nu^0(z).$$

Функции Бесселя и родственные функции. См. также ВТФ II, гл. 7 и стр. 235—236 настоящего тома.

Функции Бесселя

$$J_\nu(z)=\sum_{m=0}^{\infty}\frac{(-1)^m (z/2)^{\nu+2m}}{m!\,\Gamma(\nu+m+1)},$$

$$Y_\nu(z)=\frac{J_\nu(z)\cos\nu\pi-J_{-\nu}(z)}{\sin\nu\pi},$$

$$H_\nu^{(1)}(z)=J_\nu(z)+iY_\nu(z),$$

$$H_\nu^{(2)}(z)=J_\nu(z)-iY_\nu(z),$$

$$Ji_\nu(x)=\int_\infty^x J_\nu(t)\frac{dt}{t}.$$

Модифицированные функции Бесселя

$$I_\nu(z)=\sum_{m=0}^{\infty}\frac{(z/2)^{\nu+2m}}{m!\,\Gamma(\nu+m+1)},$$

$$K_\nu(z)=\frac{\pi}{2}\frac{I_{-\nu}(z)-I_\nu(z)}{\sin\nu\pi}.$$

Функции Кельвина и родственные им функции

$$\mathrm{ber}_\nu(z)+i\,\mathrm{bei}_\nu(z)=J_\nu[z\exp(3\pi i/4)],$$

$$\mathrm{ber}_\nu(z)-i\,\mathrm{bei}_\nu(z)=J_\nu[z\exp(-3\pi i/4)],$$

$$\mathrm{ker}_\nu(z)+i\,\mathrm{kei}_\nu(z)=K_\nu[z\exp(\pi i/4)],$$

$$\mathrm{ker}_\nu(z)-i\,\mathrm{kei}_\nu(z)=K_\nu[z\exp(-\pi i/4)],$$

$$\mathrm{ber}(z)=\mathrm{ber}_0(z),\qquad \mathrm{bei}(z)=\mathrm{bei}_0(z),$$

$$\mathrm{ker}(z)=\mathrm{ker}_0(z),\qquad \mathrm{kei}(z)=\mathrm{kei}_0(z).$$

Отметим, что определение функций $\mathrm{ker}_\nu(z)$ и $\mathrm{kei}_\nu(z)$ отличается от данного в ВТФ II, п. 7.2.3.

$$X_\nu^{(b)}(z)=\mathrm{ber}_\nu^2(z)+\mathrm{bei}_\nu^2(z),$$

$$V_\nu^{(b)}(z)=\left[\mathrm{ber}'_\nu(z)\right]^2+\left[\mathrm{bei}'_\nu(z)\right]^2,$$

$$W_\nu^{(b)}(z)=\mathrm{ber}_\nu(z)\,\mathrm{bei}'_\nu(z)-\mathrm{bei}_\nu(z)\,\mathrm{ber}'_\nu(z),$$

$$2^{-1}Z_\nu^{(b)}(z)=\mathrm{ber}_\nu(z)\,\mathrm{bei}'_\nu(z)+\mathrm{bei}_\nu(z)\,\mathrm{ber}'_\nu(z).$$

Многочлены Неймана

$$O_0(x)=\frac{1}{x};\quad O_n(x)=\frac{1}{4}\sum_{m=0}^{\leqslant n/2}\frac{n(n-m-1)!}{m!\,(x/2)^{n-2m+1}},\qquad n=1,\ 2,\ \ldots,$$

$$O_{-n}(x)=(-1)^n O_n(x),\qquad n=1,\ 2,\ \ldots$$

Функции Ангера — Вебера

$$\mathbf{J}_\nu(z) = \pi^{-1} \int_0^\pi \cos(\nu\theta - z \sin\theta)\, d\theta,$$

$$\mathbf{E}_\nu(z) = \pi^{-1} \int_0^\pi \sin(\nu\theta - z \sin\theta)\, d\theta.$$

Функции Струве

$$\mathbf{H}_\nu(z) = \sum_{m=0}^{\infty} \frac{(-1)^m (z/2)^{\nu+2m+1}}{\Gamma(m+{}^3/_2)\,\Gamma(\nu+m+{}^3/_2)} = \frac{(z/2)^{\nu+1}}{\Gamma({}^3/_2)\,\Gamma(\nu+{}^3/_2)}\, {}_1F_2(1;\ {}^3/_2,\ \nu+{}^3/_2;\ -z^2/4) =$$

$$= 2^{1-\nu}\pi^{-1/2}\,[\Gamma(\nu+{}^1/_2)]^{-1}\, s_{\nu,\,\nu}(z);$$

$$\mathbf{L}_\nu(z) = \exp[-2^{-1}(\nu+1)\pi i]\, \mathbf{H}_\nu[z \exp(i\pi/2)].$$

Функции Ломмеля

$$s_{\mu,\,\nu}(z) = \frac{z^{\mu+1}}{(\mu-\nu+1)(\mu+\nu+1)}\, {}_1F_2\left(1;\ \frac{\mu-\nu+3}{2},\ \frac{\mu+\nu+3}{2};\ -\frac{z^2}{4}\right),$$

$$S_{\mu,\,\nu}(z) = s_{\mu,\,\nu}(z) + 2^{\mu-1}\,\Gamma\left(\frac{\mu-\nu+1}{2}\right)\Gamma\left(\frac{\mu+\nu+1}{2}\right) \times$$

$$\times \left[\sin\left(\frac{\mu-\nu}{2}\pi\right) J_\nu(z) - \cos\left(\frac{\mu-\nu}{2}\pi\right) Y_\nu(z)\right].$$

Функции Ломмеля двух переменных

$$U_\nu(w,\ z) = \sum_{m=0}^{\infty} (-1)^m \left(\frac{w}{z}\right)^{\nu+2m} J_{\nu+2m}(z),$$

$$V_\nu(w,\ z) = \cos\left(\frac{w}{2} + \frac{z^2}{2w} + \frac{\nu\pi}{2}\right) + U_{2-\nu}(w,\ z).$$

Гипергеометрические функции. См. также ВТФ I, гл. 2, 4.
Обобщенный гипергеометрический ряд

$${}_mF_n(\alpha_1,\ \ldots,\ \alpha_m;\ \gamma_1,\ \ldots,\ \gamma_n;\ z) = \sum_{k=0}^{\infty} \frac{(\alpha_1)_k \cdots (\alpha_m)_k}{(\gamma_1)_k \cdots (\gamma_n)_k}\, \frac{z^k}{k!}.$$

${}_2F_1(a,\ b;\ c;\ z)$ — гипергеометрический ряд Гаусса. Его часто (например, в ВТФ I, гл. 2) обозначают через $F(a,\ b;\ c;\ z)$.

${}_1F_1(a;\ c;\ z)$ — вырожденный гипергеометрический ряд Куммера. Его часто (например, в ВТФ I, гл. 6) обозначают через $\Phi(a;\ c;\ z)$.

${}_mF_n(\alpha_1,\ \ldots,\ \alpha_m;\ \gamma_1,\ \ldots,\ \gamma_n;\ z)$ часто записывают в виде

$${}_mF_n\left[\begin{matrix}\alpha_1,\ \ldots,\ \alpha_m;\ z\\ \gamma_1,\ \ldots,\ \gamma_n\end{matrix}\right].$$

Неполная бета-функция

$$\mathrm{B}_x(p,\ q)=\int_0^x t^{p-1}(1-t)^{q-1}\,dt=p^{-1}x^p\,{}_2F_1(p,\ 1-q;\ p+1;\ x);$$

$$I_x(p,\ q)=\frac{B_x(p,\ q)}{B_1(p,\ q)},$$

$$S_n(b_1,\ b_2,\ b_3,\ b_4;\ z)=\sum_{h=1}^{n}\frac{\prod\limits_{j=1}^{n}{}'\,\Gamma(b_j-b_h)}{\prod\limits_{j=n+1}^{4}\Gamma(1+b_h-b_j)}\,z^{1+2b_h}\times$$

$$\times\,{}_0F_3(1+b_h-b_1,\ \ldots,\ *,\ \ldots,\ 1+b_h-b_4;\ (-1)^n z^2).$$

Штрих в $\prod'$ и звездочка в ${}_0F_3$ означают, что опускается член, содержащий b_h-b_h. При $n=1$ произведение $\prod$ в числителе, а при $n=4$ — в знаменателе заменяется единицей.

Вырожденные гипергеометрические функции. См. также ВТФ I, гл. 6 и ВТФ II, гл. 8 и 9. См. также Гипергеометрические функции, Ортогональные многочлены, E-функцию, G-функцию.

Функции Уиттекера

$$M_{\varkappa,\mu}(z)=z^{1/2+\mu}e^{-z/2}\,{}_1F_1(1/2+\mu-\varkappa;\ 2\mu+1;\ z),$$

$$W_{\varkappa,\mu}(z)=\frac{\Gamma(-2\mu)\,M_{\varkappa,\mu}(z)}{\Gamma(1/2-\mu-\varkappa)}+\frac{\Gamma(2\mu)\,M_{\varkappa,-\mu}(z)}{\Gamma(1/2+\mu-\varkappa)}.$$

Функции параболического цилиндра

$$D_\nu(z)=2^{\nu/2+1/4}z^{-1/2}\,W_{\nu/2+1/4,\ 1/4}(2^{-1}z^2),$$

$$D_n(z)=(-1)^n\exp(2^{-2}z^2)\,\frac{d^n}{dz^n}\left[\exp(-2^{-1}z^2)\right].$$

Функция Бейтмена

$$k_{2\nu}(z)=\frac{1}{\Gamma(\nu+1)}\,W_{\nu,\ 1/2}(2z).$$

Интегральная экспонента и родственные функции

$$-\mathrm{Ei}(-x)=E_1(x)=\int_x^\infty e^{-t}\frac{dt}{t}=\Gamma(0,\ x),\qquad -\pi<\arg x<\pi,$$

$$\mathrm{Ei}^+(x)=\mathrm{Ei}(x+i0),\quad \mathrm{Ei}^-(x)=\mathrm{Ei}(x-i0),\qquad x>0,$$

$$\overline{\mathrm{Ei}}(x)=2^{-1}\left[\mathrm{Ei}^+(x)+\mathrm{Ei}^-(x)\right],\qquad x>0.$$

Последняя функция в ВТФ II, п. 9.7 обозначается $E^*(x)$.

$$\mathrm{li}(z) = \int_0^z \frac{dt}{\ln t} = \mathrm{Ei}(\ln z),$$

$$\mathrm{si}(x) = -\int_x^\infty \frac{\sin t}{t}\, dt = \frac{1}{2i}[\mathrm{Ei}(ix) - \mathrm{Ei}(-ix)],$$

$$\mathrm{Si}(x) = \int_0^x \frac{\sin t}{t}\, dt = \pi/2 + \mathrm{si}(x),$$

$$\mathrm{Ci}(x) = -\int_x^\infty \frac{\cos t}{t}\, dt = -\mathrm{ci}(x) = 2^{-1}[\mathrm{Ei}(ix) + \mathrm{Ei}(-ix)].$$

Функции ошибок и родственные функции

$$\mathrm{Erf}(x) = 2\pi^{-1/2}\int_0^x \exp(-t^2)\, dt = \frac{2x}{\sqrt{\pi}}\, {}_1F_1\left(\frac{1}{2}, \frac{3}{2};\, -x^2\right),$$

$$\mathrm{Erfc}(x) = 2\pi^{-1/2}\int_x^\infty \exp(-t^2)\, dt = 1 - \mathrm{Erf}(x) = (\pi x)^{-1/2}\exp(-2^{-1}x^2)\, W_{-1/4,\, 1/4}(x^2)$$

$$\mathrm{Erfc}(x) = \pi^{-1/2}\,\Gamma(1/2,\, x^2),$$

$$\mathrm{Erfc}\left(x\sqrt{\pm i}\right) = \sqrt{\pm 2i}\,[C(x^2) \mp i\, S(x^2)].$$

Эти функции отличаются множителем $2\pi^{-1/2}$ от функций, введенных в ВТФ II, п. 9.9.

$$C(x) = 2^{-1/2}\pi^{-1/2}\int_0^x t^{-1/2}\cos t\, dt,$$

$$S(x) = 2^{-1/2}\pi^{-1/2}\int_0^x t^{-1/2}\sin t\, dt.$$

Неполные гамма-функции

$$\gamma(\alpha,\, x) = \int_0^x e^{-t}t^{\alpha-1}\, dt = \alpha^{-1}x^\alpha\, {}_1F_1(\alpha;\ \alpha+1;\ -x),$$

$$\Gamma(\alpha,\, x) = \int_x^\infty e^{-t}t^{\alpha-1}\, dt = \Gamma(\alpha) - \gamma(\alpha,\, x) = x^{(\alpha-1)/2}e^{-x/2}\, W_{(\alpha-1)/2,\, \alpha/2}(x).$$

Частные случаи функций Уиттекера

$$M_{-1/4,\,1/4}(x) = 2^{-1}\pi^{-1/2}x^{1/4}e^{x/2}\operatorname{Erf}(x^{1/2}),$$

$$M_{n+1/4,\,-1/4}(x) = \frac{(-2)^{-n}}{(1/2)_n}x^{1/4}e^{-x/2}\operatorname{He}_{2n}[(2x)^{1/2}],$$

$$M_{n+3/4,\,1/4}(x) = \frac{(-1)^n 2^{-n-1/2}}{(3/2)_n}x^{1/4}e^{-x/2}\operatorname{He}_{2n+1}[(2x)^{1/2}],$$

$$M_{0,\,\mu}(x) = 2^{2\mu}\Gamma(\mu+1)\,x^{1/2}I_\mu(x/2);$$

$$M_{0,\,\mu}(\pm ix) = 2^{2\mu}\Gamma(\mu+1)\,e^{\pm\mu\pi i/2}x^{1/2}J_\mu(x/2),$$

$$M_{\mu+1/2,\,\mu}(x) = x^{\mu+1/2}e^{-x/2},$$

$$M_{-\mu-1/2,\,\mu}(x) = x^{\mu+1/2}e^{x/2},$$

$$M_{\mu-1/2,\,\mu}(x) = 2\mu x^{1/2-\mu}e^{x/2}\gamma(2\mu,\ x),$$

$$M_{\mu+n+1/2,\,\mu}(x) = \frac{n!}{(2\mu+1)_n}x^{\mu+1/2}e^{-x/2}L_n^{2\mu}(x);$$

$$W_{-1/2,\,0}(x) = -x^{1/2}e^{x/2}\operatorname{Ei}(-x),$$

$$W_{-1/4,\,\pm 1/4}(x) = \pi^{1/2}x^{1/4}e^{x/2}\operatorname{Erfc}(x^{1/2}),$$

$$W_{n/2+1/4,\,\pm 1/4}(x) = 2^{-n/2}x^{1/4}e^{-x/2}\operatorname{He}_n[(2x)^{1/2}],$$

$$W_{\mu,\,\pm 1/4}(x) = 2^{-\mu}(2x)^{1/4}D_{2\mu-1/2}[(2x)^{1/2}],$$

$$W_{0,\,\mu}(x) = (x/\pi)^{1/2}K_\mu(x/2),$$

$$W_{0,\,\mu}(ix) = \frac{(\pi x)^{1/2}}{2}\exp[-(\nu/2+1/4)\pi i]\,H_\mu^{(2)}(x/2),$$

$$W_{0,\,\mu}(-ix) = \frac{(\pi x)^{1/2}}{2}\exp[(\nu/2+1/4)\pi i]\,H_\mu^{(1)}(x/2),$$

$$W_{\mu+1/2,\,\pm\mu}(x) = x^{\mu+1/2}e^{-x/2},$$

$$W_{\mu-1/2,\,\pm\mu}(x) = x^{1/2-\mu}e^{x/2}\Gamma(2\mu,\ x),$$

$$W_{\mu+n+1/2,\,\pm\mu}(x) = (-1)^n n!\,x^{\mu+1/2}e^{-x/2}L_n^{2\mu}(x).$$

***E*-функция Мак-Роберта.** См. также ВТФ I, гл. 5.
Если $p \geqslant q+1$,

$$E(p;\ \alpha_r : q;\ \rho_s : x) = \sum_{r=1}^{p}\frac{\prod\limits_{s=1}^{p}{}'\,\Gamma(\alpha_s-\alpha_r)}{\prod\limits_{t=1}^{q}\Gamma(\rho_t-\alpha_r)}\Gamma(\alpha_r)\,x^{\alpha_r}\times$$

$$\times\ {}_{q+1}F_{p-1}(\alpha_r,\ \alpha_r-\rho_1+1,\ \ldots,\ \alpha_r-\rho_q+1;\ \alpha_r-\alpha_1+1,\ \ldots,\ *,\ \ldots$$
$$\ldots,\ \alpha_r-\alpha_p+1;\ (-1)^{p-q}x),$$

где $|x|<1$ при $p=q+1$.

Если $p \leqslant q+1$, то

$$E(p;\ \alpha_r : q;\ \rho_s : x) = \frac{\prod\limits_{r=1}^{p} \Gamma(\alpha_r)}{\prod\limits_{s=1}^{q} \Gamma(\rho_s)}\, {}_pF_q(\alpha_1, \ldots, \alpha_p;\ \rho_1, \ldots, \rho_q;\ -1/x),$$

где $x \neq 0$ и $|x| > 1$ при $p = q+1$. Если $p > q+1$, последнее соотношение дает асимптотическое разложение для E-функции при больших значениях x.

$$E(\alpha :: x) = \Gamma(\alpha)(1+x^{-1})^{-\alpha},$$

$$E(: \nu+1 : x) = x^{\nu/2} J_\nu(2x^{-1/2}),$$

$$E(1/2+\nu,\ 1/2-\nu :: 2x) = \frac{(2\pi x)^{1/2} e^{x} K_\nu(x)}{\cos(\nu\pi)},$$

$$E(\alpha,\ \beta :: x) = \Gamma(\alpha)\Gamma(\beta) x^{-k} e^{x/2} W_{k,m}(x),$$

$$k = 2^{-1}(1-\alpha-\beta), \quad m = 2^{-1}(\alpha-\beta),$$

$$E\left(\alpha,\ \beta,\ \frac{\alpha+\beta}{2},\ \frac{\alpha+\beta+1}{2} : \alpha+\beta : \frac{x^2}{4}\right) =$$

$$= \pi^{1/2}\Gamma(\alpha)\Gamma(\beta)(x/2)^{-2k} W_{k,m}(ix) W_{k,m}(-ix),$$

$$k = 2^{-1}(1-\alpha-\beta), \quad m = 2^{-1}(\alpha-\beta).$$

G-функция Мейера. См. также ВТФ I, гл. V.

$$G_{p,q}^{m,n}\left(x \,\middle|\, \begin{matrix} a_1, \ldots, a_p \\ b_1, \ldots, b_q \end{matrix}\right) = \frac{1}{2\pi i}\int_L \frac{\prod\limits_{j=1}^{m}\Gamma(b_j - s)\prod\limits_{j=1}^{n}\Gamma(1-a_j+s)}{\prod\limits_{j=m+1}^{q}\Gamma(1-b_j+s)\prod\limits_{j=n+1}^{p}\Gamma(a_j-s)}\, x^s\, ds,$$

где L — путь, отделяющий полюсы функции $\Gamma(b_1 - s) \ldots \Gamma(b_m - s)$ от полюсов $\Gamma(1-a_1+s) \ldots \Gamma(1-a_n+s)$. Более подробное определение см. в ВТФ I, п. 5.3.

Из формул, содержащих G-функцию, при частных значениях параметров получаются многие интегралы, содержащие функции Бесселя, функции Лежандра и другие высшие трансцендентные функции. Следующие два списка дают выражения для некоторых частных видов G-функций через хорошо известные высшие трансцендентные функции и, обратно, выражения высших трансцендентных функций через G-функции. Эти списки не полны. См. также ВТФ I, п. 5.6.

Частные случаи G-функции

$$G_{02}^{10}(x \mid a,\ b) = x^{(a+b)/2} J_{a-b}(2x^{1/2}),$$

$$G_{02}^{20}(x \mid a,\ b) = 2x^{(a+b)/2} K_{a-b}(2x^{1/2}),$$

$$G_{12}^{11}\left(x \,\middle|\, \begin{matrix} 1/2 \\ b,\ -b \end{matrix}\right) = \pi^{1/2} e^{-x/2} I_b(x/2),$$

$$G_{12}^{11}\left(x \,\middle|\, \begin{matrix} a \\ b,\ c \end{matrix}\right) = \frac{\Gamma(1-a+b)}{\Gamma(1+b-c)} x^b\, {}_1F_1(1-a+b;\ 1+b-c;\ -x),$$

$$G_{12}^{20}\left(x \,\middle|\, \begin{matrix} 1/2 \\ b,\ -b \end{matrix}\right) = \pi^{-1/2} e^{-x/2} K_b(x/2),$$

$$G_{12}^{20}\left(x \middle| {a \atop b,\, c}\right) = x^{(b+c-1)/2} e^{-x/2} W_{k,\,m}(x),$$
$$k = 2^{-1}(1+b+c) - a, \quad m = b/2 - c/2,$$
$$G_{12}^{21}\left(x \middle| {1/2 \atop b,\, -b}\right) = \frac{\pi^{1/2}}{\cos b\pi} e^{x/2} K_b(x/2),$$
$$G_{12}^{21}\left(x \middle| {a \atop b,\, c}\right) = \Gamma(b-a+1)\,\Gamma(c-a+1)\, x^{(b+c-1)/2} e^{x/2} W_{k,\,m}(x),$$
$$k = a - 2^{-1}(b+c+1), \quad m = b/2 - c/2,$$
$$G_{04}^{10}(x \mid a,\, b,\, 2b-a,\, b+1/2) = \pi^{-1/2} x^b I_{2(a-b)}(2^{3/2}x^{1/4})\, J_{2(a-b)}(2^{3/2}x^{1/4}),$$
$$G_{04}^{10}(x \mid a,\, a+1/2,\, b,\, 2a-b) = \frac{1}{2\pi^{1/2}\cos(b-a)\pi} \times$$
$$\times x^a \left[J_{2(a-b)}(2^{3/2}x^{1/4})\, I_{2(b-a)}(2^{3/2}x^{1/4}) + I_{2(a-b)}(2^{3/2}x^{1/4})\, J_{2(b-a)}(2^{3/2}x^{1/4})\right],$$
$$G_{04}^{10}(x \mid a+1/2,\, a,\, b,\, 2a-b) = 2^{-1}\pi^{-1/2}[\sin(a-b)\pi]^{-1} \times$$
$$\times x^a \left[J_{2(a-b)}(2^{3/2}x^{1/4})\, I_{2(b-a)}(2^{3/2}x^{1/4}) - I_{2(a-b)}(2^{3/2}x^{1/4})\, J_{2(b-a)}(2^{3/2}x^{1/4})\right],$$
$$G_{04}^{20}(x \mid a,\, a+1/2,\, b,\, b+1/2) = x^{(a+b)/2} J_{2(a-b)}(4x^{1/4}),$$
$$G_{04}^{20}(x \mid a,\, -a,\, 0,\, 1/2) = -\pi^{1/2}(\sin 2a\pi)^{-1} \times$$
$$\times \left[J_{2a}(z e^{\pi i/4})\, J_{2a}(z e^{-\pi i/4}) - J_{-2a}(z e^{\pi i/4})\, J_{-2a}(z e^{-\pi i/4})\right], \qquad z = 2^{3/2}x^{1/4},$$
$$G_{04}^{20}(x \mid 0,\, 1/2,\, a,\, -a) = \pi^{1/2} i^{-1} (\sin 2a\pi)^{-1} \times$$
$$\times \left[e^{2a\pi i} J_{2a}(z e^{-\pi i/4})\, J_{-2a}(z e^{\pi i/4}) - e^{-2a\pi i} J_{2a}(z e^{\pi i/4})\, J_{-2a}(z e^{-\pi i/4})\right],$$
$$z = 2^{3/2}x^{1/4},$$
$$G_{04}^{30}(x \mid 3a-1/2,\, a,\, -a-1/2,\, a-1/2) =$$
$$= 2\pi^{1/2}(\cos 2a\pi)^{-1} x^{a-1/2} K_{4a}(2^{3/2}x^{1/4}) \left[J_{4a}(2^{3/2}x^{1/4}) + J_{-4a}(2^{3/2}x^{1/4})\right],$$
$$G_{04}^{30}(x \mid 0,\, a-1/2,\, -a-1/2,\, -1/2) =$$
$$= 4\pi^{1/2} x^{-1/2} K_{2a}(2^{3/2}x^{1/4}) \left[J_{2a}(2^{3/2}x^{1/4}) \cos a\pi - Y_{2a}(2^{3/2}x^{1/4}) \sin a\pi\right],$$
$$G_{04}^{30}(x \mid -1/2,\, a-1/2,\, -a-1/2,\, 0) =$$
$$= -4\pi^{1/2} x^{-1/2} K_{2a}(2^{3/2}x^{1/4}) \left[J_{2a}(2^{3/2}x^{1/4}) \sin a\pi + Y_{2a}(2^{3/2}x^{1/4}) \cos a\pi\right],$$
$$G_{04}^{30}(x \mid a,\, b+1/2,\, b,\, 2b-a) = \pi^{1/2} 2^{1/2} x^b K_{2(a-b)}(2^{3/2}x^{1/4})\, J_{2(a-b)}(2^{3/2}x^{1/4}),$$
$$G_{04}^{40}(x \mid a,\, a+1/2,\, b,\, b+1/2) = 4\pi x^{(a+b)/2} K_{2(a-b)}(4x^{1/4}),$$
$$G_{04}^{40}(x \mid a,\, a+1/2,\, b,\, 2a-b) = 2^3\pi^{1/2} x^a K_{2(b-a)}(2^{3/2}x^{1/4} e^{\pi i/4})\, K_{2(b-a)}(2^{3/2}x^{1/4} e^{-\pi i/4}),$$
$$G_{04}^{n0}(x \mid a,\, b,\, c,\, d) = x^{-1/2} S_n(a,\, b,\, c,\, d;\, x^{1/2}), \qquad n = 1,\, 2,\, 3,\, 4,$$
$$G_{13}^{11}\left(x \middle| {1/2 \atop a,\, 0,\, -a}\right) = \pi^{1/2} J_a^2(x^{1/2}),$$
$$G_{13}^{11}\left(x \middle| {1/2 \atop 0,\, a,\, -a}\right) = \pi^{1/2} J_a(x^{1/2})\, J_{-a}(x^{1/2}),$$
$$G_{13}^{11}\left(x \middle| {a \atop a,\, b,\, a-1/2}\right) = x^{a/2+b/2-1/4} \mathbf{H}_{a-b-1/2}(2x^{1/2}),$$
$$G_{13}^{20}\left(x \middle| {a-1/2 \atop a,\, b,\, a-1/2}\right) = x^{a/2+b/2} Y_{b-a}(2x^{1/2}),$$
$$G_{13}^{20}\left(x \middle| {a+1/2 \atop b,\, a,\, 2a-b}\right) = -\pi^{1/2} x^a J_{b-a}(x^{1/2})\, Y_{b-a}(x^{1/2}),$$
$$G_{13}^{20}\left(x \middle| {1/2 \atop a,\, -a,\, 0}\right) = \pi^{1/2} 2^{-1} (\sin a\pi)^{-1} \left[J_{-a}^2(x^{1/2}) - J_a^2(x^{1/2})\right],$$

$$G_{13}^{21}\left(x\,\middle|\,{{1/2}\atop{a,\ 0,\ -a}}\right)=2\pi^{1/2}I_a(x^{1/2})\,K_a(x^{1/2}),$$

$$G_{13}^{21}\left(x\,\middle|\,{{1/2}\atop{a,\ -a,\ 0}}\right)=\pi^{3/2}(\sin 2a\pi)^{-1}\left[I_{-a}^2(x^{1/2})-I_a^2(x^{1/2})\right],$$

$$G_{13}^{21}\left(x\,\middle|\,{{a+1/2}\atop{a+1/2,\ b,\ a}}\right)=\frac{\pi x^{(a+b)/2}}{\cos(a-b)\pi}\left[I_{b-a}(2x^{1/2})-\mathbf{L}_{a-b}(2x^{1/2})\right],$$

$$G_{13}^{21}\left(x\,\middle|\,{{a+1/2}\atop{a,\ a+1/2,\ b}}\right)=\pi x^{(a+b/2)}\left[I_{a-b}(2x^{1/2})-\mathbf{L}_{a-b}(2x^{1/2})\right],$$

$$G_{13}^{30}\left(x\,\middle|\,{{a+1/2}\atop{a+b,\ a-b,\ a}}\right)=2\pi^{-1/2}x^aK_b^2(x^{1/2}),$$

$$G_{13}^{31}\left(x\,\middle|\,{{a+1/2}\atop{a+1/2,\ -a,\ a}}\right)=\frac{\pi^2}{\cos 2a\pi}\left[\mathbf{H}_{2a}(2x^{1/2})-Y_{2a}(2x^{1/2})\right],$$

$$G_{13}^{31}\left(x\,\middle|\,{{a}\atop{a,\ b,\ -b}}\right)=2^{-2a+2}\Gamma(1-a-b)\,\Gamma(1-a+b)\,S_{2a-1,\,2b}(2x^{1/2}),$$

$$G_{13}^{31}\left(x\,\middle|\,{{a+1/2}\atop{b,\ 2a-b,\ a}}\right)=\pi^{5/2}2^{-1}\left[\cos(b-a)\pi\right]^{-1}x^aH_{b-a}^{(1)}(x^{1/2})\,H_{b-a}^{(2)}(x^{1/2}),$$

$$G_{22}^{12}\left(x\,\middle|\,{{-c_1,\ -c_2}\atop{a-1,\ -b}}\right)=\frac{\Gamma(a+c_1)\,\Gamma(a+c_2)}{\Gamma(a+b)}x^{a-1}\,{}_2F_1(a+c_1,\ a+c_2;\ a+b;\ -x),$$

$$G_{24}^{12}\left(x\,\middle|\,{{a+1/2,\ a}\atop{b+a,\ a-c,\ a+c,\ a-b}}\right)=\pi^{1/2}x^a\,J_{b+c}(x^{1/2})\,J_{b-c}(x^{1/2}),$$

$$G_{24}^{22}\left(x\,\middle|\,{{a,\ a+1/2}\atop{b,\ c,\ 2a-c,\ 2a-b}}\right)=2\pi^{1/2}x^a\,I_{b+c-2a}(x^{1/2})\,K_{b-c}(x^{1/2}),$$

$$G_{24}^{30}\left(x\,\middle|\,{{0,\ 1/2}\atop{a,\ b,\ -b,\ -a}}\right)=i2^{-2}\pi^{1/2}\left[H_{a-b}^{(1)}(x^{1/2})\,H_{a+b}^{(1)}(x^{1/2})-H_{a-b}^{(2)}(x^{1/2})\,H_{a+b}^{(2)}(x^{1/2})\right],$$

$$G_{24}^{31}\left(x\,\middle|\,{{1/2+a,\ 1/2-a}\atop{0,\ 1/2,\ b,\ -b}}\right)=\frac{\pi^{1/2}\,\Gamma(1/2-a+b)\,x^{-1/2}}{\Gamma(1+2a)}W_{a,\,b}(2x^{1/2})\,M_{-a,\,b}(2x^{1/2}),$$

$$G_{24}^{40}\left(x\,\middle|\,{{1/2+a,\ 1/2-a}\atop{0,\ 1/2,\ b,\ -b}}\right)=\pi^{1/2}x^{-1/2}\,W_{a,\,b}(2x^{1/2})\,W_{-a,\,b}(2x^{1/2}),$$

$$G_{24}^{40}\left(x\,\middle|\,{{a,\ a+1/2}\atop{b+c,\ b-c,\ b+1/2+c,\ b+1/2-c}}\right)=\pi^{1/2}2^{-k}x^{b-1/4}\exp(-x^{1/2})\,W_{k,\,2c}(2x^{1/2}),$$
$$k=1/2+2b-2c,$$

$$G_{24}^{40}\left(x\,\middle|\,{{a,\ a+1/2}\atop{a+b,\ a+c,\ a-c,\ a-b}}\right)=2\pi^{-1/2}x^a\,K_{b+c}(x^{1/2})\,K_{b-c}(x^{1/2}),$$

$$G_{24}^{41}\left(x\,\middle|\,{{0,\ 1/2}\atop{a,\ b,\ -b,\ -a}}\right)=$$
$$=\frac{-2^{-2}\pi^{5/2}}{i\sin a\pi\sin b\pi}\left[e^{-b\pi i}H_{a-b}^{(1)}(x^{1/2})\,H_{a+b}^{(2)}(x^{1/2})-e^{b\pi i}H_{a+b}^{(1)}(x^{1/2})\,H_{a-b}^{(2)}(x^{1/2})\right],$$

$$G_{24}^{41}\left(x\,\middle|\,{{1/2,\ 0}\atop{a,\ b,\ -b,\ -a}}\right)=$$
$$=\frac{2^{-2}\pi^{5/2}}{\cos a\pi\cos b\pi}\left[e^{-b\pi i}H_{a-b}^{(1)}(x^{1/2})\,H_{a+b}^{(2)}(x^{1/2})+e^{b\pi i}H_{a+b}^{(1)}(x^{1/2})\,H_{a-b}^{(2)}(x^{1/2})\right],$$

$$G_{24}^{41}\left(x\,\middle|\,{{1/2+a,\ 1/2-a}\atop{0,\ 1/2,\ b,\ -b}}\right)=$$
$$=x^{-1/2}\pi^{1/2}\,\Gamma(1/2+b-a)\,\Gamma(1/2-b-a)\,W_{a,\,b}(2ix^{1/2})\,W_{a,\,b}(-2ix^{1/2}),$$

$$G_{24}^{42}\left(x\,\middle|\,{{a,\ a+1/2}\atop{b+c,\ b-c,\ b+1/2+c,\ b+1/2-c}}\right)=$$
$$=2^{k+1}\pi^{3/2}\,\Gamma(1-2a+2b+2c)\,\Gamma(1-2a+2b-2c)\times$$
$$\times x^{b-1/4}\exp(x^{1/2})\,W_{k,\,2c}(2x^{1/2}),\qquad k=2a-2b-1/2,$$

$$G_{44}^{14}\left(x\left|\begin{matrix}a-1, & -c_1, & -c_2, & -c_3\\ -b_1, & -b_2, & -b_3, & -b_4\end{matrix}\right.\right)=\frac{\prod\limits_{h=1}^{4}\Gamma(a+b_h)}{\prod\limits_{h=1}^{3}\Gamma(a+c_h)}x^{a-1}\times$$

$$\times\,{}_4F_3(a+b_1,\ a+b_2,\ a+b_3,\ a+b_4;\ a+c_1,\ a+c_2,\ a+c_3;\ -x),$$

$$G_{pq}^{1p}\left(x\left|\begin{matrix}a_1, \ldots, a_p\\ b_1, \ldots, b_q\end{matrix}\right.\right)=\frac{\prod\limits_{j=1}^{p}\Gamma(1+b_1-a_j)}{\prod\limits_{j=2}^{q}\Gamma(1+b_1-b_j)}x^{b_1}\,{}_pF_{q-1}(1+b_1-a_1,\ \ldots,\ 1+b_1-a_p;$$

$$1+b_1-b_2,\ \ldots,\ 1+b_1-b_q;\ -x),\qquad p\leqslant q,$$

$$G_{pq}^{1n}\left(x\left|\begin{matrix}a_1, \ldots, a_p\\ b_1, \ldots, b_q\end{matrix}\right.\right)=\frac{\prod\limits_{j=1}^{n}\Gamma(1+b_1-a_j)\,x^{b_1}}{\prod\limits_{j=2}^{q}\Gamma(1+b_1-b_j)\prod\limits_{j=n+1}^{p}\Gamma(a_j-b_1)}\times$$

$$\times\,{}_pF_{q-1}(1+b_1-a_1,\ \ldots,\ 1+b_1-a_p;\ 1+b_1-b_2,\ \ldots,\ 1+b_1-b_q;\ -x),$$

$$p\leqslant q,$$

$$G_{pq}^{q1}\left(x\left|\begin{matrix}a_1, \ldots, a_p\\ b_1, \ldots, b_q\end{matrix}\right.\right)=$$

$$=x^{a_1-1}E(1-a_1+b_1,\ \ldots,\ 1-a_1+b_q;\ 1-a_1+a_2,\ \ldots,\ 1-a_1+a_p:x).$$

Функции, выражающиеся через G-функцию

$$x^{\mu}J_{\nu}(x)=2^{\mu}G_{02}^{10}(2^{-2}x^2\,|\,\nu/2+\mu/2,\ \mu/2-\nu/2),$$

$$x^{\mu}J_{\nu}(x)=4^{\mu}G_{04}^{20}(4^{-4}x^4\,|\,\nu/4+\mu/4,\ \nu/4+\mu/4+1/2,\ \mu/4-\nu/4,\ 1/2+\mu/4-\nu/4),$$

$$x^{\mu}Y_{\nu}(x)=2^{\mu}G_{13}^{20}\left(2^{-2}x^2\left|\begin{matrix}\mu/2-\nu/2-1/2\\ \mu/2-\nu/2,\ \mu/2+\nu/2,\ \mu/2-\nu/2-1/2\end{matrix}\right.\right),$$

$$x^{\mu}K_{\nu}(x)=2^{\mu-1}G_{02}^{20}(2^{-2}x^2\,|\,\mu/2+\nu/2,\ \mu/2-\nu/2),$$

$$x^{\mu}K_{\nu}(x)=$$

$$=4^{\mu-1}\pi^{-1}G_{04}^{40}(4^{-4}x^4\,|\,(\nu/4+\mu/4,\ 1/2+\nu/4+\mu/4,\ -\nu/4+\mu/4,\ 1/2-\nu/4+\mu/4),$$

$$e^{-x}I_{\nu}(x)=\pi^{-1/2}G_{12}^{11}\left(2x\left|\begin{matrix}1/2\\ \nu,\ -\nu\end{matrix}\right.\right),$$

$$e^{-x}K_{\nu}(x)=\pi^{1/2}G_{12}^{20}\left(2x\left|\begin{matrix}1/2\\ \nu,\ -\nu\end{matrix}\right.\right),$$

$$e^{x}K_{\nu}(x)=\pi^{-1/2}\cos\nu\pi\,G_{12}^{21}\left(2x\left|\begin{matrix}1/2\\ \nu,\ -\nu\end{matrix}\right.\right),$$

$$x^{\mu}\mathbf{H}_{\nu}(x)=2^{\mu}G_{13}^{11}\left(2^{-2}x^2\left|\begin{matrix}1/2+\nu/2+\mu/2\\ 1/2+\nu/2+\mu/2,\ \mu/2-\nu/2,\ \mu/2+\nu/2\end{matrix}\right.\right),$$

$$\mathbf{H}_{\nu}(x)-Y_{\nu}(x)=\pi^{-2}\cos\nu\pi\,G_{13}^{31}\left(2^{-2}x^2\left|\begin{matrix}1/2+\nu/2\\ 1/2+\nu/2,\ -\nu/2,\ \nu/2\end{matrix}\right.\right),$$

$$x^{\mu}[I_{\nu}(x)-\mathbf{L}_{\nu}(x)]=\pi^{-1}2^{\mu}\,G_{13}^{21}\left(2^{-2}x^{2}\,\middle|\,\begin{matrix}\mu/2+\nu/2+1/2\\ \mu/2+\nu/2,\ \mu/2+\nu/2+1/2,\ \mu/2-\nu/2\end{matrix}\right),$$

$$x^{\mu}\left[I_{-\nu}(x)-\mathbf{L}_{\nu}(x)\right]=\pi^{-1}2^{\mu}\cos\nu\pi\,G_{13}^{21}\left(2^{-2}x^{2}\,\middle|\,\begin{matrix}1/2+\nu/2+\mu/2\\ 1/2+\nu/2+\mu/2,\ \mu/2-\nu/2,\ \mu/2+\nu/2\end{matrix}\right),$$

$$S_{\mu,\nu}(x)=2^{\mu-1}\frac{1}{\Gamma(1/2-\mu/2-\nu/2)\,\Gamma(1/2-\mu/2+\nu/2)}\,G_{13}^{31}\left(2^{-2}x^{2}\,\middle|\,\begin{matrix}1/2+\mu/2\\ 1/2+\mu/2,\ \nu/2,\ -\nu/2\end{matrix}\right),$$

$$J_{\nu}^{2}(x)=\pi^{-1/2}\,G_{13}^{11}\left(x^{2}\,\middle|\,\begin{matrix}1/2\\ \nu,\ 0,\ -\nu\end{matrix}\right),$$

$$J_{\nu}(x)\,J_{-\nu}(x)=\pi^{-1/2}\,G_{13}^{11}\left(x^{2}\,\middle|\,\begin{matrix}1/2\\ 0,\ \nu,\ -\nu\end{matrix}\right),$$

$$x^{\sigma}J_{\mu}(x)\,J_{\nu}(x)=\pi^{-1/2}\,G_{24}^{12}\left[x^{2}\,\middle|\,\begin{matrix}1/2+\sigma/2,\ \sigma/2\\ \dfrac{\mu+\nu+\sigma}{2},\ \dfrac{\nu+\sigma-\mu}{2},\ \dfrac{\mu+\sigma-\nu}{2},\ \dfrac{\sigma-\mu-\nu}{2}\end{matrix}\right],$$

$$x^{\mu}I_{\nu}(x)\,J_{\nu}(x)=\pi^{1/2}2^{3\mu/2}\,G_{04}^{10}\left(\frac{x^{4}}{64}\,\middle|\,\mu/4+\nu/2,\ \mu/4-\nu/2,\ \mu/4,\ \mu/4+1/2\right),$$

$$I_{\nu}(x)\,J_{-\nu}(x)=\pi^{1/2}\cos\left(2^{-1}\nu\pi\right)G_{04}^{10}\left(\frac{x^{4}}{64}\,\middle|\,0,\ 1/2,\ \nu/2,\ -\nu/2\right)-$$
$$-\pi^{1/2}\sin\left(2^{-1}\nu\pi\right)G_{04}^{10}\left(\frac{x^{4}}{64}\,\middle|\,1/2,\ 0,\ \nu/2,\ -\nu/2\right),$$

$$x^{\mu}J_{\nu}(x)\,Y_{\nu}(x)=-\pi^{-1/2}\,G_{13}^{20}\left(x^{2}\,\middle|\,\begin{matrix}1/2+\mu/2\\ \nu+\mu/2,\ \mu/2,\ \mu/2-\nu\end{matrix}\right),$$

$$I_{\nu}(x)\,K_{\nu}(x)=2^{-1}\pi^{-1/2}\,G_{13}^{21}\left(x^{2}\,\middle|\,\begin{matrix}1/2\\ \nu,\ 0,\ -\nu\end{matrix}\right),$$

$$x^{\mu}K_{\nu}(x)\,J_{\nu}(x)=\pi^{-1/2}2^{3\mu/2-1/2}\,G_{04}^{30}\left(\frac{1}{64}x^{4}\,\middle|\,\mu/4+\nu/2,\ \mu/4+1/2,\ \mu/4,\ \mu/4-\nu/2\right),$$

$$x^{\sigma}I_{\nu}(x)\,K_{\mu}(x)=2^{-1}\pi^{-1/2}\,G_{24}^{22}\left[x^{2}\,\middle|\,\begin{matrix}\sigma/2,\ \sigma/2+1/2\\ \dfrac{\nu+\mu+\sigma}{2},\ \dfrac{\nu+\sigma-\mu}{2},\ \dfrac{\mu+\sigma-\nu}{2},\ \dfrac{\sigma-\nu-\mu}{2}\end{matrix}\right],$$

$$x^{\mu}H_{\nu}^{(1)}(x)\,H_{\nu}^{(2)}(x)=\pi^{-5/2}\,2\cos\nu\pi\,G_{13}^{31}\left(x^{2}\,\middle|\,\begin{matrix}1/2+\mu/2\\ \mu/2+\nu,\ \mu/2-\nu,\ \mu/2\end{matrix}\right),$$

$$x^{\mu}K_{\nu}^{2}(x)=2^{-1}\pi^{1/2}\,G_{13}^{30}\left(x^{2}\,\middle|\,\begin{matrix}1/2+\mu/2\\ \nu+\mu/2,\ -\nu+\mu/2,\ \mu/2\end{matrix}\right),$$

$$x^{\sigma}K_{\nu}(x)\,K_{\mu}(x)=2^{-1}\pi^{1/2}\,G_{24}^{40}\left[x^{2}\,\middle|\,\begin{matrix}\sigma/2,\ \sigma/2+1/2\\ \dfrac{\nu+\mu+\sigma}{2},\ \dfrac{\nu+\sigma-\mu}{2},\ \dfrac{\mu+\sigma-\nu}{2},\ \dfrac{\sigma-\nu-\mu}{2}\end{matrix}\right],$$

$$x^{2\mu}K_{2\nu}\left(xe^{\pi i/4}\right)K_{2\nu}\left(xe^{-\pi i/4}\right)=$$
$$=2^{3\mu-3}\pi^{-1/2}\,G_{04}^{40}\left(\frac{1}{64}x^{4}\,\middle|\,\mu/2,\ \mu/2+1/2,\ \mu/2+\nu,\ \mu/2-\nu\right),$$

$$x^{l}e^{-x/2}M_{k,m}(x)=\frac{\Gamma(2m+1)}{\Gamma(1/2+k+m)}\,G_{12}^{11}\left(x\,\middle|\,\begin{matrix}1-k+l\\ 1/2+l+m,\ 1/2+l-m\end{matrix}\right),$$

$$x^{l}e^{-x/2}W_{k,m}(x)=G_{12}^{20}\left(x\,\middle|\,\begin{matrix}l-k+1\\ m+l+1/2,\ l-m+1/2\end{matrix}\right),$$

$$x^{l}e^{x/2}W_{k,m}(x)=\frac{1}{\Gamma(1/2+m-k)\,\Gamma(1/2-m-k)}\,G_{12}^{21}\left(x\,\middle|\,\begin{matrix}k+l+1\\ l-m+1/2,\ m+l+1/2\end{matrix}\right),$$

$$e^{-x/2}W_{\varkappa,m}(x)=\pi^{-1/2}x^{1/2}2^{k-1/2}\,G_{24}^{40}\left(2^{-2}x^{2}\,\middle|\,\begin{matrix}1/4-k/2,\ 3/4-k/2\\ 1/2+m/2,\ 1/2-m/2,\ m/2,\ -m/2\end{matrix}\right),$$

$$e^x W_{k,m}(2x) = \frac{x^{1/2} 2^{-(k+1)} \pi^{-3/2}}{\Gamma(1/2+m-k)\,\Gamma(1/2-m-k)}\, G_{24}^{42}\left(x^2 \,\middle|\, \begin{matrix} 1/4+k/2, & 3/4+k/2 \\ m/2, & 1/2+m/2, & -m/2, & 1/2-m/2 \end{matrix}\right),$$

$$W_{k,m}(x)\, M_{-k,m}(x) = \frac{\pi^{-1/2}\,\Gamma(1+2m)}{\Gamma(1/2-k+m)}\, G_{24}^{31}\left(2^{-2}x^2 \,\middle|\, \begin{matrix} 1+k,\ 1-k \\ 1/2,\ 1,\ 1/2+m,\ 1/2-m \end{matrix}\right),$$

$$x^l\, W_{k,m}(2ix)\, W_{k,m}(-2ix) =$$

$$= \frac{x\pi^{-1/2}}{\Gamma(1/2+m-k)\,\Gamma(1/2-m-k)}\, G_{24}^{41}\left(x^2 \,\middle|\, \begin{matrix} 1/2+l/2+k, & 1/2+l/2-k \\ l/2, & 1/2+l/2, & l/2+m, & l/2-m \end{matrix}\right),$$

$$W_{k,m}(x)\, W_{-k,m}(x) = \pi^{-1/2}\, G_{24}^{40}\left(2^{-2}\, x^2 \,\middle|\, \begin{matrix} k+1,\ -k+1 \\ 1/2,\ 1,\ m+1/2,\ -m+1/2 \end{matrix}\right),$$

$${}_2F_1(a,\ b;\ c;\ -x) = \frac{\Gamma(c)\, x}{\Gamma(a)\,\Gamma(b)}\, G_{22}^{12}\left(x \,\middle|\, \begin{matrix} -a,\ -b \\ -1,\ -c \end{matrix}\right),$$

$${}_4F_3(a\ \ b,\ c,\ d;\ e,\ f,\ l;\ -x) = \frac{\Gamma(e)\,\Gamma(f)\,\Gamma(l)}{\Gamma(a)\,\Gamma(b)\,\Gamma(c)\,\Gamma(d)}\, x\, G_{44}^{14}\left(x \,\middle|\, \begin{matrix} -a, & -b, & -c, & -d \\ -1, & -e, & -f, & -l \end{matrix}\right),$$

$${}_pF_q(a_1,\ \ldots,\ a_p;\ b_1,\ \ldots,\ b_q;\ -x) = \frac{\prod_{j=1}^{q} \Gamma(b_j)}{\prod_{j=1}^{p} \Gamma(a_j)}\, x\, G_{p,\,q+1}^{1,\,p}\left(x \,\middle|\, \begin{matrix} -a_1,\ \ldots\ -a_p \\ -1,\ -b_1,\ \ldots,\ -b_q \end{matrix}\right),$$

$$p \leqslant q+1,$$

$$E(p;\ \alpha_r : q;\ \beta_s : x) = G_{q+1,\,p}^{p,\,1}\left(x \,\middle|\, \begin{matrix} 1,\ \beta_1 \ldots,\ \beta_q \\ \alpha_1,\ \ldots,\ \alpha_p \end{matrix}\right).$$

Относительно других специальных функций, выражающихся через G-функцию, в частности комбинаций функций Лежандра, а также комбинаций обобщенных гипергеометрических рядов, см. C. S. Meijer, Nederl. Akad. Wetensch., Proc. 43 (1940), 198—210 и 366—378; 44 (1941), 82—92, 186—194, 298—307, 435—451, 590—605, 1062—1070; 49 (1946), 227—235, 344—356, 457—469, 632—641, 765—772, 936—943, 1063—1072, 1164—1175; 55 (1952), 369—379, 483—487; 56 (1953), 43—49, 187—193.

Гипергеометрические ряды от многих переменных. См. также ВТФ, гл. 5. Гипергеометрические ряды от двух переменных. Во всех двойных суммах m и n меняются от 0 до ∞.

$$F_1(\alpha;\ \beta,\ \beta';\ \gamma;\ x,\ y) = \sum \frac{(\alpha)_{m+n}\,(\beta)_m\,(\beta')_n}{(\gamma)_{m+n}\, m!\, n!}\, x^m y^n,$$

$$F_2(\alpha;\ \beta,\ \beta';\ \gamma,\ \gamma';\ x,\ y) = \sum \frac{(\alpha)_{m+n}\,(\beta)_m\,(\beta')_n}{(\gamma)_m\,(\gamma')_n\, m!\, n!}\, x^m y^n,$$

$$F_3(\alpha,\ \alpha',\ \beta,\ \beta';\ \gamma;\ x,\ y) = \sum \frac{(\alpha)_m\,(\alpha')_n\,(\beta)_m\,(\beta')_n}{(\gamma)_{m+n}\, m!\, n!}\, x^m y^n,$$

$$F_4(\alpha,\ \beta;\ \gamma,\ \gamma';\ x,\ y) = \sum \frac{(\alpha)_{m+n}\,(\beta)_{m+n}}{(\gamma)_m\,(\gamma')_n\, m!\, n!}\, x^m y^n,$$

$$\Phi_1(\alpha,\ \beta,\ \gamma;\ x,\ y) = \sum \frac{(\alpha)_{m+n}\,(\beta)_m}{(\gamma)_{m+n}\, m!\, n!}\, x^m y^n,$$

$$\Phi_2(\beta,\ \beta',\ \gamma;\ x,\ y) = \sum \frac{(\beta)_m\,(\beta')_n}{(\gamma)_{m+n}\, m!\, n!}\, x^m y^n,$$

$$\Phi_3(\beta, \gamma; x, y) = \sum \frac{(\beta)_m}{(\gamma)_{m+n}\, m!\, n!} x^m y^n,$$

$$\Psi_1(\alpha, \beta, \gamma, \gamma'; x, y) = \sum \frac{(\alpha)_{m+n} (\beta)_m}{(\gamma)_m (\gamma')_n\, m!\, n!} x^m y^n,$$

$$\Psi_2(\alpha, \gamma, \gamma'; x, y) = \sum \frac{(\alpha)_{m+n}}{(\gamma)_m (\gamma')_n\, m!\, n!} x^m y^n,$$

$$\Xi_1(\alpha, \alpha', \beta, \gamma; x, y) = \sum \frac{(\alpha)_m (\alpha')_n (\beta)_m}{(\gamma)_{m+n}\, m!\, n!} x^m y^n,$$

$$\Xi_2(\alpha, \beta, \gamma; x, y) = \sum \frac{(\alpha)_m (\beta)_m}{(\gamma)_{m+n}\, m!\, n!} x^m y^n.$$

Относительно других гипергеометрических рядов от двух переменных см. ВТФ I, п. 5.7.1.

Гипергеометрические ряды от многих переменных. Все суммирования ведутся от 0 до ∞.

$$F_A(\alpha; \beta_1, \ldots, \beta_n; \gamma_1, \ldots, \gamma_n; z_1, \ldots, z_n) = \sum \frac{(\alpha)_{m_1+\ldots+m_n} (\beta_1)_{m_1} \cdots (\beta_n)_{m_n}}{(\gamma_1)_{m_1} \ldots (\gamma_n)_{m_n}\, m_1! \ldots m_n!} z_1^{m_1} \ldots z_n^{m_n},$$

$$\Phi_2(\beta_1, \ldots, \beta_n; \gamma; z_1, \ldots, z_n) = \sum \frac{(\beta_1)_{m_1} \cdots (\beta_n)_{m_n}}{(\gamma)_{m_1+\ldots+m_n}\, m_1! \ldots m_n!} z_1^{m_1} \ldots z_n^{m_n},$$

$$\Psi_2(\alpha; \gamma_1, \ldots, \gamma_n; z_1, \ldots, z_n) = \sum \frac{(\alpha)_{m_1+\ldots+m_n}}{(\gamma_1)_{m_1} \ldots (\gamma_n)_{m_n}\, m_1! \ldots m_n!} z_1^{m_1} \ldots z_n^{m_n}.$$

Эллиптические функции и интегралы. См. также ВТФ III, гл. 13. Полные эллиптические интегралы

$$K(k) = \int_0^{\pi/2} (1 - k^2 \sin^2 \varphi)^{-1/2}\, d\varphi = 2^{-1}\pi\, {}_2F_1(1/2, 1/2; 1; k^2),$$

$$E(k) = \int_0^{\pi/2} (1 - k^2 \sin^2 \varphi)^{1/2}\, d\varphi = 2^{-1}\pi\, {}_2F_1(-1/2, 1/2; 1; k^2).$$

Тета-функции

$$\theta_0(v \mid \tau) = (-i\tau)^{-1/2} \sum_{n=-\infty}^{\infty} \exp[-i\pi (v - 1/2 + n)^2 \tau^{-1}],$$

$$\theta_1(v \mid \tau) = (-i\tau)^{-1/2} \sum_{n=-\infty}^{\infty} (-1)^n \exp[-i\pi (v - 1/2 + n)^2 \tau^{-1}],$$

$$\theta_2(v \mid \tau) = (-i\tau)^{-1/2} \sum_{n=-\infty}^{\infty} (-1)^n \exp[-i\pi (v + n)^2 \tau^{-1}],$$

$$\theta_3(v \mid \tau) = (-i\tau)^{-1/2} \sum_{n=-\infty}^{\infty} \exp[-i\pi (v + n)^2 \tau^{-1}],$$

$$\theta_4(v \mid \tau) = \theta_0(v \mid \tau).$$

Указанные здесь ряды связаны с определениями, данными в ВТФ III, равенства 13.19 (10) — (13), при помощи мнимого преобразования Якоби, см. ВТФ III, равенства 13.22 (8).

Модифицированные тета-функции

$$\hat{\theta}_0(v \mid \tau) = \hat{\theta}_4(v \mid \tau) = (-i\tau)^{-1/2}\left\{\sum_{n=0}^{\infty} \exp[-i\pi(v + 1/2 + n)^2 \tau^{-1}] - \sum_{n=-1}^{-\infty} \exp[-i\pi(v + 1/2 + n)^2 \tau^{-1}]\right\},$$

$$\hat{\theta}_1(v \mid \tau) = (-i\tau)^{-1/2}\left\{\sum_{n=0}^{\infty} (-1)^n \exp[-i\pi(v - 1/2 + n)^2 \tau^{-1}] - \sum_{n=-1}^{-\infty} (-1)^n \exp[-i\pi(v - 1/2 + n)^2 \tau^{-1}]\right\},$$

$$\hat{\theta}_2(v \mid \tau) = (-i\tau)^{-1/2}\left\{\sum_{n=0}^{\infty} (-1)^n \exp[-i\pi(v + n)^2 \tau^{-1}] - \sum_{n=-1}^{-\infty} (-1)^n \exp[-i\pi(v + n)^2 \tau^{-1}]\right\},$$

$$\hat{\theta}_3(v \mid \tau) = (-i\tau)^{-1/2}\left\{\sum_{n=0}^{\infty} \exp[-i\pi(v + n)^2 \tau^{-1}] - \sum_{n=-1}^{-\infty} \exp[-i\pi(v + n)^2 \tau^{-1}]\right\}.$$

Различные функции. См. также ВТФ III, гл. 18.

$$\mu(x, a) = \int_0^{\infty} \frac{x^s s^a}{\Gamma(s+1)}\, ds,$$

$$\nu(x) = \int_0^{\infty} \frac{x^s}{\Gamma(s+1)}\, ds,$$

$$\nu(x, a) = \int_0^{\infty} \frac{x^{s+a}}{\Gamma(s+a+1)}\, ds = \int_a^{\infty} \frac{x^s}{\Gamma(s+1)}\, ds.$$

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Г. Бейтмен и А. Эрдейи

**Таблицы интегральных преобразований.
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